Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

STAT

Page Denied

Next 1 Page(s) In Document Denied

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

L.I.Sedov

BASIC METHODS AND DIMENSIONS IN MECHANICS

State Publishing House
for Technical-Theoretical Literature
Moscow 1954

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

many deficiencies. These questions are touched upon only casually and in passing. The basic concepts, even the concepts of dimensional and dimensionless quantities, the problem of the number of fundamental units of measurement, etc., are not explained in a clear manner. Moreover, befuddled and intuitive representations of the content of the concept of dimensions often serve as the starting point for the emergence of viewpoints in which the formulae for dimensions are ascribed a certain mystic or peculiarly secret significance. In certain instances such confusion has led to paradoxes which serve as an object of perplexity. We will analyse in detail one example of such a misunderstanding in connection with "Rele's" conclusions concerning the heat exchange of a body in a liquid stream. In presenting the theory of similarities, relationships and mathematical devices which are not essentially connected with this theory are frequently introduced. It is desirable to derive the constructions of the theory of dimensions and similarities, as is generally the case with any theory, with the aid of methods and basic premises appropriate to the essence of the theory. Such a construction makes it possible to clearly outline the limits and possibilities of the theory. This is especially necessary in the case of the theory of dimensions and similarities since one often encounters extreme opinions: on the one hand concerning the omnipotence of this theory and on the other, its triviality. Neither opinion can be considered correct.

It should be noted, however, that most realistic and useful results can be obtained by combining the theorems of the theory of dimensions with the propositions of general physics, which, in itself, yields interesting conclusions. Therefore, in order to illustrate various applications more completely we will consider a whole series of mechanical problems and examples of the combination of dimensional methods of various types with other qualitative mechanical and mathematical theorems.

This has likewise impelled us to concern ourselves with the problems of the turbulent movements of a liquid in more detail. In the theory of turbulence, methods of similarity are the basic working theoretical methods since, in this field, we

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

still do not have a closed system of equations which make it possible to reduce the mechanical problems to mathematical problems. In the section on turbulent movements of a liquid new results are presented which supplement and clarify several problems involved in the theory of turbulence.

In addition to the examples of the use of the methods of dimensions and similarities, we have attempted to shed some light on the resolution of a number of mechanical problems which are very important in engineering and certain of which are new and have as yet only been officially developed.

In connection with general theorems concerning the nature of various mechanical relationships and also in connection with certain independent values, we have dwelt in somewhat more detail on an examination of the basic equations of mechanics which express Newton's second law. The point of view which we will present is therefore not new, however, it is considerably different from the treatment of this basic problem in mechanics as it appears in certain widely used text-books on theoretical mechanics.

The number of known applications of the theory of dimensions and similarities in mechanics is very great and we have not touched upon many of them. The author hopes that the book will give the reader an idea of the procedures and possibilities of these methods and will help in the analysis of new problems and in the formulation and execution of new experiments.

No special preparation is required for reading the greater part of the book. In order to understand the material presented in the second half of the book, it is necessary to have a general knowledge of hydromechanics.

Moscow, 1943 L.Sedov

Preface to the Second Edition

This book is an extended and revised version of the book Methods of the Theory of Dimensions and the Theory of Similitude in Mechanics published in 1944.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

In the second edition, in addition to certain corrections and minor improvements, supplemental information has been introduced in which the theorems of the theory of dimensions are utilized to determine a series of precise derivations in the theory of a wave on the surface of a heavy ideal liquid, in the theory of the movement of a viscous fluid, and in the theory of one-dimensional nonsteady states of motion of a gas. By an analogous method it is possible to seek and establish the mechanical characteristics of motion in other problems in mathematical physics, for example, in the theory of plane-parallel and spatial steady-state motions of a gas, in the theory of the dispersion of turbulent jets, etc.

In Chapter IV, solutions of a number of problems concerning one-dimensional nonsteady-state motions of a gas, which are of significant practical interest, are given.

G.M.Bam-Zelikovich to whom I express my sincere gratitude was of great assistance to me in writing Chapter IV.

I am also grateful to I.O.Bezhayev under whose supervision the calculations in the problem concerning a powerful explosion were carried out.

L.Sedov

Moscow, January 1951

Preface to the Third Edition

Recently, theorems and methods which make use of the property of invariability of mathematical and physical laws in selecting units of measurement and physical scales for the characteristics of the phenomena being utilized have penetrated much more extensively into scientific investigations.

The practical and theoretical value and strength of these methods is being recognized more and more among scientific workers in opposition to the opinion which was widespread until just recently that the theory of similarities and dimensions was of secondary importance.

A certain analogy can be spoken of between the theory of dimensions and similar-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

ities and the geometric theory of the invariants connected with the transformation of coordinates--- the fundamental theory in contemporary mathematics and physics.

After the publication of the first edition of this book many new applications of the theory of similarities and dimensions to the most diverse problems in physics, the mechanics of complex media; to certain problems of a mathematical nature connected with the utilization of the group theory for seeking solutions of differential equations*; and to statistical problems selecting and rejecting merchandise and industrial products**.

This edition contains several corrections and additions designed to give greater emphasis to the fundamental ideas inherent in the theory of similarities and dimensions. Thus, for example, this has been done in the course of the discussion of the proof of the π-theorem. The definition of the dynamic and, in general, the physical similarity of phenomena has been presented in somewhat more detail. This new definition is still not generally used in the presentation of similarity problems, but from the practical point of view it embraces all the essential peculiarities of physically similar processes. Moreover, it can be conveniently and directly utilized and, evidently, completely satisfies all the requirements of a number of applications.

Sections 8-12 in Chapter IV and the entire Chapter V have been added. The additions in Chapter IV are devoted to problems involving explosions and the extinction of shock waves and to certain theorems in the general theory of one-dimensional gas motions. The new Chapter V is concerned with the applications of the theory of one-

---

*In this connection, we take note of the recently published book: G.Birkhoff, Hydrodynamics. A Study in Logic, Fact and Similitude, 1950. There is a Russian translation edited by N.I.Gurevich: G.Birkgof, Gidrodinamika, I.L., 1954.

**See S.Drobot and M.Warmus, Dimensional Analysis in Sampling Inspection of Merchandise. Rozprawy Matematyczne, V.Warszawa, 1954.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

dimensional, nonsteady-state gas motions and the methods of dimensional analysis to certain astrophysical problems.

It has now become clear that the basic problems of the internal structure of stars and problems connected with explaining the grandiose and surprising phenomena observed in variable stars are closely related to the study of the problems of gas dynamics. The theory, as it is presented here, gives new rational formulations of the problems and presents exact solutions for equations representing adiabatic gas motions and gas equilibrium equations which take radiation effects into account. In certain cases, appropriate idealized instances of gas motion or equilibrium can be considered as schematic processes which serve as models of gas dynamic effects occurring on stars. They can provide a basis for gaining an insight into the possible mechanisms of explosions of stars, star pulsations, the internal structure of stars; of the effect of various physical factors connected with the release and adsorbtion energy inside stars, and the role of variable density; of the influence of gravity; and of possible movements occasioned by the absence of an initial equal distribution of pressures, etc.

The theory developed in the additions to Chapter IV and in Chapter V are to a significant degree entirely new. The proposed formulations and solutions of the problems of gas dynamics can be considered as illustrative examples of applications of the methods of dimensional analysis to astronomy and as a reserve of model, simple, ideal motions which can be drawn upon and utilized in the investigation of cosmogeny problems. Some of these results were obtained by myself and a number of my young students in connection with work done at a seminar on hydromechanics at Moscow State University during the academic year 1952-53.

N.S.Burnov and S.I.Sidorkin assisted in the preparation of Sections 8 and 9 of Chapter IV; V.A.Vasil'yev and M.L.Lidov, Section 10 part 1 Chapter IV; N.S.Burnova, Section 12 Chapter IV; and I.M.Yavorskaya, Section 6 Chapter V.

I express my sincere gratitude to all of them.

Moscow, March 1954 L.Sedov

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

Chapter I

GENERAL THEORY OF DIMENSIONS FOR VARIOUS QUANTITIES

1. Introduction

A whole series of concepts, for example, energy, speed, stress, etc., which characterize observable phenomena and can be assigned and defined with the aid of numbers has been introduced into the study of mechanical phenomena.

All questions concerning motion and equilibrium are formulated as problems of defining certain functions and numerical values for quantities which characterize the phenomenon. Moreover, in solving such problems natural laws and various geometrical relationships are represented by functional equations, usually, differential equations.

In purely theoretical investigations, these equations are useful in establishing the general qualitative properties of motions and in actually calculating the functional connections being sought by means of various mathematical operations. However, it is not always possible to carry out an investigation in the field of mechanics by means of mathematical reasoning and calculations. In numerous instances in solving a problem in mechanics insurmountable mathematical difficulties are encountered. Very frequently, we do not have a general mathematical statement of the problem since the mechanical problem being investigated is so complex that there is still no satisfactory mathematical representation of it and no equations for the motions involved. We encounter such a situation in solving many very important questions in the fields of aeromechanics and hydromechanics, in problems concerning

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

the study of stability and deformations in various structures, etc. In these cases, the principal role is played by experimental methods of investigation which afford the possibility of determining the most rudimentary, empirical facts. Generally, any investigation of natural phenomena begins with the determination of rudimentary empirical facts on the basis of which the laws governing the phenomenon being observed can be formulated and recorded in the form of certain mathematical relationships.

In order to set up and conduct experiments, the results of which can be utilized to establish laws and can be applied to cases in which the experiment can not be carried out directly, it is necessary to examine the essential nature of the problem being studied and to make a general qualitative analysis. Furthermore, the formulation of experiments whose results will be represented by numerical combinations which characterize the aspects of the phenomenon being investigated, may be carried out only on the basis of a preliminary theoretical analysis. In setting up experiments and generally in practice it is extremely important to select the correct dimensionless parameters. The number of them should be minimal and the parameters decided upon should reflect the basic effects as closely as possible.

The possibility of such a preliminary qualitative-theoretical analysis and the selection of a system of specific dimensionless parameters is afforded by the theory of dimensions and similarities. It can be applied to the observation of extremely complex phenomena and significantly facilitate the performance of experiments. Furthermore, writing up and carrying out experiments without taking the question of dimensions and similarities into account is unthinkable at the present time. Sometimes the theory of dimensions is the only possible theoretical method in the initial stages of the study of certain complex phenomena. However, the possibilities for the use of this method should not be overestimated. The results which can be obtained using the theory of dimensions are limited and in many cases trivial. In conjunction with this, the rather widespread idea that the theory of dimensions can not yield any important results at all is completely untrue. The combination of the

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

theory of similarities with expressions obtained from experiments or by mathematical means from equations of motion can sometimes lead to rather essential results. Usually, the theory of dimensions and similarities is of great use both in theory and in practice. All results obtained with the aid of this theory are arrived at very simply, in an elementary manner, and with hardly any difficulty. Nevertheless, despite this simpleness and elementariness, the application of the methods of the theory of dimensions and similarities to new problems requires that the investigator have appreciable experience with and a fundamental understanding of the phenomena being studied.

With the aid of the theory of dimensions it is possible to draw especially valuable conclusions in observations of those phenomena which depend on a large number of parameters, but at the same time certain of these parameters in known instances are unessential. Later on we will illustrate such cases with examples. The methods of the theory of dimensions and similarities play an especially important role in modelling various phenomena.

2. Dimensional and Dimensionless Quantities

Quantities, the numerical values of which depend on accepted scales, i.e., on systems of units of measurement, are called dimensional or concrete quantities. Quantities, the numerical values of which do not depend on accepted units of measurement, are called dimensionless or abstract quantities.

Length, time, power, energy, moment of force, etc., are examples of dimensional quantities. Angles, the relationship between two lengths, the relationship of the square of length to a surface, the relationship of energy to the moment of force, etc., are examples of dimensionless quantities.

However, the subdivision of quantities into dimensional and dimensionless is, to a certain extent, a matter of convention; thus, for example, we have just called an angle a dimensionless quantity. But it is a known fact that angles can be measured in radians, degrees, and parts of a right angle, i.e., in various units. Con-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

sequently, a number which defines an angle depends upon the selection of a unit of measurement. Therefore, an angle can be considered as a quantity having dimensions. We define an angle as the ratio of the arc of the circumference subtending it to the radius. At the same time, this defines the unit of measurement of the angle, the radian. If, in all systems of units of measurement we now measure angles only in radians, an angle can be considered a dimensionless quantity. However, fixing the unit of measurement for angles is convenient, but for length is inconvenient. This is explained by the fact that in geometrically similar figures, corresponding angles are identical but corresponding lengths are not identical and therefore in different problems it is advantageous to select different distances for the basic length.

Acceleration is usually considered a dimensional quantity, the dimension of which is length divided by the square of time. In many problems the acceleration of the force of gravity g, which is equal to the acceleration of a falling body in a vacuum, can be considered a constant quantity (9.81 m/sec^2). This constant acceleration g can be selected as a fixed unit of measurement for acceleration in all systems of units. Then any acceleration will be measured as the relationship of its quantity to the quantity of acceleration of the force of gravity. This relationship is termed an "overload" the numerical value of which will not be changed by transition from one measurement unit system to another. Consequently, an overload is a dimensionless quantity. But, at the same time, an overload can be considered a dimensional quantity, namely as acceleration when the unit of measurement is taken as acceleration equal to the acceleration of the force of gravity. In this instance we assume that an acceleration which is not equal to the acceleration of the force of gravity can be used as the unit of measurement of the overload.

On the other hand, quantities which are abstract (dimensionless) in the generally accepted sense of this word can be expressed with the aid of various numbers. Indeed the relationship of two lengths can be expressed not only in the form of an ordinary arithmetic quotient but also in percents or by other means.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

Thus the concepts of dimensional and dimensionless quantities are relative concepts. We will introduce a certain set of units of measurement. Then the quantities for which the units of measurement are identical in all accepted systems of units of measurement we will call dimensionless. The quantities for which in experiments or in theoretical investigations it is actually or potentially permissible to use different units of measurement we will call dimensional. Using this definition it follows that certain quantities can be considered in one instance dimensional, and in others, dimensionless. We presented such examples above and will encounter others as we proceed.

3. Basic and Derived Units of Measurement

Various physical quantities are connected to one another by definite relationships. Therefore, if several of these quantities are accepted as basic and certain units of measurement are established for them, units of measurement for all remaining quantities will be expressed in a specified manner through the units of measurement of the basic quantities. The units of measurement taken for the basic quantities will be called basic or primary and all others, derived or secondary.

In practice it has proven sufficient to establish units of measurement for three quantities. This however depends on the concrete conditions of the given problem and for different problems it is efficacious to use units of measurement of different quantities as the basic units. Thus, in physical investigations it is convenient to take units of length, time, and mass as basic units, and in engineering, units of length, time, and force. But, units of measurement such as units of speed, viscosity, or density can also be used as basic units.

At the present time physical and engineering systems of units of measurements are used most extensively. In the physical system the centimeter, gram-mass, and second (from this we have the abbreviated term, the CGS unit system) are accepted as the basic units of measurement, and in the engineering system, the meter, kilogram-force, and second (similarly, the MKS unit system).

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

Units of length, one meter (equal to 100 cm); mass, one kilogram (equal to 1000 g); and time, one second have been determined by the experimental method on the basis of specific relationships. The length of a standard made of a platinum-iridium alloy which is kept in the French Palace of Measures and Weights is accepted as one meter; the mass of a standard of the same metal kept in the same place is accepted as one kilogram. A second is accepted as $\frac{1}{24 \cdot 3600}$ part of an average solar day.

As soon as the basic units of measurement have been established, the units of measurement for other mechanical quantities, for example force, energy, speed, acceleration, etc., can be obtained automatically from their definitions.

The expression of derived units of measurements through basic units of measurement is called a dimension. A dimension is described symbolically by a formula in which the symbol of the unit of length is the letter L; mass, M; and time, T (the symbol of the unit of force in the engineering system is F). For example, the dimension of a square is L^2; speed $\frac{L}{T}$ or LT^{-1}; force in the physical system, $\frac{ML}{T^2}$, and in the engineering system, F.

In the future to designate the dimension of any quantity a we will use the symbol [a].

For example, for the dimension of force F in the physical system we will write:

$$[F] = K = \frac{ML}{T^2}.$$

Dimensionality formulas are very convenient for converting the numerical value of a dimensional quantity when changing from one system of units of measurement to another. For example, in measuring the acceleration of the force of gravity in centimeters and seconds we have $g = 981$ cm/sec^2. If it is necessary to go from these units of measurement to kilograms and hours, to convert the indicated numerical values for the acceleration of the force of gravity we must use the relationships:

$$1\ cm = \frac{1}{10^5}\ km, \quad 1\ sec = \frac{1}{3600}\ hours$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

therefore

$$g = 981 \frac{cm}{sec^2} = 981 \frac{\frac{1}{10^5} km}{\left(\frac{1}{3600}\right)^2 hr^2} = 98.1 \cdot 36^2 \frac{km}{hr^2}.$$

In general, if in the new systems of units of measurement the unit of length is x times, the unit of mass y times, and the unit of time z times less than the old units, the numerical value of any physical quantity Q, which has the dimension $[Q] = L^{\alpha}M^{\beta}T^{\gamma}$ is increased in the new system $x^{\alpha}y^{\beta}z^{\gamma}$ times.

The number of basic units of measurement need not obligatorily be three. Instead of three, a larger number of basic units can be used. Thus, for example, by means of experiments it can be established that the units of measurement for the four quantities: length, time, mass, and force, are independent of one another. In this case, Newton's equation takes the form

$$F = cma,$$

where F denotes the force; m the mass; a the acceleration; and c a constant having the dimension

$$[c] = \frac{KT^2}{ML}.$$

With such a selection of basic units, four arguments will generally enter into the formula of the dimension of the mechanical quantities. The coefficient c in the equation given above is a physical constant similar to the acceleration of the force of gravity g or the gravitational constant γ in the universal gravity law

$$F = \gamma \frac{m_1 m_2}{r^2},$$

where m_1 and m_2 are the masses of two material points and r, the distance between them. The numerical value of the coefficient c will depend on the selection of the basic units of measurement.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

If the constant c is considered an abstract number (consequently, c will have identical numerical values in all systems of measurements) an equal or unequal unit, then, by the same token, the dimension of force will be defined by mass, length, and time, and the unit of measurement of force will be defined simply in relation to the units of measurement of mass, length, and time.

In general, by introducing supplementary physical constants we can select by experimental means, independent of one another, units of measurement for n quantities (n is 3), but in doing so we should introduce n, 3, dimensional physical constants. In this case the formulae of the derivative quantities will contain in general n arguments.

In studying mechanical phenomena it is sufficient to introduce only three independent basic units of measurement, i.e., for length, mass (or force), and time. One can suffice with these units also in the study of thermal and even electrical phenomena. From physics it is well known that the dimensions of thermal and electrical quantities can be expressed by L, M, and T. For example, the amount of heat and the temperature have the dimension of mechanical energy. However, in practice, in many problems in thermodynamics and gas dynamics units of measurement for the amount of heat and temperature which are independent of the units of measurement of mechanical energy are commonly used. Degrees centigrade serve as units of measurement for temperature; and calories, for measuring the amount of heat. These units of measurement can be established by experimental means independent of the units of measurement for mechanical quantities.

In studying phenomena in which conversion of mechanical energy into heat occurs, it is necessary to take into consideration two additional, physical, dimensional constants; one, the mechanical equivalent of heat

$$J = 427 \ \text{kg-m/kal} ,$$

and the other, either the heat capacity coefficient c cal/m^3 degrees, or the gas

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

constant 9 m^2/sec^2 degrees/ or the Boltzmann constant $K = 1.37.10^{-16}$ erg/degr. If we measure the amount of heat or the temperature in mechanical units, then the mechanical equivalent of heat and Boltzmann's constant will be introduced into the formula as absolute dimensionless constants and will be analogous to **transference numbers** in converting, for example, meters into feet, ergs into kilogram-meters, etc.

It is not difficult to see that the number of basic units of measurement can also be less than three. Indeed, all forces can be compared with the force of gravity, although this is inconvenient and unnatural in those instances where the force of gravity plays no part. In the physical system of units the force in general is determined by the equation

$$F = ma,$$

and the force of gravity by the equation

$$F = \gamma \frac{m_1 m_2}{r^2},$$

where γ is the gravitational constant which has the dimension

$$\gamma = M^{-1}L^3T^{-2}.$$

Similar to the manner in which the dimensional constant for the mechanical equivalent of heat can be substituted by the dimensionless constant when measuring the amount of heat in mechanical units, the gravitational constant can be considered an absolute dimensionless constant. In this way, the dimension of mass in relation to L and T can be defined:

$$[m] = M = L^3T^{-2}.$$

Consequently, in this case the change in the unit of mass is defined completely by the change in the units of measurement for length and time. Thus, considering the gravitational constant as an absolute dimensionless constant, we will have two in-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

dependent units of measurement in all.

The number of independent units of measurement can be reduced to one if we take one more dimensional physical constant, for example, the coefficient of the kinematic viscosity of water ν or the speed of light in a vacuum c, as an absolute dimensionless constant.

Finally, we can consider all physical quantities as dimensionless if we take appropriate physical constants for absolute dimensionless constants. In this case, the possibility of using different systems of units of measurement is excluded. One single system of units of measurement based on the selected physical constants, i.e., on the gravitational constant, the speed of light, or the coefficient of the viscosity of water, is obtained, the values of which are taken as absolute universal constants.

In science a tendency to introduce such a system of units can be observed. This is due to the fact that it can be used to establish units of measurement which can not be lost, like the standards for the meter and the kilogram -- quantities which are essentially **random** quantities not connected with the basic phenomena of nature*.

---

* According to the original idea of the commission of the French Academy of Sciences which was engaged in establishing the metric system of measurements, one meter was defined as $\frac{1}{4 \times 10^7}$ of the length of the Parisian meridian, and 1 kg as the weight of a cubic decimeter of distilled water at 4°C. Naturally, the measurement of the length of the meridian and the preparation of standards for the meter and the kilogram were carried out with a certain degree of inaccuracy which later accurate measurements showed to be greater than the permissible percentage of error. Since in increasing the precision of the determination of the length of the meridian or the weight of a liter of pure water new deviations could be anticipated, in order to avoid a constant changing of the standards for the meter and the (continued)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

The introduction of such a single system of units of measurements which excludes all other systems of measurements is equivalent to the complete elimination of the concept of the dimension. In the single universal system of units of measurement the numerical values of all quantitative characteristics are defined by their physical quantity.

For certain relationships, a similar universal single system of units of measurement, i.e., the use of identical measures, methods of calculating time, etc., would represent a definite convenience for practical purposes, being certain of the links in the standartization of methods of measurement.

However, for many phenomena, such special constants as the gravitational constant, the speed of light in a vacuum, or the coefficient of the kinematic viscosity of water, are completely immaterial. Therefore, a single universal system of units of measurement connected with the laws of gravity, the propagation of light, or viscous friction in water, or some other physical processes, in many cases would be artifical in nature and would be inconvenient in practice. On the other hand, in various branches of physics it would be convenient and practical to use systems of units of measurements with different basic units corresponding to the nature and the relative significance of the physical concepts entering into the phenomena being observed.

In mechanics it is convenient to use force, length, and time as basic quantities, but it is more convenient to use other units for force and length in engineering mechanics than in celestial mechanics. In electrical engineering it is more advantageous to use the strength of a current, resistance, length, and time (the amp-

---

* (continued) kilogram it was agreed to accept the quantities of prepared prototypes of the standards as the basic units of measurement and to refrain from connecting the units of measurement with the length of a meridian or the weight of a liter of water.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

ere, ohm, centimeter, and second).

Moreover, for the concrete investigation of individual special classes of phenomena, the numerical values of quantitative characteristics are frequently advantageously expressed in relation to given quantities which are most characteristic of the particular problems being considered. In different cases, these characteristic basic quantities can be different.

4. On the Formula of a Dimension

The relationship of the units of measurement of derivative quantities to the units of measurement of the basic quantities can be represented by a formula. This formula is called the formula of the dimension and it can be considered as a concise definition and characteristic of the physical nature of the derived value.

It is possible to speak of a dimension only in relation to a definite system of units of measurement. In various systems of units of measurements the formula of a dimension for a given quantity can contain a different number of arguments and can be expressed in various forms. In the system of CGS units of measurement the formulae for the dimensions of all physical quantities have the form of the powered monomial

$$L^{l}M^{m}T^{t}.$$

We will show that such a formula of a dimension is defined by the following physical condition: the ratio between two numerical values of any derived quantity should not depend on the selection of a scale for the basic units of measurement. For example, if we were to measure area in square meters or square centimeters, the ratio between two areas measured in square meters will be just the same as the ratio of these areas measured in square centimeters. For basic quantities, this condition is a component part of the definition of the unit of measurement and is itself satisfied.

Let us take any dimensional derived value y; for simplicity we will assume initially that the quantity y is geometric and therefore depends only on the length,

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

consequently,

$$y = f(x_1, x_2, \ldots, x_n),$$

where x_1, x_2,..., x_n are certain distances. Let us designate by y' that value of the quantity y which corresponds to the values of the arguments x_1', x_2',..., x_n'. The numerical value of y and also y' depends on the unit of measurement for the distances x_1, x_2,..., x_n. Let us reduce this unit or the scale of the distances by α times. Then, according to the condition formulated above, we should have:

$$\frac{y'}{y} = \frac{f(x_1', x_2', \ldots, x_n')}{f(x_1, x_2, \ldots, x_n)} = \frac{f(x_1'\alpha, x_2'\alpha, \ldots, x_n'\alpha)}{f(x_1\alpha, x_2\alpha, \ldots, x_n\alpha)}, \qquad (4.1)$$

i.e., the ratio $\frac{y'}{y}$ should be identical for any value where the scale of the length is α.

From the eq.(4.1) we obtain:

$$\frac{f(x_1\alpha, x_2\alpha, \ldots, x_n\alpha)}{f(x_1, x_2, \ldots, x_n)} = \frac{f(x_1'\alpha, x_2'\alpha, \ldots, x_n'\alpha)}{f(x_1', x_2', \ldots, x_n')}$$

or

$$\frac{y(\alpha)}{y(1)} = \frac{y'(\alpha)}{y'(1)} = \varphi(\alpha). \qquad (4.2)$$

Consequently, the ratio of the numerical values of the derived geometric quantity measured in different scales of length depends only on the ratio of the scales of length.

From the expression (4.2) the form of the function $\varphi(\alpha)$ can easily be found. In fact, we have:

$$\frac{y(\alpha_1)}{y(1)} = \varphi(\alpha_1), \quad \frac{y(\alpha_2)}{y(1)} = \varphi(\alpha_2).$$

From this we obtain:

$$\frac{\varphi(\alpha_1)}{\varphi(\alpha_2)} = \varphi\left(\frac{\alpha_1}{\alpha_2}\right), \qquad (4.3)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

since when $x_1' = x_1 a_2$, $x_2' = x_2 a_2$, ..., $x_n' = x_n a_2$ we have:

$$\frac{y(a_1)}{y(a_2)} = \frac{y_1'\left(\frac{a_1}{a_2}\right)}{y_1'(1)} = \varphi\left(\frac{a_1}{a_2}\right).$$

Differentiating eq.(4.3) for a_1 and assuming that $a_1 = a_2 = a_n$ we obtain:

$$\frac{1}{\varphi(a)}\frac{d\varphi}{da} = \frac{1}{a}\left(\frac{d\varphi}{da_1}\right)_{a_1=a_2} = \frac{m}{a}.$$

Integrating, we find:

$$\varphi = Ca^m.$$

Since when $a = 1$ we have $\varphi = 1$, then $C = 1$; consequently,

$$\varphi = a^m. \qquad (4.4)$$

This conclusion is justified for any dimensional quantity which depends on several basic quantities if we only change the scale. It is not difficult to see that if the scales α, β, γ of the three basic quantities are changed, then φ will have the form

$$\varphi = \alpha^m \beta^n \gamma^l.$$

This proves that the formulae of a dimension of physical quantities should have the form of powered monomials.

5. On the Second Law of Newton

In investigating mechanical or in general physical phenomena we will introduce first a system of concepts, i.e., quantities which characterize various aspects of the processes being studied (we will call them simply characteristics), and secondly, the system of units of measurement with the aid of which the numerical values of characteristics which have been introduced can be determined.

A number of relationships exist between the characteristics of a phenomenon.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

Certain of these relationships exist only for concrete systems and for isolated particular processes, other relationships can be justified for several classes of systems and movements. Relationships of the latter variety are especially valuable and the search for such relationships is the most important task of physical investigations.

The methods of the theory of dimensions and similarities can serve as one of the means of determining the relationships between characteristics. Our goal is to show from here on processes and procedures for applying and utilizing these methods. Before the direct presentation of these procedures let us consider examples of the existence of certain mechanical relationships and the general typical processes of obtaining them. In connection with this and in connection with a certain independent interest we will examine the basic relationship of mechanics known as Newton's second law.

Certain of the relationships between characteristics are simple corollaries which express the definition of these quantities. For example, the quantity of speed v is equal to the ratio of the distance travelled to the corresponding interval of time; the quantity of the kinetic energy of a particle of matter E is equal to $\frac{mv^2}{2}$, where m is the mass of the particle of matter, etc.

In addition to these trivial relationships, with the aid of experimental or theoretical investigations functional connections can be established between the numerical values of phenomena occuring in nature and the peculiarities of observed phenomena and classes of phenomena. Kepler's laws concerning the motions of planets and the universal law of gravity can serve as examples of such relationships. Let us briefly elucidate the connection between these laws.

On the basis of many years of general observations on the movements of planets in 1609 and 1619 Kepler formulated the following general laws:

I. The planets describe an ellipse around the sun, the sun being located at one of the foci of the ellipse.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

II. The radius-vector connecting the sun with a planet traverses equal areas in equal intervals of time.

III. The square of the periods of rotation of planets around the sun are proportional to the cubes of the corresponding average distances of the planets from the sun.

If one defines the quantity of the force of the interaction of the sun and planets as mass multiplied by acceleration then it is possible to deduce the universal law of gravity from Kepler's laws by mathematical means:

$$F = \gamma \frac{m_1 m_2}{r^2}, \qquad (5.1)$$

where F is the force of attraction; r, the distance between material points; and m_1 and m_2, their masses. This law was established by Newton in 1682 and was later tested and confirmed by comparing numerous results obtained by using it with observations in nature and in specially formulated experiments.

Hooke's law which expresses the relationship between the force F or the strain in a spring and the length of the spring x can serve as another example.

This law is derived from observations of the equilibrium and motions of a load suspended from a spring on the basis of the definition of the quantity of force as the product of mass and acceleration and in a number of cases by utilizing the laws of addition of forces.

Mathematically this law is depicted as

$$F = kx, \qquad (5.2)$$

where k is the coefficient of proportionality (the coefficient of rigidity of the spring).

Utilizing this law it is possible in various special cases (the load is suspended from several springs, the mass or rigidity of the spring is changed, the initial motion conditions are changed, etc.) to theoretically determine the laws of

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

motion, i.e., the dependence of all mechanical quantities on time, or to find the period of vibration, etc.

The solution of these and other similar problems of mechanics is based on an analysis of the equation for the motion of particles of matter

$$\bar{F} = m\bar{a}, \tag{5.3}$$

where $\bar{a}$ is the acceleration vector; m, the mass of the particle of matter; and $\bar{F}$, the force vector. The force $\bar{F}$ is sometimes presented as the vector sum of several forces:

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \ldots, \tag{5.4}$$

which characterize various effects. The possibility of substituting one force for several simultaneously acting forces, as defined in the formula (5.4) is an empirical fact.

Let us now examine the quantities entering into eq.(5.3) in more detail. Acceleration a represents a kinematic quantity which can always be derived experimentally independently of eq.(5.3). The mass m defines the property of inertia of a body. For particles of matter, the concept of mass can be introduced on the basis of Newton's third law (every action is a reaction between equal but oppositely directed forces). In fact, every particle of matter can be assigned the value of a constant quantity, i.e., its mass so that when motions of any two isolated mutually acting particles of matter M_1 and M_2 or M_1 and M_3 occur there will be the following relationship

$$m_1\bar{a}_1 + m_2\bar{a}_2 = 0, \quad m_1\bar{a}_1' + m_3\bar{a}_3' = 0. \tag{5.5}$$

Consequently, the ratio of mass can always be determined experimentally independently of eq.(5.3) by measuring the ratio of acceleration during the motion of interacting bodies.

The constantness of mass which is defined by the relationships in (5.5) for all

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

possible motions is an experimental fact which expresses a natural law which, generally speaking, could tolerate refinement.

If the motion is well-known, then the relationship in (5.3) can serve as a simple equation which defines the quantity of the total force. In practice, the eq.(5.3) very frequently can be used to calculate a force. In problems concerning the definition of a motion the expression in (5.3) can be used only when the relationship between the force and a quantity which characterizes the motion (time, the coordinates of the position of a point, speed, etc.) are known. This relationship can be arrived at either theoretically on the basis of supplementary hypotheses which must obligatorily be tested experimentally, or directly by the experimental method.

Both in theoretical discussions and in experiments the definition of the relationship of a force to various physical quantities can be arrived at with the aid of the eq.(5.3). From the observation and study of the simplest motions the relationship of the product $m\bar{a}$ to other parameters of motion has been established. Subsequently, the relationships arrived at were generalized for a more complex class of motions. The correctness of the generalizations again had to be tested experimentally by comparing the deductions obtained by the equations of motion with the results of experiments. Thus, the general way of deriving the universal gravity law from Kepler's laws is characteristic of the definition of force in relation to parameters of motion.

In an analogous manner, we can define the force of the interaction of electrical charges, Coulomb's law; magnetic intensity, "Biot - Savart" law; the force of capillarity, Weber's law; the force of friction between solid bodies, Coulomb's law of friction; the connection between stress and deformation in an elastic body, Hooke's law; the force of viscous friction within a liquid, Newton's law; etc.

The opinion has frequently been expressed that force can be defined independently of eq.(5.3) by a **static method.** Indeed, in a number of instances, in particular, when it can be stated that force depends only on position, the relationship

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

of force to coordinates can be defined by comparing the force being sought with known forces from the examination of a special case of motion, namely, the case of rest when $\bar{a} = 0$*.

However, in connection with this it is necessary to keep the following in mind: first, when comparing the force being sought with known forces in the static method of defining a force, we are utilizing eq.(5.3) when $\bar{a} = 0$; second, confirmation of the correctness of relationships found for a force at rest as well as in motion is an additional hypothesis requiring experimental proof which can only be accomplished with the aid of eq.(5.3). Frequently, experimental proof does not confirm the given proposition. Such a case occurs, for example, with friction forces. It has been shown that friction at rest (with speed $v = 0$) and during motion (with speed $v \neq 0$) can be different. The same can be said of the force of a spring acting on a suspended weight. The law which defines force in relation to stress (formula 5.2), which is correct for a spring of any mass in statistical measurements, ceases to be correct when there is motion, moreover, the error produced is greater the larger the mass of the spring. If the mass of the spring is small in comparison to the mass of the load, then the formula (5.2) can be considered correct during motion of the load.

Frequently it cannot be said at all that force can be defined as a function of time, position, speed, or acceleration. Indeed, let us examine the total force acting from the water side on a boat which is moving in a loop-shaped course on the surface of the water. The force acting from the water side on the boat depends upon the motion state of the water which is determined by all the laws of motion governing the motion of the boat. For two different motions of the boat for a certain

---

* In practice, when defining a force frequent use is made of the fact that a force system does not change if forward motion with a constant acceleration is imparted to the entire system, since acceleration does not change when this is the case.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

given moment in time (the time can be considered as at the beginning of the motion when the boat and the water are at rest) let the position, speed, and acceleration be identical. Evidently, it cannot be said that the forces acting from the water side on the boat at this moment will be identical. The force can differ significantly from one another. During the first movement in previous moments the boat can agitate the water violently, while during the second motion of the boat the motion of the water in the place being considered may be more calm. In this example it is evident that the forces acting on the boat will depend using the functional method on the law of motion, i.e., on the total motion picture. In other words a succession like phenomenon will occur.

Thus, empirical natural laws, similar to the laws of universal gravity, to Hooke's law, etc. have been arrived at by examination of broad classes of motion in which the quantity of force was defined as the product of mass and acceleration.

Consequently, in concrete problems connected with the definition of motion, we cannot introduce into the discussion forces which do not depend on the equation $\bar{F} = m\bar{a}$ which has been utilized in corresponding experiments.

The investigation of mechanical phenomena can be carried out by an analogous method if we take another concept instead of force for the basic quantity, for example, the kinetic energy of the system. We can consider the equation

$$E = \sum \frac{mv^2}{2} \qquad (5.6)$$

as the definition of the kinetic energy of a mechanical system. In investigating certain classes of motion of a given mechanical system experimentally, we can observe the relationship of the quantity of energy E to a number of other mechanical characteristics. For example, in the motion of a conservative system it has been established that the kinetic energy can be represented as a certain function of the coordinates of points of the system and of the additive constant h, the value of

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

The most important task in the mechanical and, in general, in the physical investigation of many phenomena consists of establishing laws for forces in relation to the basic characteristics of the state of motion and, in connection with this, of determining the defining characteristics, and most importantly of ascertaining similar laws for practical purposes.

The fundamental service that Newton performed was that he revealed the product of mass and acceleration as a quantity which can have an identical value for various bodies and various motions occuring at various places in space at various speeds, and principally as a quantity which can in a number of instances be defined in experiments as function of time, position, or the speed of points in a system.

However, the definition of force as a function of the simplest characteristics of motion, as we have seen, is not always possible in principle. In these cases the question arises as to whether it would be more convenient to take other characteristics of motion instead of the product of mass and acceleration and investigate their connections.

Let us briefly examine the problem of the forces of inertia. Let us take the

---

* (continued) there was sufficient data to predict all future conditions and reconstruct all that had occured in the past. However it must be remarked that even within the framework of classical mechanics the theoretical problem of constructing differential equations for motion was not considered simple and still not solved in principle. The problem of the acting forces, i.e., of the correct parts of differential equations of motion is just as much of a basic task in physical investigations as the problem of constructing motion equations is. Moreover, even under conditions where classical mechanics can possibly be used, this task is not solved in very many instances. In these cases, when it is necessary to use approximate solutions even for the simplest applications, there is constant need for more precision.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

which evolves a known subclass among all the possible motions of the system:

$$E = -V + h. \tag{5.7}$$

The quantity V designates the potential energy of the system. The eq.(5.6) and the law (5.7) which is characteristic of a conservative system, lead to the equation

$$\sum \frac{mv^2}{2} + V = h, \tag{5.8}$$

which expresses the law of conservation of mechanical energy.

At the present time, in investigations of a whole series of mechanical phenomena we still cannot define motion by eqs.(5.3) or (5.7) since the simplest problem concerning the dependence of energy and kinetic energy on the circumstances of the mechanical state of a system has still not been decisively solved.

In analytical mechanics it is always supposed that the laws for forces and the expression of the potential energy are known. The fundamental problems of analytical mechanics are connected with questions about the mathematical apparatus of the investigation: with methods of integrating motion equations and establishing various equivalent or broader principles which can be substituted for the original empirical law*.

---

* At the end of the 18th century the main attention efforts of theoretical scholars were concentrated on the investigation and surmounting of the indicated mathematical difficulties (problems of celestial mechanics, the development of a general theory of differential equations, variation principles, etc.). The original equations for motion were considered in a general way. In connection with this the point of view was prevalent that physical phenomena and mechanical motions were convergent and that mechanics as a science was complete. The basic difficulty was seen as the integration of the differential equations of mechanics. The well-known Laplace proposition was accepted: given the initial conditions, (continued)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

aggregate of various systems of coordinates acting in relation to one another. Acceleration has a different quantity and direction in two systems of coordinates which perform different motions. The connection between the accelerations of points in relation to systems of coordinates which act in relation to one another is established in kinematics.

We can establish the dependence of forces on the basic characteristics of motion experimentally in a certain specific system of coordinates, usually in a system of coordinates connected with the earth or with the center of gravity of the solar system.

If we know the laws for the forces in a certain system of coordinates, then we can easily find the product of mass and acceleration, i.e., the force in any system of coordinates, the motion of which is given in relations to the original system. As is well known, in this case we have to introduce into the discussion the so-called forces of inertia. For the observer who is stationarily connected with a mobile system, the effective forces are composed of forces defined in the system of coordinates in which the experiment is being carried out (the initial system of coordinates), and of the forces of inertia which for a mobile observer are indistinguishable from the mechanical point of view from any other forces.

6. The Structure of the Functional Connections Between Physical Quantities

Physical laws established theoretically or directly from experiments represent the functional relationships between the quantities which characterize the phenomenon being investigated. The numerical values of these dimensional physical quantities depend on the selection of a system of units of measurement not connected with the intrinsic nature of the phenomenon. Therefore the functional relationships expressing the physical facts, which do not depend on the system of units of measurement, should have a certain special structure.

Let us take the dimensional quantity a which is a function independent of the

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

dimensional quantities $a_1, a_2, \ldots a_n$?

$$a = f(a_1, a_2, \ldots, a_k, a_{k+1}, \ldots, a_n); \qquad (6.1)$$

certain of these parameters in the process being examined can be variable and others, constant.

Let us determine the structure of the function $f(a_1, a_2, \ldots, a_n)$ on the proposition that this function expresses a certain physical law, which is independent of the selection of a system of units of measurement*.

---

* Let us emphasize that according to the proposition the functional connection (6.1) expresses only one essential physical relationship which defines the quantity as a function of the mutually independent defining quantities $a_1, a_2, \ldots, a_n$, and, thus, is not the utual type of conceivable mathematical connection between a certain aggregate of dimensional quantities which are not dependent upon the selection of a system of units of measurement.

For example, for the quantities $a_1, a_2, \ldots, a_n$, g, P, and m, where g is the acceleration of the force of gravity; P, weight; and m, the mass of a certain body, in addition to (6.1) there is the true expression

$$a = f(a_1, a_2, \ldots a_n) + \Phi(a_1, a_2, \ldots, a_n) \ln \frac{P}{mg}, \qquad (6.1')$$

which for the **arbitrary function** $(a_1, a_2, \ldots, a_n)$ does not depend on the selection of a system of units of measurement. However, the eq.(6.1') breaks down into two generally different physical laws: the expression (6.1') and the connection P = mg, of which the latter in the given problem can be considered "parasitic". It would be possible to introduce numerous other artificial examples of relationships which would be independent of units of measurement and which would be exclusive of the formulation of the problem given above.

We will analyze the physical law under consideration in the form (6.1) and henceforth will use only the assumption of the existence of such a connection, which may be, in a general sense, complex.

The problems involved in theoretical or experimental methods of actually establishing this connection have no significance in the subsequent discussions. This is the reason therefore for the investigation of the relationships (cont'd)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

Among the dimensional quantities $a_1, a_2, \ldots, a_n$ first let k of the quantities ($k \leq n$) have independent dimensions (the number of basic units of measurement should be greater or equal to k).

The independence of the dimensions signifies that the formula which expresses the dimension of one of the quantities, cannot be represented as a combination in the form of a powered monomial of the formulae of the dimension for the other quantities. For example, the dimension of length L, speed $\frac{L}{T}$, and energy $\frac{ML^2}{T^2}$ are independent. The dimensions of length L, speed $\frac{L}{T}$, and acceleration $\frac{L}{T^2}$ are dependent. Among mechanical quantities there are usually no more than three with independent dimensions. We will assume that k is equal to the least number of parameters with independent dimensions, therefore the dimension of the quantities a, $a_{k+1}, \ldots, a_n$ can be expressed by the dimension of the parameters of $a_1, a_2, \ldots, a_k$.

Let us take k of the independent quantities $a_1, a_2, \ldots, a_k$ as the basic quantities and let us introduce as their dimensions the designations

$$[a_1] = A_1, \quad [a_2] = A_2, \ldots, [a_k] = A_k.$$

---

* (continued) between dimensional quantities in the implicit form

$$\Psi(a, a_1, a_2, \ldots, a_n) = 0$$

or in the form of several implicit functions for the quantities a, b, c ...

$$\Psi_1(a, b, c, \ldots, a_1, a_2, \ldots, a_n) = 0,$$
$$\Psi_2(a, b, c, \ldots, a_1, a_2, \ldots, a_n) = 0,$$
$$\Psi_3(a, b, c, \ldots, a_1, a_2, \ldots, a_n) = 0$$

is not an examination of the problem in any more general form. In such a treatment, the role of "parasitic" relationships increases, moreover, in this case complex problems, not connected with the intrinsic nature of the problem which is being examined, of the solvability of systems of implicit equations do not facilitate the simplicity of arriving at basic results concerning the theory of dimensions.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

The dimensions of the remaining quantities will have the form

$$[a] = A_1^{m_1} A_2^{m_2} \dots A_k^{m_k},$$
$$[a_{k+1}] = A_1^{p_1} A_2^{p_2} \dots A_k^{p_k},$$
$$\dots\dots\dots\dots\dots\dots$$
$$\dots\dots\dots\dots\dots\dots$$
$$[a_n] = A_1^{q_1} A_2^{q_2} \dots A_k^{q_k}.$$

Let us now change the units of measurement of the quantities $a_1, a_2, \dots, a_k$ respectively $\alpha_1, \alpha_2, \dots, \alpha_k$ times. The numerical values of these quantities and the quantities $a, a_{k+1} \dots a_n$ in the new system of units will correspondingly be equal to:

$$a_1' = \alpha_1 a_1, \qquad a' = \alpha_1^{m_1} \alpha_2^{m_2} \dots \alpha_k^{m_k} a,$$
$$a_2' = \alpha_2 a_2, \qquad a_{k+1}' = \alpha_1^{p_1} \alpha_2^{p_2} \dots \alpha_k^{p_k} a_{k+1},$$
$$\dots\dots\dots \qquad \dots\dots\dots\dots\dots$$
$$\dots\dots\dots \qquad \dots\dots\dots\dots\dots$$
$$a_k' = \alpha_k a_k. \qquad a_n' = \alpha_1^{q_1} \alpha_2^{q_2} \dots \alpha_k^{q_k} a_n.$$

In the new system of units of measurement the relationship (6.1) has the form

$$a' = \alpha_1^{m_1} \alpha_2^{m_2} \dots \alpha_k^{m_k} \cdot a = \alpha_1^{m_1} \alpha_2^{m_2} \dots \alpha_k^{m_k} f(a_1, a_2, \dots, a_n) =$$
$$= f(\alpha_1 a_1, \dots, \alpha_k a_k, \alpha_1^{p_1} \alpha_2^{p_2} \dots \alpha_k^{p_k} a_{k+1}, \dots, \alpha_1^{q_1} \alpha_2^{q_2} \dots \alpha_k^{q_k} a_n). \qquad (6.2)$$

This equation shows that the function f has the property of uniformity in relation to the scale $\alpha_1, \alpha_2, \dots, \alpha_k$.

The scales $\alpha_1, \alpha_2, \dots, \alpha_k$ are arbitrary. Let us utilize a selection of these scales in order to reduce the number of arguments in the function f. Let us assume that:

$$\alpha_1 = \frac{1}{a_1}; \quad \alpha_2 = \frac{1}{a_2}, \dots, \alpha_k = \frac{1}{a_k},$$

i.e., we select a system of units of measurement such that the values of the first k

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

arguments on the right side of the expression (6.2) are equal to unity*. In other words, utilizing the fact that the expression (6.1) in accordance with the proposition does not depend on the system of units of measurement, we establish a system of units of measurement such that k arguments of the function f have fixed constant values equal to unity.

In this relative system of units of measurement, the numerical values of the parameters a, a_{k+1}, ..., a_n are defined by the formulae

$$\Pi = \frac{a}{a_1^{m_1} a_2^{m_2} \dots a_k^{m_k}},$$

$$\Pi_1 = \frac{a_{k+1}}{a_1^{p_1} a_2^{p_2} \dots a_k^{p_k}},$$

. . . . . .

. . . . . .

$$\Pi_{n-k} = \frac{a_n}{a_1^{q_1} a_2^{q_2} \dots a_k^{q_k}},$$

where a, a_1, a_2, ..., a_n are numerical values of the quantities being examined in the original system of units of measurement. It is not difficult to see that the values Π, Π_1, ..., Π_{n-k} do not depend on the selection of the original system of units of measurement, since they have a zero dimension in relation to the units of measurement A_1, A_2, ..., A_k. It is also evident that the values Π, Π_1, ..., Π_{n-k} do not depend at all on the selection of systems of those units of measurement by which k units of measurement for the quantities a_1, a_2, ..., a_k are expressed. Consequently, these quantities can be considered dimensionless.

Utilizing the relative system of units of measurement, the relationship (6.1)

---

* For simplicity we will agree that the parameters a_1, a_2, ..., a_k are finite and are not zero. The following conclusions are expanded when a_1, a_2, ..., a_k can be converted to zero or to infinity, if the function f for these values of the arguments is continuous.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

can be presented in the form

$$\Pi = f(1, 1, \ldots, \Pi_1, \ldots, \Pi_{n-k}). \qquad (6.3)$$

Thus, the connection between n + 1 dimensional quantities a, a_1, ..., a_n, which are not dependent on the selection of a system of units of measurement takes the form of the relationship between the n + 1 - k quantities Π, Π_1, ..., Π_{n-k}, which represent dimensionless combinations of n + 1 dimensional quantities*. This general result of the theory of dimensions is known as the Π-theorem.

If it is known that a dimensionless quantity being examined is a function of a number of dimensional quantities, then this function can depend only on dimensionless combinations consisting of the defining dimensional quantities.

It is evident that in the expression (6.3) it is possible to replace the system of dimensionless parameters Π_1, Π_2, ..., Π_{n-k}, changing the form of the function f, with another system of dimensionless parameters which are functions of the n - k parameters Π_1, ..., Π_{n-k}.

It is not difficult to see that of the n parameters a_1, a_2, ..., a_n, among which there are more than k parameters with independent dimensions, more than n - k independent dimensionless powered combinations cannot be constructed. This follows directly from the result of the expression (6.3) if we take any selected dimensionless combination defined by the quantities a_1, a_2, ..., a_n as the quantity a.

Any physical relationship between dimensional quantities can be formulated as a

---

* It is necessary to correct the result if the function $f(a_1, a_2, \ldots, a_k, a_{k+1}, \ldots, a_n)$ where the values of the first k arguments equal zero or infinity is discontinuous. In the relationship (6.3) the first k arguments can be replaced by unity only at those points where a_1, a_2, ..., a_k are not zero or infinity. Therefore, in the vicinity of points, the number of existing arguments in the right side of (6.3) can exceed n - k.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

relationship between dimensionless quantities. This is actually the source of useful applications of the method of the theory of dimensions to the investigation of mechanical problems.

The less the number of parameters which define the quantity being studied, the more limited the functional relationship will be, and the simpler it will be to conduct the investigation. In particular, if the number of basic units of measurement is equal to the number of defining parameters which have independent dimensions, then with the aid of the theory of dimensions this relationship can be defined with precision up to the constant factor.

In fact, if $n = k$, i.e., all the dimensions are independent, then it is not necessary to form dimensionless combinations out of the parameters $a_1, a_2, \ldots, a_n$, and therefore the functional relationship (6.3) can be represented as

$$a = c a_1^{m_1} a_2^{m_2} \ldots a_n^{m_n},$$

where c is a dimensionless constant, and the exponents $m_1, m_2, \ldots, m_n$ can be easily defined with the formula for the dimension of a. As far as the dimensionless constant c is concerned, it can be defined either experimentally or theoretically by solving the appropriate mathematical problem.

It is evident that the more we are able to select the basic units of measurement, the more useful the theory of dimensions is.

We have seen above that the number of basic units of measurement can be selected arbitrarily, however, an increase in the number of basic units is attended by the introduction of additional physical constants which must also be figured among the defining parameters.

Increasing the number of basic units of measurement, we increase the number of dimensional constants. In general, the difference $n + l - k$, which is equal to the number of dimensionless parameters into which the physical relationship is formulated, remains constant.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

Increasing the number of basic units of measurement can be of use only in a case when from the number of additional physical relationships it is clear that the physical constants which are occasioned by the introduction of new basic units of measurement are unessential. For example, if we examine a phenomenon in which mechanical and thermal processes take place, then in order to measure the amount of heat and the mechanical energy we can introduce two different units of measurement, calories and joules, but it will then be necessary to take into consideration the dimensional constant J, the mechanical equivalent of heat. We will now assume that we are considering the phenomenon of heat-transfer in a moving, incompressible, ideal liquid. In this case heat energy will not be transformed into mechanical energy or the reverse and therefore the thermal and mechanical processes will occur independently of the value of the mechanical equivalent of heat. If we were to assume the possibility of changing the quantity of the mechanical equivalent of heat, this would be reflected in no way in the values of the characteristic quantities. Consequently, in the case under consideration, the constant J will not enter into the physical relationship, and increasing the number of the basic units of measurement makes it possible to obtain additional important information with the help of the theory of dimensions.

Further on, we will illustrate these conclusions with examples.

7. Parameters, Which Define Classes of Phenomena

In any study of mechanical phenomena we begin with schematic representations, with the determination of the basic factors which define the quantities we are interested in, and, in the broad sense of the word with the construction of a model of the processes being investigated utilizing the simplest forms and phenomena which have already been clarified and studied.

A correct schematic representation is very frequently a difficult task, requiring of the investigator much experience, intuition, and a preliminary qualitative determination of the mechanism of the processes being studied. The essence of cer-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

tain problems consists of the verification of the accuracy of hypotheses, the correctness of which is more or less problematical.

The determination of defining factors and profound penetration into the essence of mutual connections and laws is the basis for the conscious utilization and control of natural phenomena in order to resolve the manifold problems formulated in the lifetime of man.

The properties of bodies and the elementary physical laws which play an essential role and control a phenomenon are characterized by a number of quantities which can be dimensional or dimensionless, variable or constant.

A mechanical system and its motion state are defined by a number of dimensional and dimensionless parameters and functions.

Considering the aggregate of different mechanical systems which compass certain motions, we always can delimit with an appropriate form the classes of permissible systems and movements in such a way that a concrete system and its motion can be defined by a finite number of dimensional and dimensionless parameters. The delimitation of a class of permissible systems and motions can always be achieved with additional requirements concerning the fixing of abstract parameters and, in a dimensionless form, of a type of assigned functions of the problem.

The theory of dimensions makes it possible to arrive at conclusions resulting from the possibility of using arbitrary or special systems of units of measurement to describe physical laws. Therefore, in calculating the parameters which define the class of motions, it is necessary to indicate all the dimensional parameters connected with the essential nature of the phenomenon, independent of whether or not these parameters factually preserve constant values (in particular, these might be physical constants) or of whether they might be changed for different motions of a determined class. It is important that dimensional parameters can utilize different numerical values in different systems of units of measurement even though, possibly, they are identical for all the motions being examined. For example, in examining

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

STAT

Page Denied

Next 1 Page(s) In Document Denied

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

FOR OFFICIAL USE ONLY

pressure p is naturally not zero and has a dimension which is not dependent on the dimensions of the temperature and density.

Let us now assume that the state of the gas is defined by the values of the temperature, density, and one physical constant, for example, the coefficient of heat capacity c_v' measured in mechanical units of measurement $\left([c_v'] = \frac{L^2}{T^2 \Theta}\right)$. Having designated the mechanical equivalent of heat by J kg-m/cal, we have:

$$c_v' = Jc_v,$$

where c_v is the coefficient of heat capacity in thermal units of measurement $\left([c_v] = \frac{\text{cal}}{\text{mass} \times ^\circ\text{C}}\right)$. The dimension of the pressure can be expressed by the dimensions Θ, ρ, and c_v', therefore the proposition advanced is acceptable from the point of view of the theory of dimensions. Since the dimensions Θ, ρ, and c_v' are independent, from the proposition that

$$p = f(\Theta, \rho, c_v'),$$

it follows directly that Clapeyron's equation

$$\frac{p}{c_v'\rho\Theta} = c \quad \text{or} \quad p = \rho R \Theta,$$

is correct where c is a dimensionless constant, and R designates the dimensional constant $cc_v' = cJc_v$.

Thus, Clapeyron's equation can be considered the result of the single hypothesis which consisting of the fact that pressure, density, temperature, and heat capacity which are independent of the values of the other characteristics are connected with one another by a relationship which is physical in nature. Below in separate examples we will point out means of combining the methods of the theory of dimensions with expressions ensuing from symmetry, from the linearity of problems, from the mathematical properties of functions for small or large values of the defining parameters, etc.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

ORIGINAL

1. Motion of the Mathematical Pendulum

As the first example we shall examine the classical example dealing with the motion of a mathematical pendulum.

The mathematical pendulum (Fig.1) is composed of a material point (bob) weight, suspended by means of a weightless string which does not vary in length and whose other end is rigidly attached. We shall limit the totality of all the possible motions by the condition that the pendulum travels in one plane.

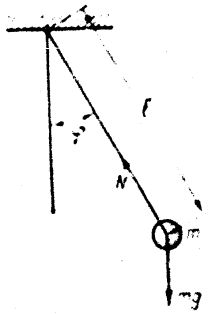


Fig.1 - A Mathematical Pendulum

Let us assign the meaning to the values: l - length of the pendulum, φ - the angle between the string and the vertical, t - time, m - the mass of the weight and N - the strain acting on the string. If the resistance forces are disregarded then the problem dealing with the motion of the pendulum can be reduced to the solution of the equations

$$\frac{d^2\varphi}{dt^2} = -\frac{g}{l}\sin\varphi. \qquad (1.1)$$

$$m\left(\frac{d\varphi}{dt}\right)^2 l = N - mg\cos\varphi \qquad (1.2)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

ORIGINAL

provided that we first assume that

$$t = 0 \quad \varphi = \varphi_0 \quad \text{and} \quad \frac{d\varphi}{dt} = 0$$

or in other words, that the initial moment of time is selected to be the one when the pendulum is deflected to the angle φ_0, and the velocity is equal to zero.

From the eqs. (1.1), (1.2) and the initially assumed conditions, it is possible to see that as governing parameters, the following system can be accepted:

$$t, l, g, m, \varphi_0$$

The numerical values of all the other dimensions are determined completely by the values of these parameters. Thus we can write:

$$\left.\begin{aligned} \varphi &= \varphi(t, \varphi_0, l, g, m), \\ N &= mgf(t, \varphi_0, l, g, m), \end{aligned}\right\} \qquad (1.3)$$

where φ and f are **non-dimensional functions.**

The numerical values of the functions φ and f should not be dependent on the system of unit measurement. The values of these functions can be determined either through the solution of the eqs. (1.1) and (1.2), or else through experimental means.

From the general considerations, developed above, is deduced the fact that the five dimensional variables of functions φ and f may be reduced to two variables, which in themselves represent non-dimensional combinations, composed of t, l, g, m and φ_0 as there are present three independent units of measurement.

From the values t, l, g, m and φ_0 it is possible to construct two independent non-dimensional combinations:

$$\varphi_0 \text{ and } t\sqrt{\frac{g}{l}}. \qquad (1.4)$$

All the other non-dimensional combinations, composed of t, l, g, m and φ_0 or simply from any arbitrary values determined by these parameters, will be the functions of combination (1.4). Thus it is possible to write:

$$\varphi = \varphi\left(\varphi_0, t\sqrt{\frac{g}{l}}\right), \qquad (1.5')$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

FOR OFFICIAL

$$N = mgf\left(\varphi_0,\ t\sqrt{\frac{g}{l}}\right). \qquad (1.5'')$$

The formulae (1.5) are developed with the aid of the dimensions method, showing that the rule of motion is not governed by the mass of the load, and that the strain acting on the string is directly proportional to the mass of the load. These deductions are also based directly from the eqs. (1.1) and (1.2). The value $t\sqrt{\frac{g}{l}}$ may be examined as an element of time, expressed in a special system of units of measurement in which the length of the pendulum and the acceleration caused by the force of gravity are accepted to be equal to the unit of one.

Let us assign the value of T to any characteristic period of time, for example, to the time required for the pendulum to move from the extreme limit of its swing to the vertical position, or between two identical phases, as for instance, the period of oscillation, etc. (The existence of the periodic motion can be accepted as a hypothesis, or as a result known from supplementary data). We thus have:

$$T = f_1(\varphi_0,\ l,\ g,\ m) = \sqrt{\frac{l}{g}}\, f_2(\varphi_0,\ l,\ g,\ m).$$

The function f_2 represents a non-dimensional value, but as it is not possible to construct a non-dimensional combination from l, g and m, then it becomes obvious that the function f_2 is independent of l, g, and m. Thus,

$$T = \sqrt{\frac{l}{g}} \cdot f_2(\varphi_0). \qquad (1.6)$$

The eq. (1.6) establishes the relationship between time T and the length of the pendulum. It is not possible to determine the nature of function $f_2(\varphi_0)$ with the aid of the dimensions theory. To determine the value of $f_2(\varphi_0)$ it is absolutely necessary to do so theoretically on the basis of eq. (1.1), or else by the experimental method.

The formula (1.6) may be developed directly from the correlations (1.5'). In

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

FOR OFFICIAL

actual practice, for the period of oscillation the correlation (1.5') gives:

$$\varphi_0 = \varphi\left(\varphi_0,\ T\sqrt{\frac{g}{l}}\right).$$

In solving this equation we receive the formula (1.6).

If T is the period of oscillation, then through the application of symmetry it becomes apparent that the value of the period of T does not depend on the sign φ_0, therefore:

$$f_2(\varphi_0) = f_2(-\varphi_0).$$

Hence, the function f_2 is the even function of the variable φ_0. Assuming that for small values of φ_0, the function $f_2(\varphi_0)$ is regular, we can write down:

$$f_2(\varphi_0) = c_1 + c_2\varphi_0^2 + c_3\varphi_0^4 + \ldots \qquad (1.7)$$

For small oscillations the terms with the ratios of φ_0^2 and higher, may be discarded, and then for the period T we receive the formula:

$$T = c_1\sqrt{\frac{l}{g}}. \qquad (1.8)$$

The solution of eq.(1.1) shows that $c_1 = 2\pi$. Thus we see that for small oscillations of the pendulum, with the aid of the dimensions theory, it is possible to derive a formula for the period of the pendulum oscillation giving accuracy up to the constant multiple.

The formulae (1.5) and (1.6) will retain the accuracy of their results even in the case when instead of the eq.(1.1) we use the equation

$$\frac{d^2\varphi}{dt^2} = -\frac{g}{l}f(\varphi),$$

where $f(\varphi)$ is any function of φ. It can be said generally that the accuracy of the formulae (1.5) and (1.6) depends on only one condition which consists of the fact that the nature of motion is determined by the parameters

$$t,\ l,\ g,\ m,\ \varphi_0.$$

For determining this system of parameters we can call on the aid of the formulae

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

ORIGINAL

of motion, but the system can also be expressed without resorting to the use of the motion formulae. In fact, to describe the nature of a pendulum it is necessary to state the values of l and m. Furthermore, it is necessary to state the value of g as the essence of the matter is determined by the force of gravity. Finally, it is absolutely necessary to state the value of φ_0 and t, as the concrete motion as well as the state of motion are determined through the angle of extreme deflection φ_0, and which is analyzed by means of the moment of time t.

2. The Flow of Heavy Fluid Through a Spillway

Let us examine the problem of the jet flow of a heavy fluid through a spillway (Fig.2), which in itself represents a vertical wall with a triangular orifice situated symmetrically in respect to the vertical, and with the angle α of the opening

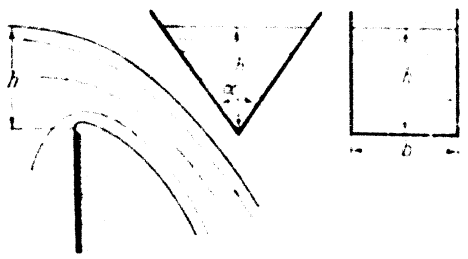


Fig.2 - The Flow of Heavy Fluid Through the Spillway

equal to 90°. The fluid flows out under pressure h, which is equal to the height of the fluid level above the top line of the triangle, for a considerable distance from the spillway opening. For the sake of simplicity, we shall assume that the vessel which contains the fluid is extremely large, and therefore, the fluid flow may be considered as uniform.

In jet fluid flow the factors of inertia and ponderability occupy basic meanings. They are characterized respectively by the density value ρ and the acceleration due to gravity g.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The nature of the fluid flow in the described spillway is completely analyzed by the parameters ρ, g, h

The weight Q of the fluid flowing through the orifice of the spillway in unit time can only be a function of these parameters:

$$Q = f(\rho, g, h).$$

The theory of dimensions permits determining the nature of this function. In fact, the dimension Q is expressed in kg/sec which is also the case for the combination $\rho g h^3 \sqrt{\frac{g}{h}}$. This gives

$$\frac{Q}{\rho g^{3/2} h^{5/2}}$$

which is an indifferent quantity. This relation contains the functional quantities ρ, g, h from which it is impossible to construct indifferent combinations. Consequently, it is preferable to write

$$\frac{Q}{\rho g^{3/2} h^{5/2}} = C$$

or

$$Q = C \rho g^{3/2} h^{5/2}, \qquad (2.1)$$

where C is an absolute constant, which can be determined simpler by experiment. The derived formula fully determines the relationship for the quantity of fluid passing as a result of the pressure h and of the density factor ρ.

The analysis of fluid flow may be extended to include spillways having variable angles α. In such cases, the system of control parameters is enlarged to include the angle α, changing eq.(2.1) to

$$Q = C(\alpha) \rho g^{3/2} h^{5/2}, \qquad (2.2)$$

i.e., the coefficient C will depend on the value of the angle α.

For a rectangular spillway of a width b, the system of control parameters will be ρ, g, h, b.

All nondimensional values are then determined by the parameter h/b, and eq.(2.1) can be replaced by

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$Q = f\left(\frac{h}{b}\right) p g^{1/2} h^{3/2}. \qquad (2.3)$$

The function $f\left(\frac{h}{b}\right)$ may be determined experimentally, having the fluid flow through a spillway with a variable width b, at a constant value of h. Having thus determined the value of the function $f\left(\frac{h}{b}\right)$, eq.(2.3) may now be applied in cases where the width function b is constant, but where the pressure function h varies, i.e., cases for which actual experimentation was not performed.

This example shows that conclusions derived by the dimensional method may be useful in setting up experiments, allowing to limit them in number and thus to obtain savings not only of funds but also of time. The change of one set of values in an experiment may be accomplished through the change of another set of values. On the basis of experiments with water, it is possible to obtain answers dealing with the pumping of crude oil, mercury, etc.

3. Fluid Flow Through Pipes

The significance of the theory of dimensions and similitude in hydraulics was first realized during the study of fluid flow in pipes. Aside from the practical importance, and the simplicity of the dimensions theory, its application to hydraulic problems, which resulted in improvements and greatly advanced hydraulics, took place only at the end of the Nineteenth Century, after the work done by Osborn Reynolds*.

For a long time, numerous empirical formulas, suggested by various authors, were

---

* Reynolds, Phil.Trans.Roy.Soc., 1883 and 1885; also see the collection of work "Problems of Turbulence", ONTI, 1936.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

used in the study of hydraulics. These formulae contained a series of dimensional constants, the values of which were governed by the specific experimental conditions and the properties of fluids.

The establishment of the dimensions theory along with a more exact and a universal approach to the problem permitted to confirm or eliminate many of the imperic laws established for the flow of various types of fluids under various temperature conditions, through pipes of varying diameters, and for various velocities of fluid flow.

Let us analyze and establish the problem. We shall classify the nature of flow by the following conditions: Pipes - cylindrical, of uniform cross-sectional design (Fig.3). Hence, the pipe and its cross-section are fully determined by the assignment of a cross-sectional area, or by the assignment of an arbitrary characteristic linear dimension a. In the case of round pipes, the radius or the diameter are usually taken as the characteristic dimension. The length of the pipe is assumed to be great, thus it is permissable to disregard the peculiarities of flow at the ends of pipes. To make the conditions ideal, we assume that the pipes are of infinite length.

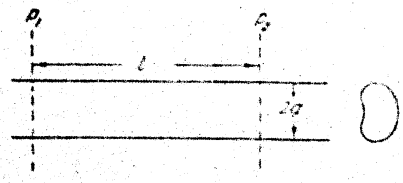


Fig.3 - The Flow of an Incompressible Fluid Through a Cylindrical Pipe

In regards to the fluid flow being studied, we shall assume that it is steady.

Furthermore, in our process of study, we shall assume the fact that the phenomena of compression does not exist, and due to this we shall study the flow of an incompressible fluid. The factors of inertia and viscosity of the fluid, expressed by density ρ and the coefficient of viscosity μ we shall take into consideration. Due to the fact that the coefficient of viscosity is effected by temperature, we also take this factor into consideration. We shall also take into account the influence

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

of temperature. *

In order to determine the fluid flow it is necessary to assign either the pressure drop along the length of the pipe, or the use of fluid per unit of time at the cross-section of the pipe, or the average velocity of the fluid along a section of the pipe, etc.

Therefore, the pipe, the fluid, and the state of fluid flow, are totally expressed by the system of parameters:

$$\rho,\ \mu,\ a,\ \bar{u}.$$

All the mechanical characteristics of flow are the functions of these parameters.

Let us examine, for an example, the pressure drop along the pipe. The pressure drop per unit length of the pipe is expressed as the value

$$\frac{p_1 - p_2}{l},$$

where p_1 and p_2 are the measured pressures in the section of a pipe, at the ends of the length l.

The combination

$$\frac{p_1 - p_2}{l \cdot \frac{\rho \bar{u}^2}{2a}} = \psi$$

is a non-dimensional value and is called the coefficient of resistance of the pipe.

The resistance in a section of the pipe with a length l is expressed in the form of:

$$P = (p_1 - p_2) S = \psi \cdot \frac{l}{a} \cdot S \cdot \frac{\rho \bar{u}^2}{2}, \qquad (3.1)$$

where S is the cross-sectional area of the pipe.

---

* In this case we shall consider the effect of temperature with the understanding that it can be considered to be uniform throughout the whole mass of the fluid.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

From the four determining parameters ρ, μ, a and u it is possible to construct only one independent non-dimensional combination

$$\frac{u \cdot a \cdot \rho}{\mu} = R,$$

which is called the Reynolds number. All the non-dimensional values, which depend on the four mentioned parameters, are functions of the Reynolds number. In particular,

$$\psi = \psi(R). \qquad (3.2)$$

The problem of determining the pipe resistance, or of determining the use of fluid in relation to the pressure drop, are narrowed down to the finding of the functional relationship $\psi(R)$. This function can be found experimentally by measuring the resistance in relation to the velocity (or in relation to the fluid used) of fluid flow in any arbitrary pipe. The data so secured can be utilized during the analysis of the flow of other fluids and in pipes of different diameter. Thus, for an example, in accordance to experimental data on the flow of water, it is possible in series of cases (when compression does not exist i.e., in cases when the velocity is considerably lower than the speed of sound) to solve many questions dealing with the flow of air in a pipe, etc. (Fig.4).

Experience shows that the flow of fluid in a pipe takes place under two regimes which are completely different from each other - the laminar regime and the turbulent regime. During the laminar flow through a cylindrical pipe, the fluid particles flow in a straight course parallel to that formed by the pipe. In the case of turbulent flow there is present a disorderly mixing of the fluid in a direction perpendicular to the course of the pipe. Turbulent flow can only be observed in part as a steady flow.

In a series of cases the laminar fluid flow in a pipe is accompanied by weak stability, or even nonstability, and gives place to turbulent flow.

The element of stability represents in itself, the characteristic of the fluid flow as a whole. Due to this, in cases of smooth pipes, this element must be deter-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

mined by Reynolds number R. Experiments confirm this deduction well. For small values of the Reynolds numbers the laminar flow is steady, for the large values - nonsteady. The regime of the flow is expressed by the Reynolds number. The limits of stability of the laminar flow are determined by a certain value of the Reynolds number which is called the critical value. In case of the round cylindrical pipes, the critical value of the Reynolds number is in the range of R_{KR} = 1,000 to 1,300.

The laminar regime is characteristic for the flow of high viscosity fluids, at small velocities, in pipes of small diameters (for example, in capillar tubes). The turbulent regime is characteristic for the flow of fluids with low viscosity, occuring at high velocities, in pipes of large diameter.

Experimental data shows that the function $\psi(R)$ has two branches, one of which corresponds to the laminar, and the other to the turbulent regime of flow. Near the critical value of the Reynolds number there exists a certain state of transition.

In the case of laminar flow through a cylindrical pipe, all the fluid particles travel along straight lines parallel to the axis of the pipe, with a constant velocity, i.e., with zero acceleration. This type of fluid flow is called the "Gagen-Pouazell" flow. The element of inertia of the fluid, as expressed by the parameter ρ, may be present only then when the acceleration is not equal to zero.* Due to this, in the case of laminar flow, the resistance is not affected by ρ. Thus, in the case of laminar flow, the right side of the eq.(3.1) does not depend on ρ. From this we draw the conclusion that, with laminar flow, the density ρ in the eq.(3.1) must be reduced, thereby the function $\psi(R)$ must take on the form

$$\psi = \frac{C}{R} = \frac{C\mu}{\rho a u}, \qquad (3.3)$$

where C is the non-dimensional constant, determined by the geometric form of the cross-section of the pipe, a is the radius of the pipe. In the case of a round pipe

* As is known, density and acceleration enter into the flow formulae only in their development.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

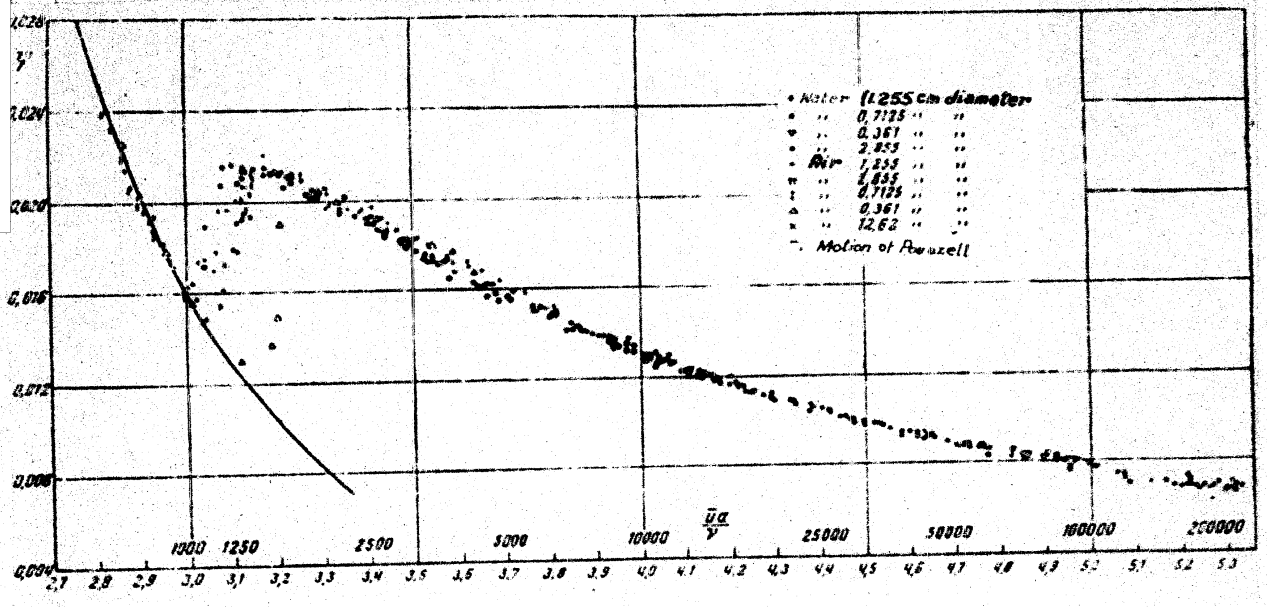


Fig.4 - Experimental Data on the Coefficient of Resistance of Round Cylindrical Pipes.

(Measurements of Stanton and Pennel; a = pipe radius)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the value for C is easily calculated from theory: C = 16.

Consequently, in the case of laminar flow, the formula for the resistance of the pipe becomes

$$P = \frac{1}{2}\frac{S}{a^2} C\mu l\bar{u} = C_1\mu l\bar{u}, \qquad (3.4)$$

where C_1 is the non-dimensional constant which is dependent on the cross-sectional shape of the pipe. The formula (3.4) can thus be derived directly, if for the determining parameters are taken only the three values a, μ, $\bar{u}$, and consideration be given to the fact that P is proportional to l.

If the pressure drop which causes the fluid flow is given, then it is more convenient to take, in the form of determining parameters, the values

$$\rho,\ \mu,\ a \text{ and } i = \frac{p_1 - p_2}{l}.$$

In this case, the flow regime is determined by the non-dimensional parameter

$$\frac{\rho i a^3}{\mu^2} = J.$$

From the formula (3.1) it is easy to see that

$$J = \frac{1}{2}\mathbf{R}^2\psi(\mathbf{R}). \qquad (3.5)$$

This equation establishes the relationship of J to R through the function (R) which we shall write as

$$Q = \bar{u}S$$

equals the volume of fluid, flowing through the cross-section of a pipe per unit of time (referred to as the volumetric discharge of the pipe). The non-dimensional equation

$$\frac{Q\rho}{\mu a} = \mathbf{R}\frac{S}{a^2}$$

becomes the function of value J, thus

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$Q = \frac{\mu a}{\rho} f(J). \qquad (3.6)$$

The nature of function f(J) for the laminar flow may be determined simply. On the basis of eqs.(3.3) and (3.5) we find:

$$Q = \frac{2}{C} \frac{S}{a^2} \frac{ia^4}{\mu} = C_2 \cdot \frac{ia^4}{\mu}, \qquad (3.7)$$

where the non-dimensional constant C_2 depends on the cross-sectional shape of the pipe. For a round pipe

$$C_2 = \frac{\pi}{8}.$$

Eq.(3.7) describes Pouazell's Law, which was confirmed experimentally by Gagen in 1839 and by Pouazell in 1840. The close similarity of the results obtained from this law to experimental data is one of the main confirmations of the correctness of the law of friction due to viscosity in a fluid, and of the initial outlining of this phenomena.

4. The Motion of a Body in a Fluid

In outlining the problem of the motion of an aeroplane, a submarine, etc., we are faced with the question of the forward motion of a solid body, at constant velocity, within a limitless fluid mass surrounding all the space outside of the body.

Let us establish the geometric shape of the surface describing the body. For the complete expression of the surface of a body it is sufficient to express only certain characteristics of length d.

Let us examine the series of the forward motions of the body, parallel to some immovable plane. Let us express through V and α respectively the velocity of motion and the angle which determines the direction of the motion (Fig.5). The values V and α may vary for different motions.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Let us further assume that the fluid can be considered incompressible. The elements of inertia and the viscosity of the fluid shall be taken into account. For the sake of simplicity we shall assume that the forces acting on the mass do not exist. The distribution of pressure acting on the surface of the body as well as the total of the forces acting from the side of the fluid onto the body depends on the nature of the agitated motion of the fluid.

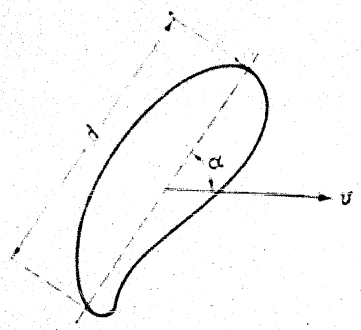


Fig.5 - Movement of a Solid Body in a Liquid

For a body with an assigned shape, the stabilized state of fluid motion is determined by a system of five parameters *: d, v, α, ρ, μ.

All the non-dimensional mechanical values, related to the state of fluid flow, may be analyzed as functions $5 - 3 = 2$ of the non-dimensional parameters, by the angle of attack α and by the Reynolds number

$$\frac{vd\rho}{\mu} = R.$$

Let us assign W to the force acting on the body from the side. [For continued

---

* The pressure at infinity p_0, which can be assigned arbitrarily, does not enter into this system of parameters for the following reason: The fluid is incompressible and therefore the change p_0 cannot show its effect on the velocity. Instead of taking the value for the full pressure, it is always permissible to take into consideration only the difference between the pressures p and p_0. From this it is evident that the valus p_0 is insignificant and therefore, it is not (cont'd)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

discussion it is immaterial whether we accept W as the total resistance force or as only one of its components, such as the frontal resistance (directed in the opposite direction from the direction of the bodily motion), or the lifting force acting perpendicularly to the motion]. From the basic theorem of the dimensions theory it develops that the non-dimensional value $\frac{W}{\rho d^2 v^2}$ acts as the function of angle α and the Reynolds number R. Therefore:

$$W = \rho d^2 v^2 f(\alpha, R). \qquad (4.1)$$

The determination of function $f(\alpha, R)$ represents the most important problem of theoretical and experimental aerodynamics and hydrodynamics.

It is obvious that for the chosen system of parameters, the effect of viscosity on the motion expresses itself only through the influence of the Reynolds number.

---

* (cont'd) necessary to introduce it as a governing parameter. However, when the fluid flow is accompanied by the cavitation phenomena, which is tied in with the occurence of fluid evaporation in the region of the lowered pressure, then to the series of governing parameters it is essential to add the value p_0 and p' where p' represents the compression force of the fluid vapor at a certain temperature. In cases dealing with compressible fluids, it is absolutely necessary to include the value of p_0, or any other parameter which may act as a substitute for p_0, in the series of determining parameters.

During motion, accompanied by cavitation, there exists yet one more non-dimensional parameter, $\chi = 2\frac{p_0 - p'}{\rho v^2}$. In experiments dealing with the study of the influence of the cavitation χ number, the change in the χ number can be affected by varying either the value of p_0 or the value of V, or by an artificial method of changing the value of p'. It is also possible to use different fluids and to change the density ρ.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

From the structure of the equation $R = \frac{vd\rho}{\mu}$ it is possible to make certain basic conclusions about the role played by the viscosity of the fluid with the increase in velocity or in the size of the body. For an example, with the increase in velocity, or in the linear dimensions of the body, the Reynolds number increases. However, for retaining the role of viscosity the Reynolds number must remain a constant. As every change in the Reynolds number can be credited to the account of the change in the coefficient of viscosity. If the product Vdp is increased, then in order to maintain the Reynolds number constant it is essential to increase the coefficient of viscosity μ. Therefore, under conditions where the velocity of the body is the same, the flow of honey (having a large $\frac{\mu}{\rho}$) induced by the motion of a large body is analogous to the flow of water (having a small $\frac{\mu}{\rho}$) induced by the motion of a small body. Or, the motion of the body through honey at a high velocity is analogous to the motion of the same body through the water at a low velocity. The analogy of motion is explained by the fact that all non-dimensional values for these motions are equal.

Furthermore, from these conclusions it is also obvious that during the motion of a body in the same type of fluid, the effect of viscosity drops with the increase of velocity and with the increase of the body dimensions.* Theoretical research and experimental data show that with large values of the Reynolds number the role of the viscosity diminishes, and in certain cases becomes nonexistent. Disregarding viscosity, i.e., assuming that $\mu = 0$, we arrive at the understanding of an ideal fluid.

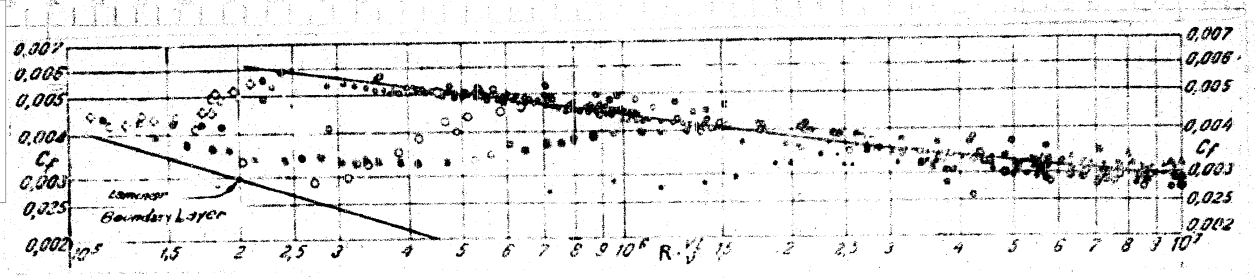
In problems on the motion of a body in an ideal fluid, the number of governing

---

* In this case we assume that with other things being equal, the effect of viscosity drops with the decrease in the coefficient of viscosity.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2



- ▲ Frud, Plates 16'; 25' and 50'(~4.88; 7.62 and 15.25 m)
- ⊙ Washington, Plates 20' and 30'(~6.1 and 9.15 m)
- ⋆ Gebers, Yubigan, Plates 0.6; 1.6; 3.6; 4.6 and 6.52 m
- ● Gebers, Vienna Plates 1.25; 2.50; 5; 7.5 and 10 m
- ✦ National Laboratory of Frud, Plates 3'; 8' and 16' (~0.915; 2.44 and 4.88 m)
- ● Washington, Smooth Plate 80'(~24.4 m)
- ● Washington, Plate with fish Plates 80'(~24.4 m)
- ● Kempf and Klasa, Plates 0.5 and 0.75 m
- ● Vizelsberger, Plates covered with cloth and lacquered to a thickness of 0.5; 1; 1.5 and 2 m (tested in air)
- ● Tsam, Board Plate 2'(~0.61 m) and cardboard 16'(~4.88 m) (tested in air)
- ▲ Gibbons, Glass Plate 9½'(~2.9 m) (tested in air)
- ◇ Shenkher, Katamaran (form of raft) flat, smooth 3'(~0.915 m)
- ● " " " " flat, rough leading edge 3'(~0.915 m)
- ○ Shenkher, Katamaran (form of raft) flat, smooth 6'(~1.83 m)
- ○ " " " " flat, under conditions or rapid development of turbulence 6'(~1.83 m)
- ▫ Kempf, an element from the shell of a vessel

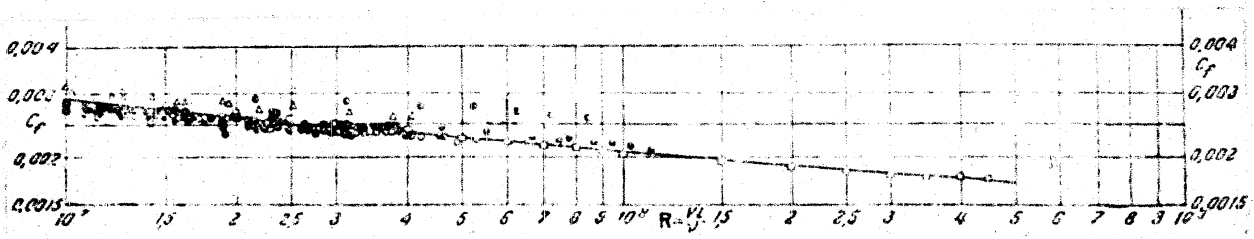


Fig.6 - Coefficient of Resistance $C_f = \frac{W}{lb\,\frac{\rho v^2}{2}}$ of Flat Square Plates, Towed Parallel to their Surface

(l = Dimension, Parallel to the Direction of Motion, b = Width of the Plate)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

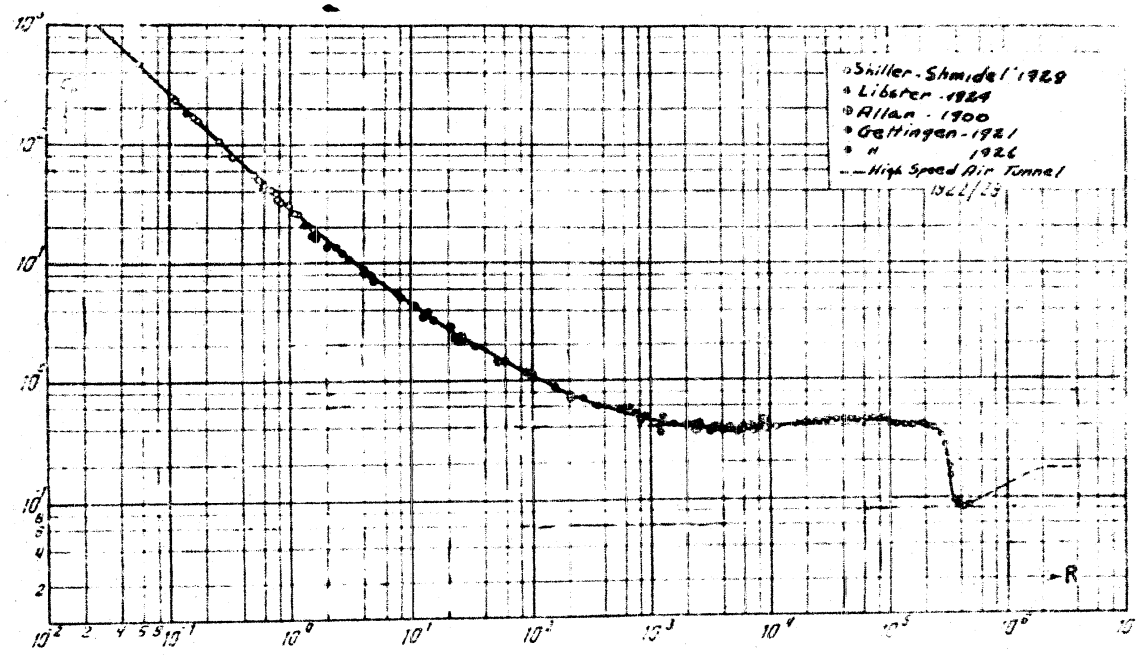


Fig.7 - Coefficient of Resistance of a Sphere $C_w = \frac{W}{\frac{\pi d^2}{4} \times \frac{\rho v^2}{2}}$ as a Function of the Reynolds Number $R = \frac{vd}{\nu}$ (d = diameter of the sphere)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2
Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

parameters is reduced to four:

$$d,\ \alpha,\ \rho,\ v.$$

In an ideal fluid all the non-dimensional characteristics are determined by the angle of attack α, and therefore, the eq.(4.1) is substituted by the equation

$$W = \rho d^2 v^2 f_1(\alpha) \qquad (4.2)$$

Consequently, during the motion of a body in an ideal, incompressible fluid the forces acting on the body from the side of the fluid are proportional to the square of the velocity. For a viscous fluid at sufficiently high Reynolds numbers this law holds only approximately true.

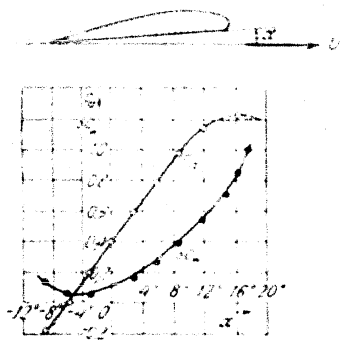


Fig.8 - Typical Curves Showing the Relationship of the Coefficient of Lift $C_a = \frac{2A}{\rho S v^2}$ and the Resistance $C_w = \frac{2w}{\rho S v^2}$ of a Wing, on the Angle of Attack α (S - Area of the Wing in the Plan)

For bodies of various shape, the functions $f(\alpha, R)$ and $f_1(\alpha)$ in eqs. (4.1) and (4.2) depend essentially, not including the angle of attack, on the abstract parameters which determine the geometric shape of the body. In Fig.6 and 7 data is presented showing the relationship between the coefficient of resistance and the Reynolds number. Figure 8 shows the degree of influence that the angle of attack has on the resistance and on the lifting power of a wing.

Let us now examine a case for the comparatively low velocity of a body, one which involves low Reynolds numbers.

In lowering the Reynolds number the role of the viscosity increases. If we neglect inertial forces in relation to viscosity then this will be the equivalent of adopting unrealistic values for the parameter ρ. In this case, the system of govern-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

ing parameters is composed of four:

$$d,\ v,\ \rho,\ \mu.$$

and therefore, all the non-dimensional characteristics will also depend only on the angle of attack α. Therefore,

$$W = \mu\, d v\, f_2(\alpha). \qquad (4.3)$$

From this it is evident that the resistance and the lifting force are propor-

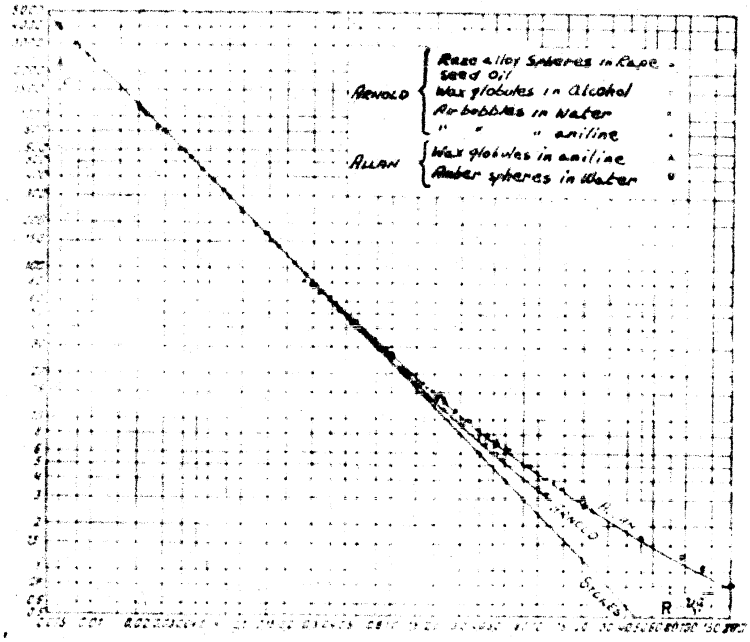


Fig.9 - Coefficient of Resistance of a Sphere for Low Reynolds Numbers

tional to the velocity, the coefficient of viscosity, and the linear dimension d. This law, which may be called the Stoke's Law, agrees well with experimental results for low velocities of small bodies, for an example, as during the sinking of small particles in a liquid.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

For the sphere, the function is $f_2(\alpha) = \text{const} = 3$, i.e., it does not depend on the angle α. The theoretic value of the coefficient for low velocities of the sphere, and under assigned conditions (which are reduced to insignificant values in the equations of Navie and Stokes, by the inertial values) has been calculated by Stokes. It was found to be equal $c = 3\pi$ (when d is the diameter of the sphere).

Thus we see that the dimensions theory allows to clarify the nature of the function $f(\alpha, R)$ in its relationship to the Reynolds number for relatively low and somewhat high values of the Reynolds number. When $R \to \infty$ then we approach an ideal fluid. In which case, the function $f(\alpha, R)$ leans toward a certain function $f_1(\alpha)$,.which is independent from the Reynolds number. The regimes of motion, corresponding to low Reynolds numbers, are characteristic of the fact that the fluid viscosity has a basic meaning, but the element of inertia has large secondary values for μ and small values for ρ. Within the limit of $\rho = 0$ the eq.(4.3) is correct. Assuming that $\rho \neq 0$, we receive from this equation:

$$W = \rho d^2 v^2 \cdot \frac{f_2(\alpha)}{c d \rho v} .$$

From this follows the fact that for low Reynolds numbers the correct version of the equation would be:

$$f(\alpha, R) = \frac{f_2(\alpha)}{R} . \qquad (4.4)$$

This relationship represents in itself, the elaboration of the Stoke's Law. The comparison between experimental data and the results obtained by Stoke's Law for a sphere is presented in Fig.9.

With the aid of the dimensions theory we have established that if we are to disregard the inertial factors in the equations of Navie and Stokes, then Stokes' Law (4.3) holds true for bodies of any shape.

The function $f_2(\alpha)$ can be determined by experiment or by theory, through the solution of the simplified Navier-Stokes equations.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

5. Heat Dissipation From a Body in a Fluid Stream

In 1915 Rayleigh applied the dimensions theory to the problem of Bussinesk on the heat transfer from the body to the fluid flowing over the body*. In the long run the analyses made by Rayleigh have been used as reference by a series of authors**, but the questions raised in these quotations have remained unanswered.

Let us examine in detail all the situations which arise when the dimensions theory is applied to this problem.

The outline of the problem is as follows: there exists a steady process of heat transfer from the body of specific form to the fluid which completely fills all the space around this body. The body is immovable, the fluid flows over the body. At a distance sufficiently ahead of the body the fluid flows forward at a constant velocity V.

Let H be the quantity of heat dissipated by the body per unit of time. Assuming that the fluid is ideal and incompressible,, Rayleigh reasons as follows: the value H is determined by the values of the following parameters: the characteristic dimension l of the body, the fluid velocity v at a distance removed from the body, the temperature gradient ϑ equal to the difference between the temperature of the body and of the fluid at a distance from the body. It is assumed that the body temperature is maintained at a constant, the thermal capacity c per unit of volume of the fluid and the coefficient of conductivity λ of the fluid.

Thus, we can write:

$$H = f(l, v, \vartheta, c, \lambda).$$

In the form of basic units of measurement Rayleigh chooses the units for the measurement of length, time, temperature, quantity of heat and mass: L, T, °C , Q, M.

---

* Rayleigh, Nature, 1915, p.66.

** Ryabushinskiy, Nature, 1915, p.591; Rayleigh, Nature, 1915, page 644, Bridgmen, Analysis of Dimensionality, ONTI, 1934.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Therefore, the dimensions of the parameters will be:

$$[l] = L,\ [v] = \frac{L}{T},\ [\vartheta] = C^\circ,\ [c] = \frac{Q}{L^3C^\circ},\ [\lambda] = \frac{Q}{LC^\circ T};$$

let us note that all of these dimensions are not related to mass.

From the five governing dimensional parameters it is possible to develop only one independent nondimensional combination

$$\frac{lvc}{\lambda}.$$

The dimension H will be:

$$\frac{Q}{T}.$$

It is easy to see that the combination $\frac{H}{\lambda l \vartheta}$ represents in itself a nondimensional value, and thus

$$H = \lambda l \vartheta f\left(\frac{lvc}{\lambda}\right). \tag{5.1}$$

This formula was developed by Rayleigh. From it develops the fact that the flow of heat is proportional to the temperature gradient v and has one and the same value for the various values of v and c, but with a constant product of v c.

Ryabushinskiy made the following observation. In view of the fact that the quantity of heat and the temperature have energy dimension (in the kinetic theory of gases the temperature is determined as the average kinetic energy of the molecules in chaotic motion), then for the basic units of measurement it is possible to take only the units for the measurement of length, time and mass. Then the dimensions of the governing parameters will be:

$$[l] = L,\ [v] = \frac{L}{T},\ [\vartheta] = \frac{ML^2}{T^2},\ [c] = \frac{1}{L^3},\ [\lambda] = \frac{1}{LT}.$$

Now from the governing parameters it is possible to compose two independent nondimensional combinations:

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\frac{tvc}{\lambda} \text{ and } cl^3.$$

Consequently, in this case the dimensions theory leads to the formula

$$H = \lambda 10 f\left(\frac{tvc}{\lambda}, cl^3\right), \tag{5.2}$$

which, evidently gives less information than the formula (5.1).

In a personal reply to Ryabushinskiy, Rayleigh wrote*:

"The question, raised by Ryabushinskiy, pertains to logic rather than to the method of applying dimensionality, which so interests me. The question greatly deserves further study. My conclusion was reached on the strength of the basic Fourier equations for heat conductivity, in which the temperature and the quantity of heat are accepted as values 'sui generis'. We would be dealing with a paradox, if the depth of our knowledge of the nature of heat in the molecular theory would lead us into a weaker position than we were in previously during the analysis of the common problem. The solution of the paradox lies apparently in the fact that in equations of Fourier is contained an assumption on the nature of heat and temperature which is not present in Ryabushinskiy's argument". Pausing on this answer, Bridgmen** drew a correct conclusion that Rayleigh's reply would hardly satisfy anyone, but he himself does not, however, present a clarification of the question.

The dilemma is answered in the following manner: in the system of the basic units, adopted by Rayleigh for the development of formula (5.1) there are found three separate units for the measurement of energy - erg $\frac{ML^2}{T^2}$, degree C^0 and calorie Q. The determination of heat capacity and temperature as a mechanical energy is presented in the kinetic theory of gases. The conversion of heat capacitance and temperature into mechanical units of measurement is tied in with the

---

* Nature, 1915, page 644.

** Bridgmen, Analysis of Dimensionality, p 17, ONTI 1934.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

value of constants - mechanical equivalent of heat, $\left(J = 427 \text{ kgm/cal } [J] = \frac{ML^2}{T^2Q}\right)$ and the Bol'tsman constant $k = 1.38 \cdot 10^{-16}$ ergs/degree $\left([k] = \frac{ML^2}{T^2C^0}\right)$. For the independent units of measurement of mechanical energy, heat capacity and temperature it is imperative to analyze these constants as physical constants.

From the conditions introduced by incompressibility and ideal fluid follows the fact that the subject of velocities is determined through kinematic principles and the phenomena is not accompanied by transformation between the thermal and the mechanical energies. The mechanical processes take place independently of the thermal ones. From this it follows that the density of the fluid is immaterial for all thermal values, and that the value of the mechanical equivalent of heat is usually immaterial in view of the absence of conversion of heat energy into mechanical energy. Furthermore, if we assume that the density ρ and the value J have no effect on the heat transfer process being studied, then from the dimensions theory arises the fact that the value of the Bol'tsman constant k also has no significance, as the dimensionality of the constant k contains the symbol of unit mass, from which the dimension of H and the governing factors are independent. The insignificance of values ρ, J and k in the proposed thinking can also be easily realized through the mathematical formulation of the question on determining the quantity of heat dissipated by the body of the fluid. These circumstances justify the absence of ρ, J and k from the governing parameters as selected by Rayleigh*. However, if we retain the thinking on the insignificance of density ρ** and do not go further with the supposition that J and k are also insignificant, which is the result of supplementary thinking, then to the Rayleigh's list of the governing parameters it is

---

* If we study this same problem for the case of a viscous and a compressible fluid then the values ρ, J and k become significant. Then these parameters or their substitute parameters must be included in the list of the governing dimensions.

** In Ryabushinskiy's discussions this assumption is adhered to.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

essential to add the values k and J. After this we obtain the following system of governing parameters:

$$l, c, \delta, c, \lambda, J, k.$$

From these seven dimensional values it is possible to develop only two independent nondimensional combinations:

$$\frac{l\delta}{c} \text{ and } \frac{Jcl^2}{k}.$$

In this case the formula (5.1) is substituted by the following one:

$$H = \varphi_0 \left(\frac{l\delta}{c}, \frac{Jcl^2}{k} \right). \tag{5.3}$$

The developed formula (5.3) is reduced to formula (5.1) if we adopt into our thinking the insignificance of the mechanical equivalent for the heat value of J, and thus also the insignificance of the parameter

$$\frac{Jcl^2}{k}$$

If now, along with Dyabushinskiy's thinking, we utilize the thermal values as expressed in mechanical terms, then k and J will become nondimensional universal constants and the formula (5.3) will be changed to formula (5.2). The conclusion becomes rather weak because in this method of analysis the added considerations in the mechanics of the phenomena are not taken into account.

6. Dynamic Similarity and Modeling of Phenomena

The theory of dimensions and similarity possesses great value in the modeling (simulation) of various phenomena. In modeling we substitute for the study of the phenomenon in nature that interests us the study of a similar phenomenon on a model of less or greater scale, ordinarily under special laboratory conditions. The principal meaning of modeling consists in the fact that the results of experiments with models can give the necessary answers concerning the character of the effects and concerning the various quantities that are connected with the phenomenon under

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

natural conditions. In most cases the modeling is based on a consideration of physically similar phenomena. The study of natural phenomenon of interest to us we replace by the study of physically similar phenomenon which is more convenient and advantageous to realize.

Mechanical or generally physical similarity can be considered as a generalization of geometrical similarity. Two geometrical figures are similar if the ratios of all the corresponding lengths are identical. If one knows the coefficient of similarity, that is, the scale, then by the simple multiplication by the magnitude of the scale of the dimensions of one geometric figure one obtains the dimensions of the other figure, namely its similar geometrical figure.

There exist various methods for the determination of the mechanical or physical similarity. We shall give below the definition of physical similarity in that form which is necessary for practical purposes and which is convenient for immediate applications.

Two phenomena are similar if from the given characteristics of one phenomenon it is possible to obtain the characteristics of the other phenomenon by simple conversion or recalculation which is similar to the transition from one system of units of measurement to another system.

In order to realize the conversion it is necessary to know the "transitional scales".

The numerical characteristics for two different but similar phenomena can be considered as the numerical characteristics of one and the same phenomenon, which are expressed in two different systems of units of measurement. For each set of similar phenomena all the dimensionless characteristics (dimensionless combinations of dimensional quantities) possess identical numerical value. It is not difficult to see that the inverse conclusion is also correct, i.e., if all the dimensionless characteristics for two motions are identical, then the motions are similar.

The set of mechanically similar motions is determined by the regime of the

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

motion.

The similarity of two phenomena sometimes can be understood in a wider sense by assuming that the above indicated definition refers only to a certain special system of characteristics, which completely defines the phenomenon and permits one to find any other characteristic that cannot, however, be obtained by simple multiplication by suitable scales during the transition from one to another "similar" phenomenon. For example, in this sense two arbitrary ellipses can be considered similar in the case of the utilization of Cartesian coordinates which are directed along the main axes of the ellipses. By the indicated conversion one can obtain the Cartesian coordinates of the points of any ellipse through the coordinates of the points of any one ellipse (affine similarity).

In order to preserve the similarity during modeling it is necessary to observe certain conditions. However, very often in practice these conditions which ensure the similarity of phenomenon in the whole are not fulfilled, and then the problem arises concerning the magnitude of the errors (scale effect) which occur during the transfer to nature of the results obtained from the model.

After the establishment of a system of parameters which determine the evolved class of phenomena, it is not difficult to establish the conditions for the similarity of two phenomena.

In fact, let the phenomenon be defined by n parameters, part of which can be dimensionless, and certain are dimensional physical constants. We assume further that the dimensions of the variable parameters and of the physical constants are expressed by k principal units of measurement ($k < n$). In the general case it is evident that out of the n quantities one can compose not more than n-k independent dimensionless combinations. All the dimensionless characteristics of the phenomenon can be considered as functions of these n-k independent dimensionless combinations composed of the defining parameters. Consequently among all the dimensionless quantities composed of the characteristics of the phenomenon one can always speak of

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

a certain basis, that is, a system of dimensionless quantities which determine all the remaining quantities.

The class of phenomena which is defined by a suitable statement of the problem contains phenomena which in general are not similar to each other. The isolation from it of the subclass of similar phenomena is effected with the aid of the following condition:

The necessary and sufficient condition for the similarity of two phenomena will be the constancy of the numerical values of the dimensionless combinations that form the basis. The conditions on the constancy of the basis of the abstract parameters composed of the given quantities defining the phenomenon are called the criteria of the similarity.

In the problem of steady plane-parallel progressive motion of a body in an incompressible viscous fluid all the dimensionless quantities are determined by two parameters: the angle of attack α and the Reynolds number R. The conditions of physical similarity, the criteria of similarity, are represented by the following relations

$$\alpha = \text{const} \quad \text{and} \quad R = \text{const}$$

During modeling of this phenomenon the results of experiments with a model can be transferred to nature only for identical α and R. The first condition is always easily realized in practice. It is more difficult to satisfy the second condition (R = const.), especially in those cases where the streamlined body possesses large dimensions, such as e.g. the wing of an airplane. If the model is less than nature, then in order to preserve the magnitude of the Reynolds number it is necessary either to increase the velocity of the circulating stream, which in practice is ordinarily unrealizable, or to substantially change the density and viscosity of the fluid.

In practice these circumstances impose greater difficulties in the study of aerodynamic resistance. The necessity for the constancy of the Reynolds number has

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

led to the construction of gigantic aerodynamic tubes in which one can create the blowings of airplanes in nature (Figs.10 and 11), and also to the construction of tubes of the closed type in which compressed, i.e. denser, air circulates with great velocity (Fig.12).

Special theoretical and experimental investigations show that in a number of cases the Reynolds number for bodies of streamlined form influences markedly only the dimensionless coefficient of frontal resistance and sometimes influences very weakly the dimensionless coefficient of lifting force and certain other quantities that play a very important role in various practical problems. Consequently, a

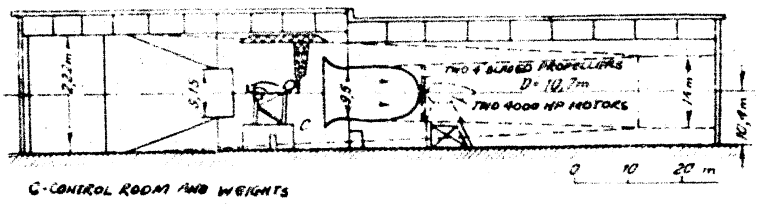


Fig.10 - Longitudinal Cross Section of an NACA Aerodynamic Tube for Testing in Nature. The width of the tube is 18.3 meters. The return channels are not shown.

difference in the value of the Reynolds number for the model and for the phenomenon in nature is not essential in certain problems.

We have indicated the conditions for similarity of the motion of a wing without taking into account the property of compressibility of air, which is nonessential for velocities small in comparison with the velocity of sound. In what follows we shall consider the conditions for similarity taking into account the compressibility of air.

Let us as another example consider the problem of modeling elastic constructions or designs.

Let us have any structure consisting of a homogeneous material, for example, a bridge girder. The elastic properties of isotropic material are determined by two

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

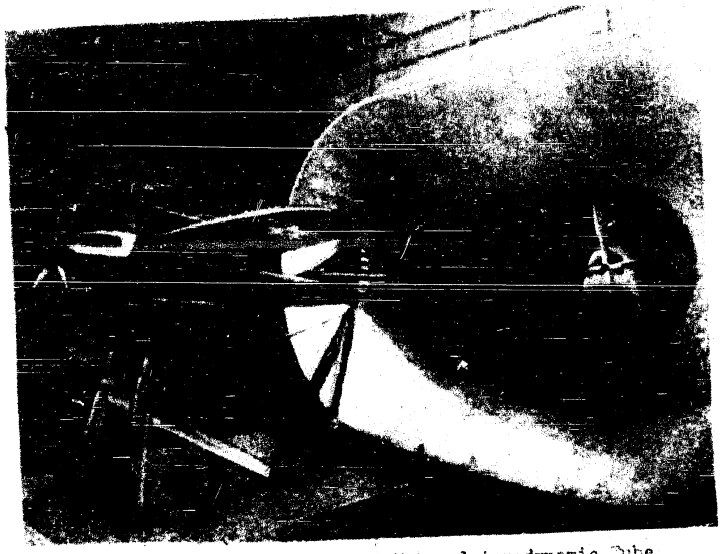


Fig.11 - A Photograph of a Natural Aerodynamic Tube

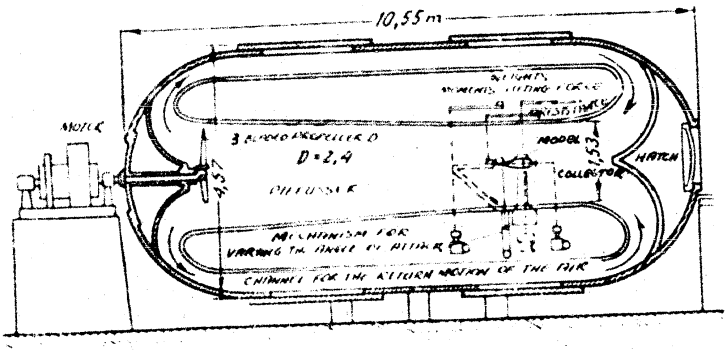


Fig.12 - The Cross Section of an NACA Aerodynamical Tube of Variable Density. The pressure within the tube can attain 21 atmospheres, and the velocity of the air flow can reach 23 meters/second.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

constants: Young's modulus $E \frac{kg}{m^2}$ and by the dimensionless coefficient of Poisson σ. Let us consider geometrically similar constructions and compile a table of the defining parameters.

In order to determine all the dimensions of the model it is sufficient to assign a certain dimension B which is characteristic of it. If in the considered state of equilibrium the weight of the construction is essential, then the specific weight $\gamma = \rho g \frac{kg}{m^3}$ must figure as a determining parameter. Besides the force of gravity of the parts of the structure, there act in addition external loads which are distributed by a certain definite method over the elements of the construction. Let the magnitude of these loads be determined by the force P (kg). Thus the system of defining parameters will be the following:

$$\sigma, E, B, \rho g, P$$

In this case we have $n = 5$ and $k = 2$; consequently the basis for the mechanically similar states of elastic equilibrium will be the following three dimensionless parameters:

$$\sigma, \quad \frac{E}{\rho g B}, \quad \frac{P}{EB^2}.$$

The criteria of similarity consists in the constancy of these parameters in the model and in nature. For the fulfillment of these conditions all deformations (strains) will be similar. If the model is n times less than nature, then in the model the strains will be n times less than in nature.

If the model and the structure in nature are fulfilled as to being made from one and the same material, then the values of σ, ρ and E are identical in model and nature, and therefore it is necessary to satisfy the following condition for mechanical similarity

$$gB = \text{const.}$$

Under usual conditions we have $g = \text{const.}$; consequently we must have $B = \text{const.}$ for the observation of mechanical similarity, i.e. the model must coincide with

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

nature. In other words, modeling is impossible in the case of constant g*.

Variations in g can be attained by an artificial method, if we force the model to rotate with constant angular velocity, namely by placing the model in a so called centrifugal machine (Fig.13). In the case of sufficiently small dimensions of the model and large radius of rotation the centrifugal forces of inertia of the elements of the model can be considered parallel. By executing rotation around a vertical axis we obtain that in a state of relative equilibrium of the model (relative to the centrifugal machine) there will act constant mass forces which are similar to the force of gravity, but only with a different acceleration. By choice of the angular velocity of rotation one can obtain any large values for the acceleration.

The idea of employing the centrifugal machine for the modeling of various processes was promoted in 1932 by N.N.Davidenko** and G.I.Pokrovskiy***. Still earlier than this, in 1929, Davidenko**** proposed using for the same purpose a container that falls upon a stiff spring; however, this method is inconvenient and was discarded.

At the present time there have been constructed centrifugal machines which are used for the investigation on models of various processes***** occurring in grounds.

---

* The increase required in practice in the specific weight ρg in the case of decreased size of the model can sometimes be realized by the application to the elements of the model of additional loads.

** N.N.Davidenko, Zhurnal Tekhnicheskoy Fiziki (Journal of Technical Physics), No 1, 1933.

*** G.I.Pokrovskiy, Zhurnal Tekhnicheskoy Fiziki, No 4, 1933.

**** See G.I.Pokrovskiy, Vestnik Voyen.-Inzh. Akademii RKKA im. V.V.Kuybyshev, No 30, 1940.

***** The condition $\frac{E}{\rho g \beta} = \text{const.}$ must be fulfilled during modeling of processes in which besides other essential parameters one encounters the parameters (cont'd)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Let us consider the stress τ (kg/m^2) arising during the deformation of an elastic construction under the action of a weight and given distribution of loads. By τ we shall understand the maximum value of any component of the stress or in general a certain component of the stress for a definite element of the construction.

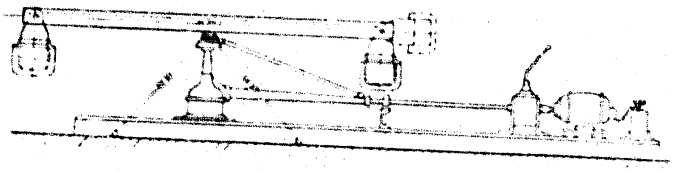


Fig.13 - Schematic outline of the centrifuge for the testing of models

The combination $\frac{1}{E}$ is dimensionless; therefore we can write the following:

$$\frac{\tau}{E} = f\left(\sigma, \frac{E}{\rho g B}, \frac{P}{EB^2}\right).$$

If the model and structure in nature are made of the same material, then we have E = const. Therefore, for mechanically similar states the stresses in corresponding points will be identically.

If we assume that the stressed states are mechanically similar and that rupture is determined by the values of the maximum stresses, then it is evident that in the model and in nature the ruptures will have their onset for similar states. If the magnitudes of the external loads are large, and the dead weight of the construction is small so that it can be disregarded, then the parameter $\gamma = \rho g$ and consequently also the parameter $\frac{E}{\rho g B}$ are nonessential. In this case the foregoing relation acquires the following form

$$\frac{\tau}{E} = f\left(\sigma, \frac{P}{EB^2}\right).$$

***** (cont'd) parameters ρ, g, B and E. Therefore in all such cases it is possible to model with the aid of the centrifugal machine.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

and the conditions for similarity are represented by the following two conditions:

$$\sigma = \text{const}, \text{ and } \frac{P}{EB^2} = \text{const}.$$

Hence it follows that during modeling with preservation of the properties of the materials the external loads must vary in proportion to the square of the linear dimensions.

Let us designate by the letter λ the variation in length during deformation of a certain element of the elastic system. For constructions of the above defined class there holds a relation of the following form

$$\frac{\lambda}{B} = \varphi\left(\sigma, \frac{\rho g B}{E}, \frac{P}{EB^2}\right).$$

In a number of cases, from physical considerations it is evident at once that the quantity $\frac{\lambda}{B}$ decreases during increase of Young's modulus E (stiffer material), i.e. during decrease of the parameter $\frac{\rho g B}{E}$.

Let us now take two geometrically similar constructions of different dimensions and prepared of one and the same material (E and σ are identical). We assume that the magnitudes of the external loads vary in proportion to the square of the dimensions, i.e.

$$\frac{P}{EB^2} = \text{const}.$$

It is evident in this case that the parameter $\frac{\rho g B}{E}$ decreases with decrease in the dimensions of the construction; consequently the mechanical similarity will be disrupted. In a construction of less dimensions the relative deformations will be less; therefore the construction of small dimensions will possess larger strength. However, this conclusion is correct only in the case where the specific weight of the material $\gamma = \rho g$ plays an essential role. If the proper weight (γ) is not substantial and we have $\frac{P}{EB^2} = \text{const.}$, then the relative deformations possess identical values for bodies of different scales.

Let us consider also the case where γ is nonessential, and is known that for a given construction the ratio $\frac{\lambda}{B}$ decreases during decrease of the external load P.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

If the external loads will be proportional to the cube of the linear dimensions, then it is evident that for a construction of small dimensions the ratio $\frac{1}{D}$ will be less than for a construction of large dimensions. Consequently in this case the decrease of the dimensions also increases the strength.

An interesting example in which the scale of the model is determined uniquely is the example on hydrostatic models of dirigibles and aerostats*.

In practice it is very important to know the form and deformations of the elements of the fabric envelop of an aerostat balloon after filling it with gas. The geometrical form of the aerostat determines its hydrostatic and aerodynamic properties; knowledge of the deformation of the material is necessary in order to ensure the strength of the balloon.

Let us note the parameters which determine the static state of the aerostat.

Within the limits of the difference in heights for various points of the aerostat it is possible to consider that the air and the gas possess constant specific weight. Within the aerostat and in its envelop the differences between the pressure of the air and the pressure of the gas (in the problem under consideration only the differences of these pressures are essential) are determined by the quantity γ^*, which is equal to the difference of the specific weight of the air and the gas $\gamma^* = \gamma_{air} - \gamma_{gas}$.

Experiments in the investigation of the dependence between the stresses and deformations in the materials indicate that for a given material the relative deformations are identical, when the stresses are identical. In fabric envelops the stresses are defined as the forces computed per unit length of the cross section.

---

* See A.G.Vorob'yev, Hydrostatic Test of Models of Aerostats, Sbornik Leningradskogo Instituta Inzh. Putey Soobshcheniya (Symposium of the Leningrad Institute of Engineering Means of Communication), No.XVIII, 1928; V.V.Katanskiy, Proyektirovaniye Ballono-Takelazhnykh Konstruktsiy i t.d. (Planning of Balloon Rigging Constructions etc), ONTI, 1936.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Let us designate by the Greek letter τ (in units of kg/m) the characteristic stress and by the latin letter l the characteristic linear dimension. Let us further introduce into the consideration the weight per unit area of the material q (kg/m^2) and the external given concentrated forces Q (kg) that are applied to various elements of the envelop (the direction of these forces in various cases is correspondingly various).

Thus for geometrically similar envelops made of materials with identical ratios between the deformations and the directions, we obtain the following system of determining parameters:

$$\tau^*, \tau, l, q, Q$$

The conditions for similarity are represented in the following form

$$\frac{\tau_1^* l_1}{\tau_1} = \frac{\tau_2^* l_2}{\tau_2}, \qquad (a)$$

$$\frac{\tau_1^* l_1}{q_1} = \frac{\tau_2^* l_2}{q_2}, \qquad (b)$$

$$\frac{\tau_1^* l_1^2}{Q_1} = \frac{\tau_2^* l_2^2}{Q_2} \qquad (c)$$

Condition (c) is easily satisfied by means of a suitable choice of the external forces Q which can be realized with the aid of loads and blocks. If the model and nature (l_2 / l_1) are made of one and the same material, then we have $q_1 = q_2$ and $\tau_1 = \tau_2$ in consequence of which the conditions (a) and (b) contradict one and another. Therefore let us consider the case where the forces of the own weight of the envelop is not taken into consideration, thanks to which the condition (b) drops out.

In order to satisfy the condition (a) it is necessary to set the following ratio:

$$\frac{l_1}{l_2} = \sqrt{\frac{\tau_1^*}{\tau_2^*}};$$

whence it follows that in the case of decrease in the scale $l_2 < l_1$ it is necessary

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

to increase the specific lifting force $\gamma^*_2 - \gamma^*_1$. This effect can be realized by means of the application of a heavy fluid and by the fastening to the model of "upward braces". On this are based the hydrostatic models of dirigibles and aerostats. For the "gas" of the model one can take water, mercury, glycerin etc. A general view of such a model is shown in Fig.14.

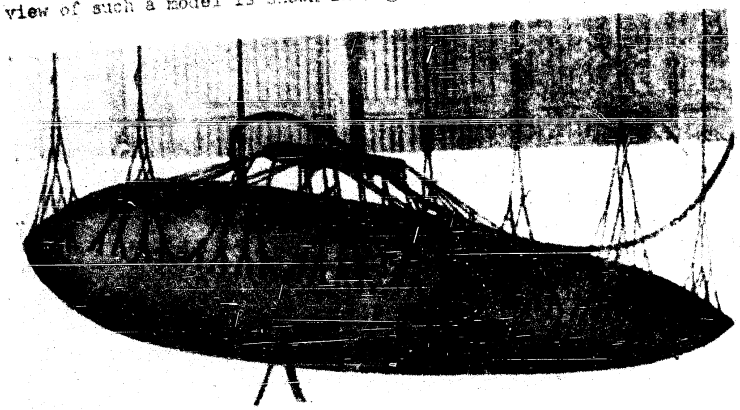


Fig.14 - Testing of the Model of a Dirigible

If for nature there are known the altitude of flight, the temperature, the pressure and gas being employed, then having selected the fluid for the model we obtain the completely determined ratio of the specific weights, which determines the scale of the model.

For example, under normal conditions we have the following values:

for hydrogen	$\gamma^* = 1.1$	kg/m^3
for water	$\gamma^* = 1000$	kg/m^3
for mercury	$\gamma^* = 13,600$	kg/m^3

Hence, when the water is utilized, we find the following value for the scale of the model:

$$\frac{l_1}{l_2} = n = \sqrt{\frac{1000}{1.1}} = 30.1:$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

if mercury is utilized we then obtain:

$$n = \frac{1}{1.1} \quad \text{III}$$

Modeling with the aid of mercury leads to very small models which in general are inconvenient.

The weight of the envelop in nature and the weight of the envelop of the model act in opposite sides; therefore the influence of the weight will destroy the similarity. Taking into account the influence of ponderability of the envelop can be realized by means of special devices for the application of external forces which act vertically upwards and which are distributed over the envelop in correspondence with the requirement of the condition on similarity (b) and with account for the own weight of the envelop directed downward.

In certain cases the modeling can be conducted by utilizing experiments with nonsimilar phenomena known to be so, when certain criteria of similarity π_1, π_2, ... possess different values in the model and in nature, but in addition when additional considerations earlier known give the form of the dependence of the sought for dimensionless quantities upon these defining dimensionless parameters π_1, π_2, ... In such cases during modeling it is necessary to maintain the constancy of only those criteria of similarity whose dependency is unknown.

Sometimes the indicated method of modeling can be employed when the form of the dependency of the sought for quantities upon the parameters π_1, π_2, ... is proposed as a working hypothesis, which can be confirmed or verified after the carrying out of the modeling investigations.

As has been indicated above, an example of such a modeling in a number of cases is modeling in the case of different values of the Reynolds number, when its influence upon the sought for characteristics is nonessential; however, the same method can be applied also in those cases where the Reynolds number is considerable, but the dependence upon the Reynolds number is known beforehand.

Investigation with the aid of models is often the only possible method for the

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

experimental study and solution of the most important practical problems. Such is the case in the study of natural phenomena which occur in the course of tens, hundreds or even thousands of years; under conditions of modeling experiments similar phenomenon can be prolonged in all several hours or days. Such a position we encounter in modeling the phenomena of infiltration of petroleum which is being worked and extracted through wells.

Also possible is the reverse case where in place of the investigation of phenomenon proceeding extremely rapidly in nature one can study more convenient phenomenon that occurs in the model tremendously more slowly.

Modeling is a responsible scientific task possessing general theoretical and perceptive value, but it is necessary to consider it only as the initial base for the main task. The latter consists in the physical determination of the laws of nature, in the search for the general properties and characteristics of various classes of phenomena, in the development of experimental and theoretical methods of investigation and in the solving of various problems, and finally in the obtaining of systematic materials, methods, rules and recommendations for the solution of concrete practical problems.

7. Steady-State Motion of a Solid Body in a Compressible Fluid

Let us consider the general problem concerning steady progressive motion of a solid body with constant velocity within a fluid filling the entire space outside the body. The properties of inertia, viscosity, compressibility and heat conduction of the fluid we shall take into attention. For simplicity we shall not take into account the property of ponderability* of the fluid and the transmission of heat by way of radiation.

---

* In a number of cases the property of ponderability of a fluid is considerable, since it can cause intense convective motion arising in consequence of nonuniform heating of the fluid.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

For the purpose of the determination of the system of determining parameters let us formulate the problem mathematically. First of all we shall write down the equations of motion of a compressible viscous fluid which we shall consider a perfect gas. We have:

1) The Navier-Stokes equation

$$\rho \frac{d\bar{v}}{dt} = -\operatorname{grad} p - \frac{2}{3}\operatorname{grad}\mu \operatorname{div}\bar{v} + 2\operatorname{div}\mu \operatorname{def}\bar{v}, \qquad (7.1)$$

where def $\bar{v}$ is the tensor of the velocities of deformation.

2) The equation continuity

$$\frac{\partial\rho}{\partial t} + \operatorname{div}\rho\bar{v} = 0; \qquad (7.2)$$

3) The equation of state of the gas (the Clapeyron equation)

$$p = \rho R\Theta, \qquad (7.3)$$

where R is the gas constant.

4) The equation of the flow of heat

$$Jc_v\rho\frac{d\Theta}{dt} + p\operatorname{div}\bar{v} = \operatorname{div} J\lambda \operatorname{grad}\Theta - \frac{2}{3}\mu(\operatorname{grad}\bar{v})^2 + \\ + \mu\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \\ + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right\}. \qquad (7.4)$$

The left-hand side of this equation gives the variation in the internal energy because of the variation in the temperature and work of the forces of pressure. The right-hand side gives the variation of energy which is due to the flow of heat from thermal conduction and from the work of the internal forces of friction.

Further, in order to take into consideration the dependence of the coefficients of viscosity and thermal conduction upon the temperature we make use of the formula of Seterland (Southerland)*:

$$\frac{\lambda}{\lambda_0} = \frac{\mu}{\mu_0} = \frac{1+\frac{C}{273,1}}{1+\frac{C}{\Theta}}\sqrt{\frac{\Theta}{273,1}}, \qquad (7.5)$$

* Jeans, The Dynamical Theory of Gases, Cambridge, 1925, page 284.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

where λ_0 and μ_0 are the coefficients of heat conduction and viscosity at the temperature θ less 273.1° corresponding to zero of the scale of Celsius, and C is the constant of Southerland, which possesses the dimensionality of temperature. The constant C possesses various values for different gases. For air we have approximately $C \approx 113^\circ C$.

In addition to the indicated equations we have also the conditions at infinity and the boundary conditions: at infinity in front in the directions perpendicular to the velocity of the body, the velocity is at rest $(\bar{v})_\infty = 0$, and the density and absolute temperature possess the assigned values ρ_0 and θ.

The form of the surface of the body is fixed; therefore all the dimensions of the body are determined by the value of a certain characteristic dimension l.

The kinematic conditions of circulation are completely determined by the assigned value of the forward velocity of the body v and by the two angles α and β, which are formed by the velocity vector with the axes of the system of coordinates connected with the body.

Finally, there must be given the boundary conditions on the surface of the body for the temperature*. Let us assume that the temperature of the body on the surface is constant and equal to θ_1.

The equations (7.1)-(7.5), the conditions at infinity and the boundary conditions show that the system of determining parameters is represented by the following table

$$l,\ \alpha,\ \beta,\ v,\ \theta_0,\ \theta_1,\ \rho,\ \mu_0,\ R,\ c_v,\ J,\ \lambda_0,\ C.$$

* In many technical problems and particularly in aerodynamics we encounter problems of the circulation of a fluid strongly heated or cooled around bodies; for example, the circulation around the radiators of motors, and also the problem of investigating the circulation around bodies of flying apparatus which are to be cooled, etc.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The dimensions of the enumerated thirteen quantities are expressed by means of five basic units of measurement L, T, M, Q, and C^0.

The number of parameters can be shortened by unity if considering that the constants J, c_v and λ enter into eq.(7.4) in the form of the products Jc_v and $J\lambda$. Replacing the three parameters J, c_v and λ by Jc_v and $J\lambda$, the dimensions of all quantities are expressed by only four basic units L, T, M, and C^0; thus, such a shortening of the parameters is nonessential. The motion and the set of similar motions are determined by eight independent dimensionless parameters:

$$\frac{R}{Jc_v} = \gamma - 1, \quad \frac{\mu c_p}{\lambda} = P, \quad \alpha, \quad \beta, \quad \frac{\rho_0 l v}{\mu_0} = \mathbf{R}, \quad \frac{v}{\sqrt{\gamma R \Theta_0}} = \mathbf{M}, \quad \frac{\Theta_1}{\Theta_0}, \quad \frac{C}{\Theta_0}.$$

The kinetic theory of gases shows that the constants γ and P depend only on the structure of the gas molecules. For a perfect gas, the following theoretical formulas apply:

$$\frac{c_p}{c_v} = \gamma = 1 + \frac{2}{m}; \quad \frac{\lambda}{\mu c_v} = \frac{9\gamma - 5}{4}$$

(A.Euchen)

where m is the number of degrees of freedom of the gas molecule. For monoatomic gases we have m = 3, and for diatomic gases m = 5. For many gases (helium, hydrogen, etc) the experimental values are close to the theoretical values; however, in wide temperature and pressure variations these parameters become effective*. In practice the following applies at ordinary temperatures and pressures:

$$\gamma = 1{,}4; \quad \frac{\lambda}{\mu c_v} = P - 1 = 1{,}9, \quad \frac{\mu c_p}{\lambda} = 0{,}737.$$

The condition for mechanical similarity of motion of geometrically similar bodies consists in the constancy of the above eight parameters. Let us designate by W the drag and by A the lift. For gases with one and the same constant values

* See J.Hilsenrath and Y.S.Touloucian. Transaction of the ASME, v.76 (1954) No.6 In this work are contained many references to investigations on this problem.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

of the numbers γ and ρ we can write the following:

$$\frac{2W}{\rho l^2 v^2} = c_x = f_1\left(\alpha,\ \beta,\ \mathbf{R},\ \mathbf{M},\ \frac{\Theta_1}{\Theta_0},\ \frac{C}{\Theta_0}\right),$$
$$\frac{2A}{\rho l^2 v^2} = c_y = f_2\left(\alpha,\ \beta,\ \mathbf{R},\ \mathbf{M},\ \frac{\Theta_1}{\Theta_0},\ \frac{C}{\Theta_0}\right).$$

In the study of the distribution of pressures, the temperature field, the velocity field etc., besides the indicated eight dimensionless parameters, it is necessary still to introduce the dimensionless coordinates of the points of space $\frac{x}{l}$, $\frac{y}{l}$, $\frac{z}{l}$ (for definiteness we assume that the system of coordinates is connected with the body). For example, for the distribution of pressures and temperature we shall have the functional dependences of the following form:

$$\frac{2p}{\rho_0 v^2} = f_3\left(\frac{x}{l},\ \frac{y}{l},\ \frac{z}{l},\ \alpha,\ \beta,\ \mathbf{R},\ \mathbf{M},\ \frac{\Theta_1}{\Theta_0},\ \frac{C}{\Theta_0}\right),$$
$$\frac{\Theta}{\Theta_0} = f_4\left(\frac{x}{l},\ \frac{y}{l},\ \frac{z}{l},\ \alpha,\ \beta,\ \mathbf{R},\ \mathbf{M},\ \frac{\Theta_1}{\Theta_0},\ \frac{C}{\Theta_0}\right).$$

The determination of the form of the functions f_1, f_2, f_3, and f_4 makes up the principal problem of experimental and theoretical aerodynamics.

The parameter $\frac{C}{\Theta_0}$ can turn out to be essential only in the case where the heat conduction and viscosity play a marked role; here, the phenomenon is characterized by marked variations in the temperature. But even in the cases where the thermal conduction and viscosity are considerable, the influence of the temperature upon the coefficients of heat conduction and viscosity can be represented ordinarily with good approximation in place of the formula (7.5) by the following formula which does not contain the dimensional constant C:

$$\frac{\lambda}{\lambda_0} = \frac{\mu}{\mu_0} = \sqrt{\frac{\Theta}{273.1}},$$

or the more general formula

$$\frac{\lambda}{\lambda_0} = \frac{\mu}{\mu_0} = \left(\frac{\Theta}{273.1}\right)^k,$$

where k is a certain abstract constant number. These considerations show that in practice the parameter $\frac{C}{\Theta_0}$ ordinarily does not play an essential role and conse-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

quently it can be left out of consideration during modeling.

In most cases the temperatures θ_1 and θ_2 differ but little; therefore in various experiments the ratio $\frac{\theta_1}{\theta_2}$ is always approximately the same and close to unity.

In the preceding discussions we assumed that the temperature of the body θ_1 possesses an assigned value. If one can disregard the transmission of heat between the body and the fluid, then the boundary condition at the surface of the body can take the form

$$\frac{\partial \theta}{\partial n} = 0;$$

This corresponds to the case where the surface of the body can be considered heat-impermeable. Under this condition the parameter $\frac{\theta_1}{\theta_0}$ is excluded. The very same thing is obtained also in this case if in general one disregards the heat-conduction and considers adiabatic processes.

Sometimes the parameter $\frac{\theta_1}{\theta_0}$ is excluded because the temperature θ_1 becomes a definite quantity. For example, in certain cases independently of the thermal properties of the body the temperature of the body, as a result of heat transmission between the fluid and the body, becomes established and obtains a certain value different from the temperature of the fluid at infinity. We encounter a similar case when we measure with the fluid thermometer the temperature of a gas which is moving with a very great velocity. The readings of the thermometer depend in general upon R and M and upon the method used in setting up the thermometer relative to the flow; the temperature of the body (thermometer) will differ from the temperature of the nondisturbed flow far from the body.

If in the setting up of the problem under consideration we take into account the indicated additional data, then it is evident that for the similarity of the disturbed motions of the gas the principal value is the constancy of the Reynolds numbers and the number M; here, the number M is essential only in the case of marked effects of the property of compressibility.

All of the preceding derivations are easily extended to the case of the motion

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

of a screw in a fluid. In the problem on the screw, in addition to the forward motion of the screw, there also exists the rotation of the screw around the axis. Therefore in the case of the steady-state motion of a screw with constant forward and angular velocities still another parameter is added, namely the angular velocity of rotation, which can be given by the number of turns of the screw n per unit time.

To the characteristics of the regime of forward motion of a body for the case of a screw one adds the dimensionless parameter $\frac{v}{nl}$, which is called the relative progress of the screw. In the study of aerodynamic or hydrodynamic properties of screws this parameter is fundamental.

If the fluid can be considered ideal and incompressible, then in the case of forward motion of the screw along its axis and for the case of a fixed pitch the only dimensionless parameter defining the regime of the motion is the progress or step.

8. Nonsteady Motion within a Fluid

The conclusions of the theory of dimensions which were obtained in the preceding paragraph for the case of steady-state motion can be generalized to the case of nonsteady motion.

In the general case in the study of nonsteady motion we must introduce among the number of defining parameters the time t, which represents a variable quantity. In the consideration of mechanically similar motions we encounter the variation of the numerical value of a parameter time t because of the variation of the scales and because of the variation of the time in the process of motion. In connection with this we shall linger at the start on certain remarks concerning kinematically similar nonsteady motions.

Let us consider nonsteady kinematically similar motions. In kinematic similarity all of the corresponding dimensionless combinations which are composed of kinematic quantities are identical for all motions. The class of motions can be extended and considered as the set of kinematically nonsimilar motions, if one assumes

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

that certain dimensionless kinematic parameters characterizing the entire motion can take on various values for different motions.

Among the kinematically similar motions each concrete motion and state of motion is determined by three parameters. Two parameters yield the motion of the system in the whole, and one parameter fixes a definite state of the motion.

As parameters determining the concrete motion in the whole one can take a certain length characterizing the geometrical dimensions and a certain characteristic time. These parameters determine the kinematic scales of the law of motion.

For example, in the case of motion in a circle one naturally selects for the characteristic dimension the radius of the circle; in the case of oscillatory motion one chooses the amplitude of oscillation of a certain point etc. For the characteristic time of periodic processes one naturally selects the period. In place of the characteristic time one can take the velocity for a certain definite state or mean velocity etc.

The instantaneous state of motion of a given nonsteady motion can be determined by the value of the time t^*.

Let d, v and t be the characteristic dimension, characteristic velocity and the considered moment of time. The similar, or, in other words, the corresponding states of motion, for the motions which are similar in the whole are determined by the value of the dimensionless variable parameter $\frac{vt}{d}$, which can be considered as the dimensionless time.

The equality $\frac{v_1 t_1}{d_1} = \frac{v_2 t_2}{d_2}$ which is fulfilled for similar states of the motion of two systems can be considered as the condition for conversion of the time in the case of the transition from one system to another system.

---

* The origin of reckoning of the time, just as also the origin of reckoning for the linear coordinates, is chosen so that for various motions the position of the systems and the state of motions at the moment of time t = C be similar.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

As an example let us consider the system of harmonic oscillations of a point along the arc of a circle or circumference of radius d. The law governing the motion is represented by the following formula:

$$s = a \cos kt,$$

where s is the length of the arc. For kinematic similar motions we must have:

$$\frac{a}{d} = \text{const}.$$

The concrete motion is determined by the amplitude a and the frequency k. The state of the motion is determined by the moment of time t.

The similar states for the various motions are characterized by identical value of the dimensionless parameter kt.

If for two motions the ratios $\frac{a_1}{d_1}$ and $\frac{a_2}{d_2}$ possess different values, then such motions are kinematically not similar. The ratio $\frac{a_1}{a_2}$ determined the scale of length. For kinematically similar harmonic oscillations in a circle we must have $\frac{a_1}{a_2} = \frac{d_1}{d_2}$. For a straight line it is impossible to indicate the characteristic dimension; therefore it is evident that each two harmonic rectilinear oscillations are kinematically similar.

Let us take still further the following rectilinear motion

$$s = vt + a \cos kt,$$

which represents the uniform motion combined with harmonic oscillation. In this case the class of similar motions are characterized by the constancy of the numerical values of the dimensionless parameter $\frac{ak}{v} = S$ called the number of Strukhal (Struhal). The similar states of the motion can be characterized by the value of the parameter kt or by the value of the parameter $\frac{vt}{a} = \frac{kt}{S}$.

We can extend the class of motions and can consider the nonsimilar motions, by assuming that in the case of the conservation of the principal form of the law of oscillations the Strukhal number can take various values for different motions.

Let us now consider the nonsteady motion of a body in a fluid under the assumption that the law governing the motion of the body is given. As the dimensional

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

parameters which yield the definite law of motion we take a certain length d and the velocity v. In comparison with the case of steady-state motion in the case of nonsteady motion with given law of motion the system of defining parameters is supplemented only by the value of the length d which characterizes the law governing the motion and by the parameter of the time t. Therefore the system of dimensionless parameters which determine the motion in the whole and each state of the motion are supplemented only by two parameters $\frac{d}{l}$ and $\frac{vt}{l}$; here, for similarity of the two motions it is necessary to ensure the condition $\frac{d}{l}$ = const.; the constancy of the parameter $\frac{vt}{l}$ determines only the corresponding values of the time (the scale of time) for similar states of motions.

If the nonsteady motions represent certain oscillations with definite form and with frequency k which can be assigned arbitrarily, then the table of defining parameters is supplemented by the parameter k, in consequence of which the Strukhal number is added as the dimensionless parameter defining the regime of the motion:

Let us have nonsteady motions of a body in a fluid which represent certain progressive motions characterized by the velocity v, and the oscillatory motions with a definite form of oscillations, but possibly with different frequency k. For similarity of the various motions it is necessary to ensure the constancy of the Strukhal number if k, l and v are assigned earlier in the sense of the problem under consideration. If on the other hand the frequency k is a definite quantity, then the constancy of the Strukhal number is obtained as a consequence of the conditions of similarity which are composed of the assigned quantities. In a number of cases we encounter the study of nonsteady motion of a body in a fluid, where the motion of the motion is not known beforehand. As a similar problem let us consider the problem of the oscillations of an elastic wing in a progressive flow of a fluid (flutter of the wing).

Let there be placed in the flow of a fluid an elastic wing. For simplicity let us assume that the wing which possesses a longitudinal plane of symmetry be held

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

rigidly at the center section.

The elastic properties of the wings which are made of homogeneous material and which have completely definite construction and design are determined by two physical constants.

Let us linger awhile on the analysis of practical methods for establishing the characteristics of the elastic properties of the construction of wings employed in airplanes. A detailed and careful statement of the task permits one to characterize the elastic properties of the various constructions by certain functions and by parameters which are only essential from the point of view of the problem under consideration. In this sense we can isolate the classes of wings which are equivalent in elastic properties, but with different geometrical properties and having in general different construction.

In the general statement of the Hooke law all elastic characteristics of each wing made of different systems of geometrically similar wings equivalent with respect to properties of elasticity are determined by the value of the characteristic dimension l, the Young modulus E and the modulus of displacement G.

The distribution of masses can strongly influence the oscillations. The law governing the distribution of masses can be expressed so that the mass of each element be proportional to the total mass and be determined by the value of a certain mass m.

As is well known, the dynamic properties of an absolutely hard body depend only upon certain total characteristics of the distribution of masses. Namely, the dynamic properties of an absolutely hard body are completely determined by the values of the total mass of the body, by the position of the center of gravity and by the tensor of inertia for the center of gravity of the body.

In the case of a practical approximate statement of the problem a detailed consideration shows that the elastic wings with certain different distributions of masses, just as also in the case of an absolutely hard body, can be dynamically

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

equivalent.

In practice one establishes a system of abstract parameters which characterize the properties of the laws governing the distribution of masses, which determine the occurring oscillatory motions (the position of the center of gravity of the various cross sections of the wing, the moments of inertia of the cross sections, etc.). All the succeeding conclusions can be extended to the case of different but dynamically equivalent distributions of masses for an elastic wing.

Let us assume that in the phenomenon under consideration the fluid can be considered ideal, homogeneous and incompressible. In this case the mechanical properties of the fluid are completely determined by the density ρ. The property of ponderability of the fluid cannot be taken into consideration. Let us further assume that the fluid fills the entire space outside the wing. The middle cross section of the wing is immobile. At infinity the fluid moves progressively with constant velocity v in time, parallel to the longitudinal plane of symmetry of the wing; here, the velocity v possesses a fixed direction relative to the immobile cross section of the wing.

Summing up, we obtain that within the framework of the assumptions made the nonsteady motion of the system consisting of wing and fluid is determined by the following parameters:

$$l, \bar{z}, \bar{c}, m, \rho, v$$

and by the quantities assigning the initial disturbance. Moreover, each individual state of motion is determined by the moment of time t.

The properties of the disturbed nonsteady motions which are correct for all possible nonsteady motions arising during any sufficiently small initial disturbances, evidently, do not depend upon the parameters that characterize the initial disturbances.

Consequently, these properties for dynamically similar motions are determined by the following system of dimensionless parameters

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\frac{\rho v^2}{E}, \quad \frac{G}{E}, \quad \frac{m}{\rho l^3}, \quad \frac{vt}{l};$$

Here it is understood that the distribution of masses and the construction of the wings are identical in the various cases.

The latter circumstance can be considered in the extended sense also to require only the equivalences of the wings with respect to elastic and dynamic properties in the case of the conservation of geometrical similarity of the external surface of the wing which is adjacent to the fluid.

It is evident that the properties which characterize the motion in the whole and which are independent of the individual states of each motion are not dependent upon the parameter $\frac{vt}{l}$.

Experience indicates that the state of the steady circulation around an elastic wing can possess properties of stability and instability.

In the case of stable motion the amplitudes of displacements for the disturbed oscillations of a wing in a stream decrease. In the case of nonstable circulation there takes place the growth of the amplitudes, as a result of which the wing can be destroyed.

The stability and instability of circulation for sufficiently small disturbances can be considered as the properties which are not dependent upon the initial conditions of motion and upon the individual states of the motions. Therefore the properties of the stability of motions must be determined by the following parameters

$$\frac{\rho v^2}{E}, \quad \frac{G}{E}, \quad \frac{m}{\rho l^3}.$$

In the general statement of the problem it is impossible to divide the set of all motions into motions with increasing and with decreasing amplitude; cases are possible where during the oscillations the maximum slopes are either constant or variable, but in practice they preserve for any time t sufficiently small values.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

If the regimes of the motion with increasing and decreasing amplitudes are expressed explicitly, then the boundary between these regimes is characterized by the relation

$$F\left(\frac{\rho v^2}{E},\ \frac{G}{E},\ \frac{m}{\rho l^3}\right)=0,$$

which can be represented in the form

$$v_{cr}=\sqrt{\frac{E}{\rho}}\,f\left(\frac{G}{E},\ \frac{m}{\rho l^3}\right).$$

This formula determines the critical velocity of flutter. In the case of the variation of the velocity of the leading flow and constant values of the remaining parameters the value of the critical velocity separates the stable and the instable regimes of the circulation.

The coefficients of rigidity of the wing are proportional to the quantities E and G. The variation of E and G by n times is equivalent to the variation of the coefficients of rigidity by n times. From the obtained formula for v_{cr} it is seen that in the case of the conservation of mass, form and dimensions of the wing the critical velocity during the variation of the rigidity of the wing by n times varies by $\sqrt{n}$ times.

The Strukhal number $\frac{kl}{v_{cr}}$ corresponding to the critical velocity is determined by the values of the following abstract parameters

$$\frac{G}{E}\ \text{and}\ \frac{m}{\rho l^3},$$

by which are determined also all the abstract quantities that do not depend upon the initial data and that characterize the critical motion.

9. The Motion of a Ship

The problems of the theory of dimensions and similarity possess great practical value in various computations and especially in the experimental solution of many mechanical problems concerning sailing on the surface of water.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Let us consider the steady-state rectilinear progressive motion of a ship over the surface of a fluid which fills the entire lower half-space and which is at rest at a great depth and at remote distances ahead of the ship. The motion of the floating body causes the disturbance of the free surface. The disturbed motion of the fluid possesses a wave character which is due to the property of ponderability.

We shall consider the properties of inertia ρ, ponderability g, and viscosity of the water, which play a very important role. The property of compressibility of the water does not possess any practical value; therefore in the considered phenomenon the water can be considered incompressible. The property of capillarity also is nonessential for the motions of ordinary ships.

The dimensions and the form of the body of the ship exert a substantial influence upon the most important mechanical characteristics. Let us consider first the motion of the ship with a definite form of the body. All the geometrical dimensions are determined by the value of the length of the ship L. To different L correspond geometrical similar ship bodies. For ordinary ships of the heavy type one can consider that the total weight completely determines the position of the ship relative to the water. It is evident that the position of the ship relative to the water influences the resistance etc. Therefore the weight or the water displacement of the ship A we shall take as the defining parameter*.

In place of the water displacement A, which is expressed in tons, one can take the volumetric displacement D, which is expressed in cubic meters, since $A = \rho g D$, where ρ is the density of the water. For fresh water we have

$$\rho g = 1 \quad m/m^3.$$

The velocity of the ship we designate by the letter v. The system of defining

---

* If different motions occur for similar disposition of the ship body relative to the level of the water, then the weight is proportional to the cube L^3. In this case, in place of the two parameters L and A it is sufficient to take only one.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

parameters will be the following*:

$$L,\ D,\ \rho,\ g,\ \mu,\ v.$$

All of the geometrical and mechanical quantities, for example the wet area S**, the resistance W etc., can be considered as functions of these parameters. The dynamically similar motions and the regime of motion are determined by the following three dimensionless parameters:

$$\psi = \frac{L}{\sqrt[3]{D}} \quad \text{(coefficient of sharpness***)}$$

$$\mathsf{F} = \frac{v}{\sqrt{gL}} \quad \text{(Froude number)}$$

$$\mathsf{R} = \frac{\rho v L}{\mu} \quad \text{(Reynolds number)}$$

Thus, we can write for the resistance the following:

$$W = f(\psi,\ \mathsf{F},\ \mathsf{R})\,\rho S v^2. \tag{9.1}$$

The criteria of similarity are determined by the following relations

$$\frac{L_1}{\sqrt[3]{D_1}} = \frac{L_2}{\sqrt[3]{D_2}},\quad \frac{v_1}{\sqrt{gL_1}} = \frac{v_2}{\sqrt{gL_2}},\quad \frac{v_1 L_1 \rho_1}{\mu_1} = \frac{v_2 L_2 \rho_2}{\mu_2}.$$

It is not difficult to see that if for the model and for nature one and the

---

* The motion occurs on the surface of separation between the water and the air; therefore it would be necessary to indicate still further the density and the viscosity of the air as defining parameters (at ordinary velocities of motion the compressibility of the air is nonessential). However, these parameters turn out to be of small influence upon the phenomenon, and their consideration does not change the succeeding conclusions, since here the dimensionless quantities ρ'/ρ and μ'/μ are added, which can be considered constants for each class of motions.

** The quantity S differs but slightly from the magnitude of the wet area under the static state, which is determined only by two parameters L and D.

*** The coefficient of sharpness can be considered as the length of the geometrically similar ship's body having one ton of water displacement.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

same fluid is employed, then we have the ratios: $\frac{\mu_1}{\rho_1} = \frac{\mu_2}{\rho_2}$, and the modeling of the phenomenon is impossible. Indeed, from the constancy of the Froude number it follows that in the case of a decrease in the linear dimensions of the ship the velocity must be decreased, and from the constancy of the Reynolds number it follows that in the case of the decrease of the linear dimensions of the ship the velocity must be increased. Therefore, during modeling of this phenomenon with variation of the scale of length complete similarity is not observed, in consequence of which the magnitude of the coefficient of resistance of the model does not equal the magnitude of this coefficient for nature.

Determination of the resistance of the ship with the aid of tests with models is based upon the practical possibility of the separation of the resistance into two components: one defined by the property of viscosity, and the other defined by the property of ponderability. It turns out that the formula (9.1) can be approximately replaced by the following formula:

$$W = W_1 + W_2 = c_f(\mathrm{R})\frac{\rho S v^2}{2} + c_w(\psi, \mathrm{F})\rho g D. \qquad (9.2)$$

The form of the formula (9.2) can be based on certain qualitative considerations of a theoretical character, which will not be touched on here by us.

The resistance $W_1 = c_f \rho \frac{Sv^2}{2}$ is called the resistance of friction. For the model and for nature the resistance of friction is determined by calculation which is carried out with the aid of various semiempirical formulas. The value of the coefficient of friction is determined by the Reynolds number R. In addition, this coefficient depends upon the roughness and in a certain degree upon the form of the outline of the ship's body. With increase in the Reynolds number the coefficient of friction decreases. In practice the coefficient c_f is taken to be equal to the coefficient of friction of flate plates. The experimental data on the coefficient of friction of flate plates is presented in Fig.6.

For smooth plates the value of the coefficient c_f is determined in accordance

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

with the Prandtl-Schlichting formula

$$c_f = \frac{0{,}455}{(\log R)^{2{,}58}}. \qquad (9.3)$$

The value of the coefficient of friction c_f for a rough plate will be considerably larger.

The resistance W_2 is called the residual resistance. The coefficient $c_w = \frac{W_2}{\rho g D}$ gives the residual resistance per ton of the water displacement. This coefficient can be determined experimentally by means of tests with geometrically similar models together with observation of the following conditions:

$$\frac{L_1}{\sqrt[3]{D_1}} = \frac{L_2}{\sqrt[3]{D_2}} \quad \text{and} \quad \frac{v_1}{\sqrt{gL_1}} = \frac{v_2}{\sqrt{gL_2}}.$$

These conditions constitutes the law of similarity by Froude.

The residual resistance depends upon the shape of the ship's body. In an investigation of the influence of the shape of the body it is necessary to extend the class of motions and to study the motion of a family of bodies which is formed in accordance with a certain law in dependence upon geometrical parameters whose variation characterizes the geometrical peculiarities of the ship's outline under study.

For practical purposes it is very important to distinguish, from the infinite variety of parameters which characterize the geometrical properties of the shape of the body, such parameters as possess a determining value for the residual resistance. Experiments indicate that for all possible geometrically nonsimilar outlines of the bodies of ships of ordinary types the principal parameters that determine the coefficient c_w are the Froude number and the coefficient of tapering. In place of the Froude number with respect to length $F = \frac{v}{\sqrt{gL}}$ one can take the Froude number with respect to the volumetric water-displacement

$$F_D = \frac{v}{\sqrt{g\sqrt[3]{D}}} = F\sqrt[6]{\psi}.$$

In Fig.15 is represented the graph of Doyère*, in which is given the averaged

* Doyère, Theorie du navire, Paris, 1927.

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

experimental data on the dependence of the coefficient c_w upon Ψ and F_D for bodies without projecting parts (rudders, bracket arms for the axles of the screws etc.). With the aid of the Doyère graph and the value of the coefficient of friction as a

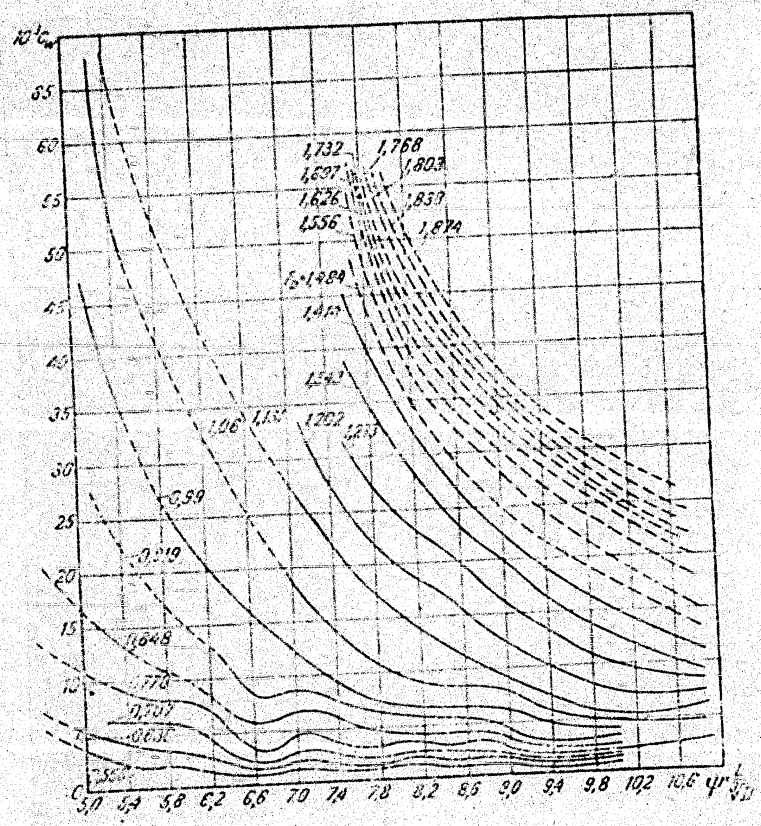


Fig.15 – The Residual Resistance per Ton of Water Displacement as a Function of the Coefficient of Tapering Ψ and the Froude Number F_D with Respect to Water Displacement.

function of the Reynolds number it is easy to calculate the resistance of the ship's body as a function of the velocity of motion. This calculation often gives in the first approximation very good results.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

For more accurate determination of the residual resistance it turns out that the following parameters* are most essential:

$$\chi = \frac{D}{Lx} \text{ AND } \frac{B}{T},$$

where x is the area of the midship, B is the width of the ship's body, and T is the setting or immersion. The coefficient χ is called the prismatic coefficient of volume or fullness.

Besides the indicated principal parameters we can take into account those parameters which determine the influence upon the resistance of various quantities that characterize the shape of the midship, prow, stern, etc.

In a study of the motion of ships we consider the tow resistance of a ship without screws W' and the resistance W'' in the presence of revolving screws, which cause a support (tug), under the action of which the motion occurs. In the last case the resistance W is equal, for uniform steady-state motion, to the horizontal component of the support (thrust) of the screws. The resistance W' is characteristic of the ship's body, which does not depend upon the property of the mover (engine). Thanks to the interaction between the body and the screw the following inequality: $W' < W''$ ordinarily holds.

If the body, the screw and its extension, the velocity of motion, the water displacement and all the dimensions are given, then these determine the resistance, and also the necessary number of turns of the screw n and the necessary power at the shaft of the screw E. In place of the velocity as the defining quantity one can take the power E or the number of turns n; in such a case the velocity becomes the quantity underlying the determination.

The product W''_2 determines the effective power expended in the propulsion of

---

* The experimental data on the influence of F, ω, χ and $\frac{B}{T}$ upon the residual resistance for a certain series of ship's bodies is contained in the book: Taylor, Speed and Power of Ships, Washington, 1933.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the ship. This power is always less than the power E developed at the shaft of the screw, since part of the power E is expended on the additional disturbances of the water by the screw during the creation of the support (thrust).

The ratio $\frac{W'v}{E} = \eta$, is called the propulsive coefficient. The quantity η characterizes the quality of the body, the quality of the screw and its operation in interaction with the ship's body. The best ships are characterized by enhanced values of the propulsive coefficient.

For ships with assigned shape of the outline, with assigned coefficient ψ, with defined screw and its defined extension relative to the body, the propulsive coefficient can be considered for constant ratio $\frac{d}{L}$ (d is the diameter of the screw) as a function of the Froude number and the Reynolds number or as a function of the step or progress of the screw $\frac{v}{nd}$ and the Reynolds number; in the case of small variations of the Reynolds number the influence of the Reynolds number is insignificant. When the shape of the ship's body and the geometrical data of the screw are varied, the value of η will depend upon the parameters that determine the shape of the body and the screw. In certain cases these influences are realized through the variation of the resistance of the body which can be considered independently of the operation of the screw; in other cases, through the characteristics of the screw which do not depend upon the shape of the ship's body. Finally, problems are encountered in which the value of the propulsive coefficient is connected with the interaction of the body and the screw.

In modern ship building there is observed the tendency toward the construction of large ships. Let us present certain simple considerations which speak well for this tendency. To the formula (9.2) one can ascribe a somewhat different form:

$$W = [c_f(R) + c'_w(\psi, F)]\rho\frac{Sv^2}{2}, \qquad (9.4)$$

where

$$c'_w = 2c_w\frac{D}{LS}\cdot\frac{1}{F^2}.$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Let us take two geometrically similar ships with water displacements that are proportional to the cube of the linear dimensions:

$$\frac{D_1}{L_1^3} = \frac{D_2}{L_2^3},$$

which is equivalent to a natural assumption concerning the similarity of the underwater portions. It is evident that the wet area S is proportional to the square of the linear dimensions:

Let L_1 and L_2 be the characteristic corresponding lengths, where $L_2 > L_1$. For one and the same velocity of motion we have the following inequalities:

$$R_2 > R_1 \text{ and } F_2 < F_1$$

From experience it is known that with increase in the Reynolds number the coefficient c_f decreases see Formula (9.3) and that with decrease in the Froude number the coefficient c_w' also decreases (at least in the range of small values of the Froude number $F < 0.5$ which are characteristic in practice). A typical dependence c_w' upon the Froude number is shown in Fig.16.

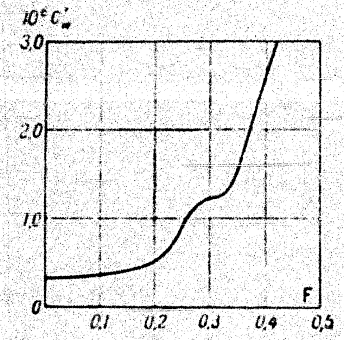


Fig.16 - A Typical Curve for the Coefficient of the Residual Resistance c_w' as a Function of the Froude Number **F**.

If the motion occurs with identical speed, then the ratio of the resistances $\frac{W_1}{W_2}$ is equal to the ratio of the powers or, in the case of identical coefficient of useful action, to the ratio of expenditures of fuel per unit time. The quantity of transported load is proportional to the water displacement, i.e. to the cube of the linear dimensions. The cost of the transportation of one ton is determined by the ratio of the weight of the fuel expended to the weight of the load transported. For identical velocity of motion the ratio of the costs Q_2 and Q_1 of transportation of one ton per distance of one kilo-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

meter is represented by the following formula

$$\frac{Q_2}{Q_1} = \frac{W_2 l_1}{W_1 l_2}.$$

The preceding ratio can be considered as one of the most important elements which characterize the efficiency of utilization. On the basis of the formula (9.4) we can write the following:

$$\frac{Q_2}{Q_1} = \frac{c_f(R_2) + c'_w(F_2)}{c_f(R_1) + c'_w(F_1)} \cdot \frac{l_1}{l_2} = \varkappa \cdot \frac{l_1}{l_2}, \qquad (9.5)$$

where $\varkappa$ is a certain quantity less than unity and which decreases with increase in the ratio $\frac{l_2}{l_1}$.

The eq. (9.5) shows that the quantity Q_2 decreases more rapidly than inversely proportionally to the increase in the dimensions of the ship. For identical velocity of motion the ratio of the powers grows more slowly than proportionally to the square of the linear dimensions.

By similar considerations we can show that if the power increases with increase in the dimensions proportionally to the cube of the linear dimensions, then the velocity increases, but the time of delivery and the cost of transportation per ton of load per one kilometer decrease.

The above presented considerations can refer not only to ships moving over the surface of the water, but also to airplanes, since the resistance of the air for fixed velocity of flight increases proportionally to the square of the linear dimensions, but the weight of the airplane and the useful load grow approximately proportionally to the cube of the linear dimensions. In this connection the relative reserve of fuel and the duration of flight of the airplanes grow with their dimensions. This explains the increase in the dimensions and the weight of airplanes intended for distant flights.

Besides these matters, there is great practical interest in the problem concerning the rational (most efficient) dimensions of aircraft engines, hydraulic machines, etc.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The ratio of the weight of reactive engines (jet engines) to the thrust developed by them ("specific weight") is the most important characteristic of an engine from the view point of its applications in airplanes.

In order to obtain a given thrust it is most convenient to use several engines with less dimensions than one engine of larger dimensions, since with increase in the dimensions of the engine its specific weight grows; therefore the thrust grows proportionally to the square of the linear dimensions, but the weight of the engine is proportional approximately to the cube of the linear dimensions.

Thus, if one keeps in mind the specific weight of engines, the expenditure of critical materials, and productional operations, then it is more advantageous to construct several small engines than one large engine. For engines of very small dimensions the indicated considerations lose their virtue, since with sharp decrease in the dimensions there is loss of mechanical similarity; here, the thrust and useful power decrease very rapidly.

Besides these general qualitative considerations of the theory of similarity, the choice of the advantageous dimensions of engines and of hydraulic machines is connected also with certain limitations, with properties and dimensions of auxiliary mechanisms of automatics, with the analysis of economic, technological, constructional and certain other requirements which must be taken into consideration for the obtaining of final conclusions.

The problem of the rational (most efficient) dimensions of aviational engines, of hydraulic turbines and of many other machines must be analyzed and studied from all sides. In this analysis the considerations of similarity also possess the most important value.

10. Gliding on the Surface of Water

Planing represents a skimming or slipping on the surface of water. During planing the supporting force is almost entirely due to the dynamic reaction of the water. During the motion of water-displacing boats the supporting force, just as

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

at rest, represents the Archimedean force which is due to the increase of the hydrostatic pressure at a depth.

The principle of planing is advantageous for rapidly traveling boats. The modern-day speedy torpedo cutters are planers. The run of hydroplanes in water during take off and the run after settling down are accompanied by planing.

The phenomenon of planing of a boat with given shape can occur for various orientations of the boat relative to the water's surface. The orientation of the boat relative to the water possesses essential significance.

The number of parameters that determine the motion of a hydroplane or a seaplane with given geometrical shapes is greater than in the above considered case of the motion of water-displacing ships. During planing, besides the draft or the moistened area, it is also necessary to give the trim angle θ (the angle with the horizon of a certain fixed direction in the boat). In place of the draft in the water and the trim angle θ one can give the load on the water Δ, the position of the center of gravity of the boat and the moment relative to the center of gravity of the external forces, but not hydrodynamic forces, but, for example, aerodynamic forces. In practice it is convenient to select as the determining quantities the load on the water and the angle of the different.

Leaving the general statement of the problem of planing just as in the case of the motion of a ship, we proceed to the conclusion that the phenomenon of steady-state planing of a boat with given geometrical shape can be determined by the following system of parameters:

$$B,\ \Delta,\ \theta,\ v,\ \rho,\ g,\ \mu.$$

The set of dynamically similar motions and all of the dimensionless combinations that are formed from the various mechanical quantities are determined by the values of the following dimensionless parameters:

$$\theta,\quad \frac{\Delta}{\rho g B^3} = C_\Delta,\quad \frac{vB\rho}{\mu} = \mathrm{R},\quad \frac{v}{\sqrt{gB}} = \mathrm{F}. \qquad (10.1)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In comparison with the motion of water-displacing ships it has been necessary to introduce into the consideration an additional angle of travel θ, which can possess various values for comparable motions. For water-displacing ships a difference in the angles of travel for motions whose comparison possesses practical interest can be considered nonessential.

The property of ponderability of the water is told through the means of parameters containing the acceleration of gravity g. In the system (10.1) there are two parameters containing g, namely C_Δ and F. This system can be replaced by the following system

$$\theta, C_B = \frac{2\Delta}{\rho B^2 v^2} = \frac{2C_\Delta}{F^2}, \quad R, F, \tag{10.2}$$

in which the acceleration of gravity g enters only through the Froude number $F = v/\sqrt{gB}$.

The phenomenon of planing bears an explicitly expressed shock character. Ahead of the gliding boat the water is practically at rest; afterwards in a short interval of time the water is brought into motion by the moving bottom of the boat. This gives the reason for the assumption that the inertial forces are the main forces, in comparison with which the forces of the weight of the particles of water are small and can be disregarded.

The assumption concerning the nonessentialness of the weight property of the particles of water is equivalent to the assumption of the nonessentialness of the parameter g, and consequently also the Froude number in the system of eqs. θ, C_B, R and F.

For a regime of planing (great numbers of the Froude number) the nonessentialness of the influence of the weight property of the water on a number of the principal characteristics of motion has been established theoretically*. Much experi-

---

* L.I.Sedov, Trudy Konferentsii po Teorii Volnovogo Soprotivleniya (Works of the Conference on the Theory of Wave Resistance), pages 7-30, published (cont'd)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

mental data** also confirms well the correctness of this position. Therefore, in the modeling of the regimes of pure planing we can depart from Froude's law of similarity.

It is not difficult to see that in the system of parameters θ, C_Δ, R and F in the case of the absence of the influence of the weight property of the fluid the conclusion concerning the simultaneous nonessentialness of the Froude number F and of the coefficient C_Δ which contain the acceleration of gravity g is untrue. The parameters C_Δ and F form the combination $\frac{2C_\Delta}{F^2} = C_B$, which does not contain g. The parameter C_B can possess important significance in a number of problems in which the weight property of the water and the parameter g are completely nonessential.

The dimensionless quantities that do not depend upon the Froude number can depend upon the load Δ and upon the velocity v only through the combination $\frac{2\Delta}{\rho B^2 v^2} = C_B$. Thanks to this, the investigation of the influence of the velocity can be replaced by the investigation of the influence of the load and vice versa. This circumstance possesses great significance when the intended velocities are insufficient.

Making use of this, we can obtain by an experimental method a number of results which are correct for regimes not accessible to direct experiment.

The property of viscosity of the water is noticeable only in the immediate neighborhood of the bottom of the boat (the boundary layer); therefore the Reynolds number does not influence essentially the distribution of pressure, the moment of hydrodynamic forces, the shape of the wetted surface, etc. The influence of the

---

* (cont'd) by TsAGI (Central Aero-Hydrodynamics Institute), Moscow, 1937; N.Ye. Kochin, Trudy TsAGI (Works of Central Aero-Hydrodynamics Institute), No.356, 1938; Yu.S.Chaplygin, Trudy TsAGI, No.508, 1940.

** L.A.Epshteyn, Trudy TsAGI, No.508, 1940; L.I.Sedov, and A.P.Vladimirov, Doklady Akademii Nauk SSSR (Reports of the Academy of Sciences of the USSR), Vol.33, No.2 and 3, 1941.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

viscosity of the water upon the damping of disturbances is noticeable practically only at very remote distances from the boat.

The force of the resistance depends essentially upon the forces of friction at the surface of the boat's bottom; therefore the viscosity and the Reynolds number influence the dimensionless coefficients of the forces of resistance.

The hydrodynamic characteristics of steady-state planing depend, besides on the indicated mechanical parameters, to a strong degree upon the geometrical shape of the boat's bottom.

The difference in nature of the supporting force causes a sharp difference in the shape of planing ships from the shape of water-displacing ships. The boat outlines of planing ships are characterized by the flat-bottom shape of the bottom, by sharply emphasized cheeks and by the presence of radons representing cross-sectional steps at the bottom of the boat. The flat-bottom shape is necessary for the creation of large vertical forces for small wet surface. The sharp cheeks and radon cause, during planing, a disruption in the jets of water, in consequence of which the lateral surface of the boat and a certain considerable portion of the lower part of the bottom does is not wet by the water, which decreases the resistance of friction.

The various peculiarities of the bottom can be characterized by certain dimensionless parameters. The influence of the variation of these parameters can be determined by way of systematic tests of a series of profiles.

In the problem of the planing of a plate having the shape of flat wedge, we come up against a very interesting fact whose essence is closely connected with mechanical similarity and analysis of dimensions. Let us have a flat-keel prismatic plate gliding along the surface of the water. Let the longitudinal plane of symmetry passing through the keel of the plate be vertical and let the motion occur parallel to the plane of symmetry. The rear end of the plate - the step (literally: trench) - represents a plane perpendicular to the plane of symmetry. Let us

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

consider the case where the length of the plate and the width of the side piece of the wedge are sufficiently large so that for all comparable motions the boundaries of the wet surface are connected in no way with the constructive width and length of the plate. The geometrical width and length of the plate for all comparable motions can be taken equal to infinity. The geometrical shape of the plate is completely determined by the angle between the side pieces $\pi - 2\beta$ (β is the keel angle) between the keel line and the plane of the face. These angles can be taken for the geometric parameters of the shape. For simplicity we shall consider the class of motions in which these angles are fixed.

It is not difficult to see that in this case the number of defining parameters is shortened, since the linear dimension characterizing the plate is absent.

The steady-state planing of a keel plate on an incompletely moistened edge is determined by the following parameters

$$\Delta,\ \theta,\ v,\ \rho,\ g,\ \mu.$$

The moistened edge of the plate along the step and the moistened length along the keel are completely determined by the indicated parameters.

The class of similar motions and the regime of motion are characterized by the following three dimensionless quantities:

$$\theta,\quad \frac{v}{\sqrt{g\sqrt[3]{\frac{\Delta}{\rho g}}}} = F_1,\quad \frac{\rho v\sqrt[3]{\frac{\Delta}{\rho g}}}{\mu} = R_1. \qquad (10.3)$$

The numbers F_1 and R_1 can be considered as the Froude number and the Reynolds number with respect to the load.

The Reynolds number for the regimes of planing (greater than F_1) influences only the quantities which depend upon the circumstances governing the motion of the fluid in the boundary layer. In particular, one can assume that the moistened length along the keel l does not depend upon the property of viscosity, that is

$$l = f(\theta, \Delta, v, \rho, g). \qquad (10.4)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In the dimensionless form this relation can be represented thus:

$$l = \sqrt{\frac{\Delta}{\gamma b^3}}\, f(\vartheta, F_1). \qquad (10.5)$$

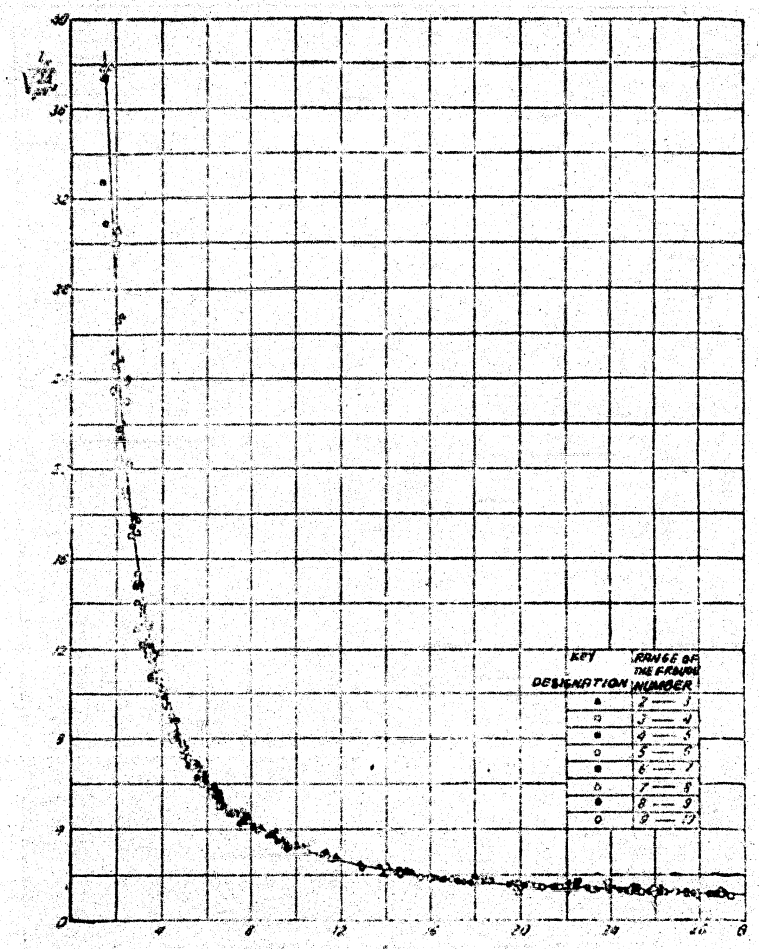


Fig.17 – Planing of Keel Plates

If one assumes still further that the property of weight does not influence the quantity l, i.e. that the acceleration g in the relation (10.5) is nonessential,

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

then we obtain the following:

$$l = \sqrt{\frac{\Delta}{\rho b^2}} \cdot f(\Phi). \qquad (10.6)$$

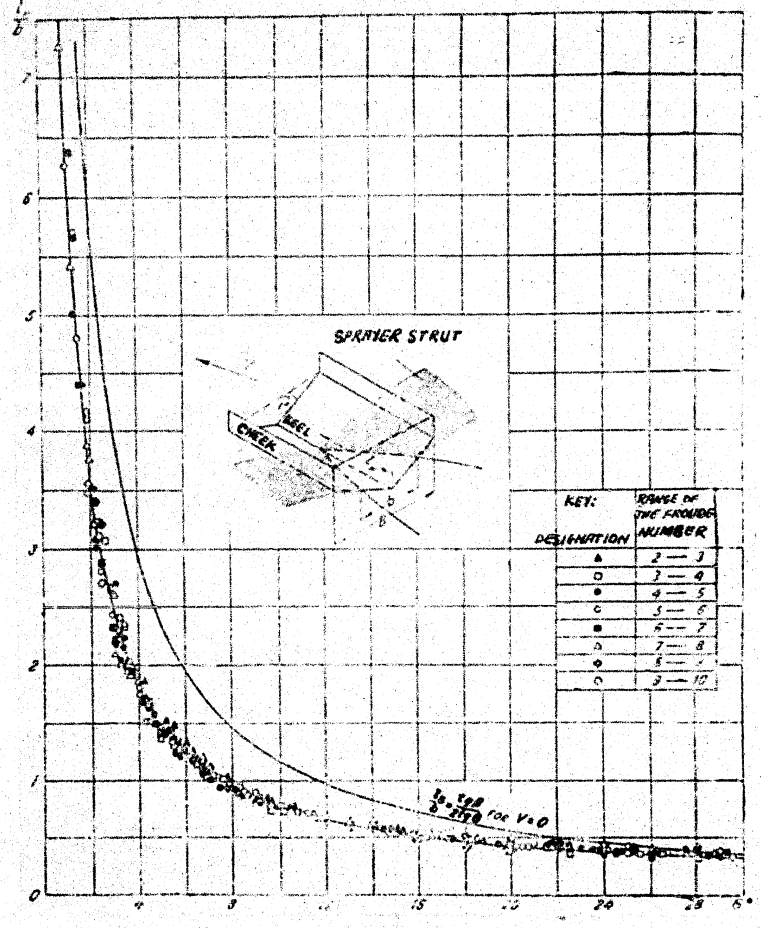


Fig.18 - The Experimental Data Shows That the Influence of the Weight of the Water is Nonessential for Froude Numbers $F_1 > 2$.

Under these assumptions it is evident that all of the linear dimensions (for

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

example, the moistened edge on the step etc.) are proportional to $\sqrt{\Delta/\rho v^2}$. For a fixed trim angle θ, but for different loads on the water Δ and for different velocities of gliding v the moistened surfaces will be geometrically similar.

Experiments confirm well the correctness of these conclusions for large values of the Froude number $F_1 > 2$* (see Figs.17, 18, and 19).

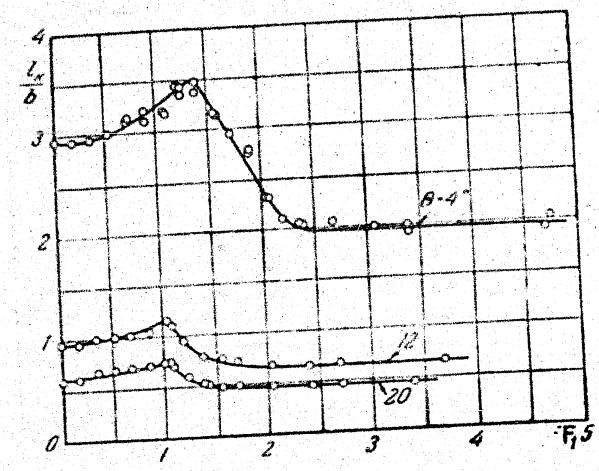


Fig.19 - Planing of Keel Plates. The Influence of the Weight is Essential Only for Froude Numbers Less Than 2: $F_1 < 2$.

In the system of determining parameters we have taken as the essential parameters the load on the water l and the velocity of motion v, since in the experiments it is namely these quantities that are assigned beforehand, and the remaining quantities are measured. In place of the load Δ one can take as the determining quantity the moistened length l or the moistened edge on the step b. For example, one can take the following system of dimensional determining parameters:

$$\theta,\ l,\ v,\ \rho,\ g,\ \mu$$

---

* Sedov L.I. and Vladimirov A.M., Doklady Akademii Nauk SSSR (Reports of the Academy of Sciences of the USSR), v.33, No.2, 1941.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

and the corresponding system of dimensionless parameters

$$0, \quad \frac{c}{\sqrt{gl}}, \quad \frac{\rho g l}{p}.$$

In this case the relations (10.4) and (10.6) can be considered as the relations that determine the load on the water Δ.

11. Impact on Water

In a number of practical problems, in particular in the case of the landing of seaplanes on water it becomes necessary to cope with the phenomenon of impact on water. In the study of this phenomenon particular interest is attached to clarification of the properties of the reaction force of water and investigation of the phenomenon of water ricochets.

Many have observed how flat stones, to which have been imparted velocity with a strong horizontal component and rotation sufficient to maintain slight inclination of the plane of the stone to horizontal, upon contact with the water skip off the water, sometimes several times. It is obvious that in the phenomenon of such a water ricochet, horizontal velocity plays a fundamental part. In the absence of horizontal velocity a flat, heavy stone can not skip off of water. Multiple ricocheting indicates a small loss of horizontal velocity at the time of contact with the water. Ricocheting of projectiles is also a well-known phenomenon. Thus, for example a circular projectile with a diameter of 0.16 meter with an initial velocity of 455 m/sec can make more than 22 ricochets on water*.

At the present time, in the artillery ricochet firing is sometimes done intentionally.

The phenomenon of water ricochet can take place during the landing of seaplanes on water. Water ricocheting of airplanes is dangerous in practice and is regarded as a highly undesirable occurrence.

---

* Jonquières, Compt. Rend. 1883, No.23, pp.1278-1281

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

We shall examine, with the help of the theory of dimensions and similarity, the problem of impact on water with application to the landing of seaplanes and to experiments with plates - profile models of ships and airplane floats.

Let us establish the geometric form of the body in motion. We shall make the scale, which can be given by the value of some linear dimension, variable. As a characteristic linear dimension it is natural to take the beam of a ship. For simplicity, we shall be limited by examination of the case when the body can be considered absolutely solid. Distinction in the aerodynamic system can be strongly reflected in the course of the entire phenomenon. The experimental investigation of the problem can be begun with an examination of the motion of bodies with identical geometric form and, consequently, with identical aerodynamic properties.

We shall assume that the body has a longitudinal plane of geometrical and dynamic symmetry, whereby motion of the body takes place in a longitudinal direction parallel to the plane of symmetry. Further, we shall assume that nonsteady-state motion of the body takes place with two degrees of freedom: vertical movements of the center of gravity and rotation around the center of gravity can take place. We shall consider the horizontal velocity of the center of gravity as constant. In reality, as a result of resistance forces there occurs a decrease of horizontal velocity; however, in view of the brevity of the most interesting processes it is possible to disregard this decrease*.

---

* Constancy of horizontal velocity is not essential for applying dimension theory to the general statement of the problem of impact on water. During the modeling of this phenomenon there arises the necessity of fixing the horizontal velocity. Under laboratory conditions constancy of horizontal velocity can be secured by movement of the towing carriage; the center of gravity of the model can slide along a vertical runner fastened to the carriage which is moved at a given constant velocity. The friction forces which arise in the driving apparatus can be made negligible.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In the general statement of the problem we shall take into account the properties of inertia, viscosity and weight (literally: ponderability) of the fluid. Compressibility and capillarity of the fluid we shall disregard. The wave motion of water can exert a substantial effect on the phenomenon under consideration; however, we shall assume that the water is smooth until contact with the body.

From the foregoing considerations it follows that the motion of the system, body-water, is defined by the following system of parameters which, within known limits, can be set arbitrarily:

I. A parameter which defines the scale: the beam B of the ship (in the case of a model, the width of the plate).

II. Kinematic parameters: the moment of time t under examination (the initial moment of time t = 0 corresponds to the moment of the body's contact with the surface of the water); horizontal velocity U and the initial vertical velocity v_0; the initial trim angle (angle of attack) θ_0; the initial angular velocity Ω_0*.

III. The dynamic parameters of the body: the coordinates of the center of gravity ζ, η in some system of coordinates connected with the body; the moment of inertia J relative to the transverse axis passing through the center of gravity; mass m; the vertical component A of the given external forces (for a free body, A = mg). In laboratory experiments, with the help of artificial equilibration the quantities A and mg can be made independent**.

IV. Physical constants: the acceleration of gravity g; the density ρ and viscosity μ of water.

---

* We assume that the initial state of motion at the moment of landing is, practically, fully defined by the parameters

$$U,\ v_0,\ \theta_0 \text{ and } \Omega_0.$$

** We assume that the supplementarily assigned external forces are applied at the center of gravity. The gross aerodynamic forces and moments can be considered quantities determined by the form and motion of the body.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

All the dimensionless quantities which we can set up in connection with the problem under study are functions of the following system of dimensionless parameters which define the system of motion and state of motion:

$$\tau = \frac{Ut}{B}, \quad \frac{|r_0}{U}, \quad \theta_0, \quad \frac{\omega_0 B}{U}, \quad \frac{\xi}{B}, \quad \frac{\tau_1}{B}, \quad \frac{J}{\rho B^5}, \quad \frac{m}{\rho B^3},$$

$$C_B = \frac{2A}{\rho B^2 U^2}, \quad F = \frac{U}{\sqrt{gB}}, \quad R = \frac{\rho U B}{\mu}.$$

The effect of the Reynolds number R can be shown by way of friction forces conditioned by the viscosity of water, which (forces) are generally small by comparison with lift force and are usually close to horizontal in direction. However, in some cases friction forces can noticeably affect the magnitude of the rotation moment. But if we consider the comparatively weak dependence of friction forces on the Reynolds number, apparently it is entirely legitimate to disregard the effect of the Reynolds number on the characteristics of vertical and angular motion and, in particular, on the phenomenon of water ricochet.

The effect of Froude's number F on hydrodynamic forces, the form of the water-contact surface and other factors is connected with the effect of the weight property of water on the turbulent motion of the water close to the body. In the case of high horizontal velocity the phenomenon bears the character of impact. Hence the forces of the reaction of water can be considered independent of Froude's number. It is nevertheless necessary to keep in mind that a sufficiently large value of Froude's number, starting with which the effect of this number is negligible, depends on the character of the mechanical quantity being examined and is connected with the values of the other defining parameters.

If A = mg, then the parameters $\frac{m}{\rho B^3}$, C_B and F become dependent. In this case it is sufficient to retain only two parameters, $\frac{m}{\rho B^3}$ and C_B, since through the coefficient of C_B there is simultaneous consideration of the weight of the model and water. We shall use Y to designate the vertical component of the total hydrodynamic

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

force. From the foregoing, the validity of the formula

$$\frac{Y}{A} = f\left(\tau, \frac{v_0}{U}, \vartheta_0, \frac{\Omega_0 B}{U}, \frac{\xi}{B}, \frac{\eta}{B}, \frac{J}{\rho B^5}, \frac{m}{\rho B^3}, C_B\right). \quad (11.1)$$

follows.

In the case of the motion of a concrete model, the parameters $\frac{\xi}{B}$, $\frac{\eta}{B}$, $\frac{J}{\rho B^5}$ and $\frac{m}{\rho B^3}$ are constant. If the trim angle is fixed and only the vertical velocity is varied, then we have motion only with one degree of freedom - vertical progressive motion. In this case $\Omega_0 = 0$ and the parameters ξ, η and J are negligible. The formula (11.1) takes the form

$$\frac{Y}{A} = f\left(\tau, \frac{v_0}{U}, \vartheta, \frac{m}{\rho B^3}, C_B\right). \quad (11.2)$$

In this formula the trim angle is preserved as the constant angle of the set-up, which can be different in different experiments.

The parameter τ is determined, if we will examine in differing experiments the maximum magnitude of the ratio $\frac{Y}{A}$ or the value generally of $\frac{Y}{A}$ for certain characteristic moments of time:

$$\frac{Y_{max}}{A} = f^*\left(\frac{v_0}{U}, \vartheta, \frac{m}{\rho B^3}, C_B\right). \quad (11.3)$$

It is obvious that during ricochet the mean values of all the quantities at the time of contact with the water, the dimensionless time of contact, $\tau_1 = \frac{Ut_1}{B}$, the dimensionless maximum depth of submersion, $\frac{h_{max}}{B}$, and others, are not connected with the parameter τ.

The entire aggregate of the distinguished class of motions can be divided into two parts corresponding to the presence and absence of skipping off the water (the presence or absence of ricochets). The boundary between these two regimes is characterized by relationship

$$\Phi\left(\frac{v_0}{U}, \vartheta_0, \frac{\Omega_0 B}{U}, \frac{\xi}{B}, \frac{\eta}{B}, \frac{J}{\rho B^5}, \frac{m}{\rho B^3}, C_B\right) = 0, \quad (11.4)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

and in the case of progressive motion with one degree of freedom, by the relationship

$$\Phi\left(\frac{v_0}{U},\ \theta,\ \frac{m}{\rho B^3},\ C_B\right)=0. \tag{11.5}$$

At the present time there are experimental data* on the form of the function Φ for the landing of a flat plate on water.

While examining the problem of the steady planing of a keel plate, we explained the possibility of reducing the number of defining parameters in the case when the water-contact surface does not depend on the dimensions of the plate, as a result of which the parameter B is excluded.

In the problem of impact of a wedge on water (at a fixed angle), the form of which was described in the preceding section, in place of formulas (11.3) and (11.5) the following formulas are valid:

$$\frac{Y_{\max}}{A}=f^*\left(\frac{v_0}{U},\ \theta,\ \frac{2A}{\rho^{1/3}m^{2/3}U^2}\right) \tag{11.6}$$

and

$$\Phi\left(\frac{v_0}{U},\ \theta,\ \frac{2A}{\rho^{1/3}m^{2/3}U^2}\right)=0. \tag{11.7}$$

In place of the parameters $\frac{v_0}{U}$ and $\frac{2A}{\rho^{1/3}m^{2/3}U^2}$ it is possible to employ the equivalent parameters $\frac{v_0}{\sqrt{\frac{A}{m}\sqrt[3]{\frac{m}{\rho}}}}$ and $\frac{U}{\sqrt{\frac{A}{m}\sqrt[3]{\frac{m}{\rho}}}}$ which have a symmetrical form and, in the case of the constants A and m, characterize distinctly the effect of vertical and horizontal velocities.

We noted above the system of defining parameters and the form of certain important relationships. Some of the indicated parameters are negligible in a number of cases; this is explained by way of special investigations which go beyond the limits of the theory of dimensions and similarity.

The phenomenon of ricocheting on the surface of water is closely connected with

---

* Sedov, L.I., DAN, Vol.37, No.9, 1942.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the phenomenon of longitudinal instability of planing. For seaplanes and hydroplanes and in experiments with models, we encounter the phenomenon of instability of planing. At the present time it is well known that for each seaplane and for each model there are unstable cycles of motion. In the case of these cycles, there arise strong longitudinal oscillations which are extremely troublesome and dangerous. Just as in the problem of landing on the water, investigation of the phenomenon of instability in planing is complicated by a large number of parameters the effect of which needs to be clarified.

It is not difficult to discern that a system of dimensionless parameters which define the stability of planing, is obtained from the system of parameters defining the phenomenon of impact on water, if we make $v_0 = \Omega_0 = 0$. To the concept of the boundary of ricocheting there corresponds an analogous concept concerning the boundary of stability separating the stable and unstable cycles of planing.

If the effect of the weight of water is lacking then the boundaries of stability depend upon the load and upon the planing velocity only by way of the coefficient of $C_B = \frac{2A}{\rho B^2 U^2}$. This is well confirmed by experimental data* for a number of practically significant cycles.

Just as in the problem of impact on water, the number of parameters which define the planing stability of flat-bottomed plates on part of the beam is reduced**.

We shall examine again, especially, the problem of vertical descent onto water.

The phenomenon of impact on water in the case of vertical descent, when the body moves progressively***, is defined by the parameters

$$t,\ v_0,\ m,\ B,\ A,\ g \text{ and } \rho.$$

---

* Sedov L.I., Tekhnika vozdushnogo flota. No.4, 1940.

** Sedov L.I., Vladimirov A.N., Izvestiya AN SSSR, Otdeleniye Tekhnicheskikh Nauk, No.1, 1943.

*** If the body is nonsymmetrical, then vertical, progressive motion can be accomplished with the help of special runners.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

As dimensionless parameters defining the system and state of motion, we conceive the following four quantities:

$$\frac{tv_0}{\sqrt[3]{\frac{m}{\rho}}}, \quad \frac{m}{\rho B^3}, \quad \frac{v_0}{\sqrt{g\sqrt[3]{\frac{m}{\rho}}}}, \quad \frac{v_0}{\sqrt{\frac{A}{m}\sqrt[3]{\frac{m}{\rho}}}}. \tag{11.8}$$

If $A = mg$, then only three parameters remain.

It is obvious that the dimensionless quantities being studied, taken for some characteristic moments of time (maximum values or mean values over a period of time), are defined by only the following two parameters at $A = mg$:

$$\frac{m}{\rho B^3}, \quad \frac{v_0}{\sqrt{g\sqrt[3]{\frac{m}{\rho}}}}.$$

For example, for maximum impact force and for an impulse which affects the body from the water side over a certain characteristic interval of time, formulas in the following form are valid:

$$P_{\max} = f_1\left(\frac{m}{\rho B^3}, \frac{v_0}{\sqrt{g\sqrt[3]{\frac{m}{\rho}}}}\right)\rho\left(\frac{m}{\rho}\right)^{2/3} v_0^2, \tag{11.9}$$

$$I = f_2\left(\frac{m}{\rho B^3}, \frac{v_0}{\sqrt{g\sqrt[3]{\frac{m}{\rho}}}}\right) mv_0. \tag{11.10}$$

If the contact velocity v_0 is large and the form of the wetted surface of the body is close to a horizontal plane, then the phenomenon of submersion in water bears a sharply expressed impact character. In this case the weight property of the water and the weight of the body are unessential. Hence, in the case of descent onto water, when the quantity

$$\frac{v_0}{\sqrt{g\sqrt[3]{\frac{m}{\rho}}}}$$

is sufficiently large, the following formulas must be valid:

$$P_{\max} = f_1\left(\frac{m}{\rho B^3}\right)\rho^{1/3}m^{2/3}v_0^2, \tag{11.11}$$

$$I = f_2\left(\frac{m}{\rho B^3}\right) mv_0. \tag{11.12}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In the case of constant v_0 and m, it is obvious that with an increase in the dimensions of the body (parameter B), the reaction of the fluid is increased. Hence, the functions $f_1\left(\frac{m}{\rho B^3}\right)$ and $f_2\left(\frac{m}{\rho B^3}\right)$ must increase when the parameter tends toward zero. Formulas (11.11) and (11.12) demonstrate that the maximum force is proportional to the square of the velocity, and the impulse is proportional to the first power of the descent velocity*.

The parameter $\frac{m}{\rho B^3}$ is excluded in the problem of the descent of a cone onto a water surface (cutting of the cone by a horizontal plane can be at any point), since in this case there is no assigned characteristic linear dimension.

Thus, in the problem of the descent of a cone onto water, in place of formulas (11.11) and (11.12) we obtain:

$$P_{max} = c_1 \rho^{1/3} m^{2/3} v_0^2, \tag{11.13}$$

$$I = c_2 m v_0. \tag{11.14}$$

These formulas define the relationship of P_{max} and I to mass. The constants c_1 and c_2 depend on the form of the cone. It is interesting to note that in formulas (11.13) and (11.14) the law of the effect of mass does not depend on the form of the cone. The form of the cone affects only the values of the constants c_1 and c_2.

However, a general conclusion that the maximum force is proportional to $m^{2/3}$ for bodies of any form, generally speaking, would be invalid. Likewise, let us examine the descent onto water of a long flat wedge with a small keel angle. Assume that the plane of symmetry of the wedge is vertical and the velocity of descent is large. By disregarding the weight property of the water and the wedge, we obtain the following system of defining parameters:

$$t, v_0, \frac{m}{L} = m_1, L, \rho,$$

---

* The experimental data confirm this conclusion; see Kreps R.L., Trudy TsAGI, No.438 and Kreps R.L., Trudy TsAGI, No.513. (TsAGI - Central Aerohydrodynamics Institute).

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

where L is the length of the wedge along the keel, m_1 is the mass of the wedge per unit length.

If L is very large then we can examine the limit case $L = \infty$. In the limit case of a flat, infinitely long wedge, when the wet surface of the wedge does not reach the edges of the wedge, the linear dimension is excluded, owing to which the phenomenon is defined only by the four dimensional quantities:

$$t,\ v_0,\ m_1,\ \rho.$$

All of the dimensionless characteristics are defined by the one dimensionless quantity

$$\frac{tv_0}{\sqrt{\frac{m}{\rho L}}}$$

The maximum and mean values of the dimensionless mechanical characteristics will be dimensionless constants.

Thus, in this case the following formula is valid for the maximum force of impact

$$\frac{P_{\max}}{L} = c_3\sqrt{\rho m_1 v_0^2},$$

or

$$P_{\max} = c_3\sqrt{\rho m L}\, v_0^2.$$

Hence, for extremely elongated bodies the maximum force is proportional to the square root of the mass of the body. The constant c_3 depends on the heeling factor; it can be viewed as a function of the careening angle β.

It is not difficult to see that c_3 increases as the careening angle decreases. For small careening angles it is possible to make $c_3 = \frac{c_4}{\beta}$, where c_4 at small β can be considered as not dependent on the careening angle. In the general case of a finite wedge the following formula can be given for c_3:

$$c_3 = \frac{f\left(\frac{m}{\rho L^3}, \beta\right)}{\beta}$$

It is convenient to treat the results of the experiment, by determining the function $f\left(\frac{m}{\rho L^3}, \beta\right)$, since at small and for very elongated wedges or small m this

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

function will vary slightly dependent upon its arguments.

12. Submersion of a Cone and a Wedge at Constant Velocity in a Fluid

We will consider the problem of nonsteady-state motion of an uncompressible fluid caused by submersion into the fluid of a solid body having the shape of a cone or wedge. The shape of the cone in the case of the three-dimensional problem and the shape of the flat wedge of infinite span in the case of the plane problem are interesting in that their surface is fixed completely by the one requirement of geometrical similarity. The aggregate of geometrically similar cones come to one single cone. The surface of the cone and the surface of the flat wedge are determined completely by dimensionless geometrical quantities.

We will assume that the fluid occupies an entire lower semispace bounded by a horizontal plane and that we can neglect the weight property and viscosity of the fluid. Therefore we will consider that the fluid is incompressible, homogenous, and ideal.

Let $t = 0$ be the initial moment of time in which originates the contact by the body with the fluid at rest. The body, submerging into the fluid, achieves forward motion at velocity v, constant in value and in direction.

We will assume that on the free surface, pressure p has a constant value p_o. From the incompressibility of the fluid it follows that the value p_o on the free surface cannot influence the turbulent motion of the fluid. Instead of pressure p we can consider the difference $p - p_o$; in this case the parameter p_o is immaterial. Therefore the mechanical properties of the fluid are determined by a single parameter - by density ρ. From what I have said, it follows that all the mechanical characteristics of the motion of the fluid at each point are determined by the quantities

$$\rho, t, v, \alpha, \beta, x, y, z$$

where α and β are angles determining the direction of velocity relative to the body, and x, y, and z are coordinates of the point under consideration, either in a

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

stationary system of coordinates, the origin of which coincides with the point of contact of the point of the cone with the surface of the fluid, or in a moving system of coordinates which is fixed to the body, and whose origin coincides with the point of the cone. It is obvious that all the dimensionless quantities which can be connected with the phenomenon under consideration are determined by the parameters

$$\alpha, \beta, \frac{x}{vt}, \frac{y}{vt}, \frac{z}{vt},$$

whereupon the parameters $\frac{x}{vt}, \frac{y}{vt}, \frac{z}{vt}$ can affect only the quantities depending upon the position within the fluid of the point under consideration. The overall dimensionless parameters (for example, the total force of reaction of the fluid etc) or the parameters independent of the position of the point in space, depend only on the angles α and β. If the direction of the velocity is fixed (for example, the velocity is vertical) then we can consider all the dimensionless overall characteristics as absolute constants, depending only on the shape of the cone.

We will designate by $\varphi(x, y, z, t)$ the potential of the velocities of the turbulent motion of a fluid for the case when the velocity of the cone is given according to the magnitude and direction. In view of the fact that the motion of the fluid is nonsteady-state motion, the problem of determining the turbulent motion of the fluid reduces to the determination of the potential of velocities as a function of four independent variables x, y, z, t.

On the basis of the theory of dimensions it is easy to reduce the four independent variables to three. Indeed, the quantity $\frac{\varphi}{v^2 t}$ is dimensionless, therefore the formula of the following form is valid:

$$\varphi = v^2 t f\left(\frac{x}{vt}, \frac{y}{vt}, \frac{z}{vt}\right). \tag{12.1}$$

For the velocity of particle of the fluid , there is a formula having the form

$$\mathfrak{v} = v f\left(\frac{x}{vt}, \frac{y}{vt}, \frac{z}{vt}\right). \tag{12.2}$$

The quantities of the total force of the reaction P of water and the wetted area S

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

are represented by the formulas

$$\left.\begin{aligned} P &= c_1 \rho v^4 t^2, \\ S &= c_2 v^2 t^2, \end{aligned}\right\} \qquad (12.3)$$

whereby the coefficients c_1, c_2 and the direction of force P depends only on the shape of the cone and upon the direction of the velocity of the motion of the cone. The first of the formulas (12.3) shows that the force of the reaction of the water is proportional to the density of the fluid, the fourth power of the velocity, and the square of the time. The wetted area is proportional to the square of the velocity and the square of the time. It is obvious that the two different states of one and the same motion are dynamically similar.

In the case of the two-dimensional problem of the submersion of the flat wedge (the plane of motion is the plane xy) for the potential of the velocities and for distribution of velocities, the following formulas are valid:

$$\varphi = v^2 t f\left(\frac{x}{vt}, \frac{y}{vt}\right),$$

$$\bar{v} = v \bar{f}_1\left(\frac{x}{vt}, \frac{y}{vt}\right).$$

For the force per unit of length of the wedge and for the wetted length along the flange we have the formulas

$$\left.\begin{aligned} P_1 &= c_1' \rho v^3 t. \\ l &= c_2' v t. \end{aligned}\right\} \qquad (12.4)$$

From formulas (12.3) and (12.4) it is evident that during submersion of the body in the water at a constant velocity, the relation of the quantity of the force of reaction of the water to the velocity of motion turns out different for bodies of different shape.

The constants c_1' and c_2' depend on the angle of careening of the wedge, upon the angles of attack of the plane of symmetry of the wedge, and velocity of the wedge relative to the nonturbulent level of the free surface.

For the plane problem, there are approximate theoretical solutions for the vertical submersion and for the submersion of a plate inclined slightly to the level

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

of the fluid when the horizontal component of the velocity of the plate is large*.

13. Small Waves on the Surface of a Non-Compressible Liquid

Considering the Cauchy-Poisson problem concerning waves on the surface of a heavy non-compressible liquid, N.Ye.Kochin** employed the dimension theory expressions and gave this classical problem a new elegant mathematical form.

Developing the dimension theory expressions, a whole class of new solutions for wave problems can be found in a clear and simple manner***. N.Ye.Kochin's solution is a special case in this class of solutions obtained.

This method and the solutions arrived at can be expanded and generalized in cases concerning spatial problems.

A plane problem concerning potential waves with an infinitely small amplitude on the surface of a heavy non-compressible liquid occupying an ever diminishing semispace can be formulated in the following manner.

Let us take the Descarte's system of coordinates; the x axis coincides with the level of the liquid at rest, the y axis extends upward vertically. The potential of the velocities $\varphi(x, y, t)$ when $y \leq 0$ is a regular harmonic function, i.e.

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \qquad \text{if} \qquad y < 0. \tag{13.1}$$

Let us then examine the motions which take place when the movement of the liquid

---

* See Wagner, ZAMM, 1932, No.4; Sedov, L.I. Trudy TsAGI, Vyp 252, 1936 (See Wagner, Journal Applied Mathematics and Mechanics, 1932; - Works of the Central Aerohydrodynamics Institute, No.252, 1936)

** Kochin, N.Ye., On the Cauchy-Poisson Wave Theory. Trudy Matematicheskogo Instituta imeni V.A.Steklova (Works of the Mathematics Institute imeni V.A. Steklov), Vol 9, 1935.

*** Sedov,L.I., On the Theory of Waves on the Surface of an Incompressible Fluid. Vest.Mosk.Univ. No.11, 1948.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

disappears after penetrating into the depths of the liquid, i.e.

$$|\operatorname{grad}\varphi| \to 0 \qquad \text{at} \quad y \to -\infty. \tag{13.2}$$

In linearized form, a condition in which there is a constant pressure on a free surface can be represented as:

$$\frac{\partial^2\varphi}{\partial t^2} + g\frac{\partial\zeta}{\partial y} = 0 \qquad \text{at} \quad y = 0 \text{ и } t \geqslant 0, \tag{13.3}$$

where g equals the acceleration of the force of gravity. In order to determine the potential of the velocities φ (x, y, t), besides the conditions (13.1), (13.2), and (13.3), it is necessary to set forth several additional requirements.

Let us take as such additional conditions, for example, the starting conditions: when t = 0 the problem concerns the form of a free surface and dissemination is due to impulsive pressures.

In formulating the initial conditions according to the linearized theory we can proceed from the equation:

$$\zeta = -\frac{1}{g}\frac{\partial\varphi}{\partial t}\Big|_{y=0} \text{ and } \frac{p_t}{\rho} = -\varphi, \tag{13.4}$$

where ζ (x,t) represents the height of points on the free surface above the undisturbed level, p_t is the impulsive pressure, and ρ is the density of the liquid.

The starting conditions can be formulated as follows: when t = 0, we have:

$$-\frac{1}{g}\frac{\partial\zeta}{\partial t}\Big|_{y=0} = f(x), \quad \varphi|_{y=0} = F(x). \tag{13.5}$$

In the function f and F simultaneously are not null or infinity, moreover, if f ≠ kx, it is evident that these functions, except the variable x, must still depend on certain dimensional constants. Since the problem is formulated in kinematic quantities, it is evident that there can be no more than two dimensional constants with independent dimensions in the functions f and F.

We satisfy the Laplace equations if we assume:

$$\varphi(x, y) = \operatorname{Re} w(x + iy), \tag{13.6}$$

where w (z) (z = x + iy) is a characteristic function of the flow of the liquid. We will then assume that the characteristic function w (z) is simple, finite, and

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

regular when $t > 0$ and the finites x and $y < 0$.

From the conditions (13.2) it follows that the derivitive $\frac{\partial w}{\partial z}$ disappears when $y \to -\infty$.

The boundary condition (13.3) can be represented as

$$\mathrm{Re}\,\mathrm{cl}\left(\frac{\partial^2 w}{\partial t^2} + ig\frac{\partial w}{\partial z}\right) = 0 \qquad \text{at} \quad y = 0. \tag{13.7}$$

This condition makes it possible to extend the combination $G(z) = \frac{\partial^2 w}{\partial t^2} + ig\frac{\partial w}{\partial z}$ to the upper semiplane, from this we see that the function $G(z)$ is simple in the entire plane of the complex variable $z = x + iy$. From the accepted hypotheses concerning the general character of the movement of a liquid it follows that the singular points $G(z)$ lie on the real axis.

Let us seek a solution for which the characteristic function $w(z)$ depends linearly on the dimensional constants entering into the supplementary conditions which determine the potential of the velocities. We will not be able to give concrete form to these conditions.

From the linearity of the problem it follows that it is sufficient to consider cases where we have only one dimensional constant a, on which the characteristic function $w(z)$ depends linearly (the constant a can be complex).

$$[a] = \mathrm{L}^p\mathrm{T}^q.$$

From the assumptions made we see that a complete system of determining parameters can be represented by the tabulation

$$z = x + iy, \quad t, \quad g, \quad a.$$

Let us now assume

$$\omega = az^{\alpha}g^{\beta}\chi(z, t, g);$$

the exponents α and β have been selected so that χ is an abstract quantity. Since $[a] = \mathrm{LPT}^q$, it is evident that for this to be so:

$$p + \alpha + \beta = 2, \quad q - 2\beta = -1,$$

from which

$$\beta = \frac{1+q}{2}, \quad \alpha = \frac{3-2p-q}{2}. \tag{13.8}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

From the dimension theory, it follows that the function $\chi(x, t, g)$ depends only on the combination

$$\lambda = \frac{gt^2}{x},$$

i.e.

$$w = ag^{\beta}x^{\alpha}\chi(\lambda). \qquad (13.9)$$

On the basis of the formula (13.9) we obtain the following formulae:

$$\frac{\partial w}{\partial t} = ag^{\beta}x^{\alpha}\chi'(\lambda)\frac{\partial \lambda}{\partial t},$$

$$\frac{\partial^2 w}{\partial t^2} = ag^{\beta}x^{\alpha}\left[\chi''(\lambda)\left(\frac{\partial \lambda}{\partial t}\right)^2 + \chi'(\lambda)\frac{\partial^2 \lambda}{\partial t^2}\right],$$

$$\frac{\partial w}{\partial x} = ag^{\beta}x^{\alpha}\left[\frac{\alpha}{x}\chi(\lambda) + \chi'(\lambda)\frac{\partial \lambda}{\partial x}\right]$$

and

$$G(x) = ag^{\beta}x^{\alpha}\left[\chi''\left(\frac{\partial \lambda}{\partial t}\right)^2 + \chi'\left(\frac{\partial^2 \lambda}{\partial t^2} + ig\frac{\partial \lambda}{\partial x}\right) + \frac{ig\alpha}{x}\chi\right];$$

since

$$\frac{\partial \lambda}{\partial t} = 2\frac{\lambda}{t}, \quad \frac{\partial^2 \lambda}{\partial t^2} = 2\frac{\lambda}{t^2}, \quad \frac{\partial \lambda}{\partial x} = -\frac{\lambda}{x},$$

thus

$$G = 4ag^{\beta+\alpha}t^{2\alpha-2}\lambda^{2-\alpha}\left[\chi'' + \left(\frac{1}{2\lambda} - \frac{i}{4}\right)\chi' + \frac{i\alpha}{4\lambda}\chi\right]. \qquad (13.10)$$

The function $G(\lambda)$ is a simple function of a complex variable λ over the entire plane. The singular points can only be located on the real axis.

Since when λ is a real function, $G(\lambda)$ is purely imaginary, it is evident that for any singular point using this function the coefficients in the Laurent series are purely imaginary.

Let us designate: $a = a_0 e^{i\delta}$ and $G = 4a_0 g^{\beta+\alpha}t^{2\alpha-2}G_1(\lambda)$. From (13.10) we see that:

$$e^{-i\delta}G_1(\lambda) = \lambda^{2-\alpha}\left[\chi'' + \left(\frac{1}{2\lambda} - \frac{i}{4}\right)\chi' + \frac{i\alpha}{4\lambda}\chi\right]. \qquad (13.11)$$

We can satisfy the conditions within a liquid and on a free surface if we take as the function $G_1(\lambda)$, any simple function which has singularities only on the real

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

axis on which it is purely imaginary.

To determine the characteristic function of a corresponding wave motion it is necessary to integrate the differential equation (13.11) for the function $\chi(\lambda)$. In the most general case, wave motions which have singularities on the free surface of a liquid are arrived at.

If we state that when $t > 0$, the movement of the liquid at the free boundary is regular, then from the finiteness of the combination $\frac{\partial^2 \omega}{\partial t^2} + ig\frac{\partial q}{\partial z}$ it follows that the function $C_1(\lambda)$ becomes an imaginary constant, which we must assume is equal to null on the basis of condition (13.2) and additional conditions governing the constant while pressure is exerted when $y = -\infty$.

Thus, we come to the problem of integrating a normal differential equation

$$\chi'' + \left(\frac{1}{2\lambda} - \frac{i}{4}\right)\chi' + \frac{\alpha}{4\lambda}\chi = 0. \qquad (13.12)$$

In this equation α is an arbitrary constant. It is not difficult to see that when α is complex, the solution of the equation (13.12) also yield a certain wave motion. The basic solution examined by N.Ye.Kochin corresponds to the case where the value of $\alpha = -\frac{1}{2}$.

Equation (13.12) after substituting the variable

$$\lambda = \frac{i}{4}\mu$$

becomes

$$\mu\frac{d^2\chi}{d\mu^2} + \left(\frac{1}{2} - \mu\right)\frac{d\chi}{d\mu} - \alpha\chi = 0. \qquad (13.13)$$

The solution of equation (13.13) is expressed in terms of the confluent hypergeometric functions $y = M(k, \gamma, x)$ which satisfy the differential equation*

$$xy'' + (\gamma - x)y' - ky = 0.$$

A general solution of equation (13.13) takes the form

$$\chi = C_1 M\left(-\alpha, \frac{1}{2}, \mu\right) + C_2\mu^{1/2} M\left(-\alpha + \frac{1}{2}, \frac{3}{2}, \mu\right).$$

---

* See Ye.Yanke and F.Emde, Table of Functions With Formulae and Curves. GTTI, 1949.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Utilizing this solution for the characteristic function of a wave motion, we can write

$$w = A_1 w_1 + A_2 w_2, \tag{13.14}$$

where

$$w_1(z, t, \alpha) = z^\alpha M\left(-\alpha, \frac{1}{2}, \frac{igt^2}{4z}\right), \tag{13.15}$$

$$w_2(z, t, \alpha) = z^\alpha \sqrt{\frac{igt^2}{4z}}\, M\left(-\alpha, \frac{1}{2}, \frac{3}{2}, \frac{igt^2}{4z}\right). \tag{13.16}$$

The arbitrary constants A_1 and A_2 can be complex.

The partial solutions of the wave problem (13.15) and (13.16) which depend on the one arbitrary constant α can be generalized by substituting $t - t_0$ for t and $z - z_0$ for z. The constants t_0 and z_0 (z_0 is real) define a special change in the initial moment of time and a shift of the singular points corresponding to the initial coordinates to the plane of motion of the liquid. Proceeding from the solutions obtained, in summing up, it is possible to construct more general solutions: thereby, the constants A_1 and A_2 can be considered functions of the parameters α, t_0, and z_0.

From formulae (13.15) and (13.16) the following properties of the functions w_1 and w_2 can easily be seen.

When $t = 0$ we have:

$$w_1(z, 0, \alpha) = z^\alpha, \quad \left(\frac{\partial w_1}{\partial t}\right)_{t=0} = 0 \tag{13.17}$$

and

$$w_2(z, 0, \alpha) = 0, \quad \left(\frac{\partial w_2}{\partial t}\right)_{t=0} = \sqrt{\frac{ig}{4}} \cdot z^{\alpha - \frac{1}{2}}. \tag{13.18}$$

Let us now consider wave motions for which the characteristic functions are determined by the formulae

$$\Omega_1(z, t) = -\frac{e^{\pi i \alpha}}{\pi i} \int_{-\infty}^{+\infty} f(x_0)\, w_1(z - x_0, t, \alpha)\, dx_0, \tag{13.19}$$

$$\Omega_2(z, t) = -\frac{2e^{\pi i}\left(\alpha - \frac{1}{2}\right)}{\sqrt{ig}\cdot\pi i} \int_{-\infty}^{+\infty} F(x_0)\, w_2(z - x_0, t, \alpha)\, dx_0, \tag{13.20}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

where $f(x_0)$ and $F(x_0)$ are definite functions for which the integrals in formulae (13.19) and (13.20) coincide.

$$\Omega_1 = -\frac{1}{\pi i}\int_{-\infty}^{+\infty}\frac{f(x_0)\,dx_0}{(x_0-z)^{-\alpha}}, \qquad \frac{\partial\Omega_1}{\partial t}=0, \tag{13.21}$$

$$\Omega_2 = 0, \quad \frac{\partial\Omega_2}{\partial t} = -\frac{1}{\pi i}\int_{-\infty}^{+\infty}\frac{F(x_0)\,dx_0}{(x_0-z)^{\frac{1}{2}-\alpha}}. \tag{13.22}$$

The first equation (13.21) when $\alpha = -1$ and $z = x$ yields:

$$\Omega_1(x) = \Phi_1 + i\Psi_1 = f(x) - \frac{1}{\pi i}\,\text{v. p.}\int_{-\infty}^{+\infty}\frac{f(x_0)\,dx_0}{x_0-x}.$$

Thus, if f(x) is real, then when $\alpha = -1$ formula (13.19) yields a solution for wave motion given the following initial conditions:

$$\text{that} \quad t=0 \quad \Phi_1(x_0) = f(x), \quad \frac{\partial\Phi_1}{\partial t}=0.$$

The following initial conditions are obtained for the function $\psi(x, y)$:

$$\text{that } t=0 \qquad \Psi_1(x,\,0) = \frac{1}{\pi}\,\text{v. p.}\int_{-\infty}^{+\infty}\frac{f(x_0)\,dx_0}{x_0-x} \text{ and } \frac{\partial\Psi}{\partial t}=0.$$

If f(x) is purely imaginary, then the initial conditions take an analogous form, and only the functions Φ_1 and ψ_1 change their roles.

In the case where $\alpha = -\frac{1}{2}$ the second equation (13.22) when $t = 0$ and $z = x$ yields:

$$\frac{\partial\Omega_2}{\partial t} = \frac{\partial(\Phi_2+i\Psi_2)}{\partial t} = F(x) - \frac{1}{\pi i}\,\text{v. p.}\int_{-\infty}^{+\infty}\frac{F(x_0)\,dx_0}{x_0-x}.$$

From this it follows that if F(x) is real, then the following initial conditions are obtained:

$$\text{that} \quad t=0 \text{ and } y=0 \quad \Phi_2 + i\Psi_2 = 0, \quad \frac{\partial\Phi_2}{\partial t} = F(x).$$

This case was analyzed by N.Ye.Kochin. Let us now explain the character of the initial conditions in a general case where $\alpha \neq -1, -\frac{1}{2}$.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Let us examine the initial conditions for the function $\Omega_1(z, t)$.

We have:

$$\Omega_1(z, 0) = -\frac{1}{\pi i}\int_{-\infty}^{+\infty}(x_0 - z)^{\alpha} f(x_0)\,dx_0.$$

If $\alpha > 0$ an is a whole number, then it is evident that $\Omega_1(z, 0)$ is a polynomial of α degree. In general if $\alpha > 0$, then the function $\Omega_1(z, t)$ will be converted into infinity when $z \to \infty$. Therefore when $\alpha \geq 1$ the conditions governing the disappearance of the velocities when $y \to -\infty$ are not satisfied.

If $\alpha = -(1 + s)$, where s is a whole positive number, then we have:

$$\Omega_1(z, 0) = -\frac{1}{\pi i}\int_{-\infty}^{+\infty}\frac{f(x_0)\,dx_0}{(x_0 - z)^{1+s}}. \tag{13.23}$$

Let us assume that the integral $\int_{-\infty}^{+\infty} f(x_0)\,dx_0$ is finite* and let us take the function

$$\omega(z) = -\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{f(x_0)}{x_0 - z}\,dx_0. \tag{13.24}$$

into consideration.

From the assumptions made about the function $f(x_0)$ it is easy to see the correctness of the equations

$$\frac{d^s\omega}{dz^s} = \omega^{(s)}(z) = \Gamma(s + 1)\,\Omega_1(z, 0), \tag{13.25}$$

$$\omega(z) = \Gamma(s+1)\int_{-i\infty}^{z} dz_s \int_{-i\infty}^{z_s} dz_{s-1} \cdots \int_{-i\infty}^{z_2} \Omega(z_1)\,dz_1 =$$

$$= \int_{-i\infty}^{z} (z - u)^s\,\Omega_1'(u)\,du. \tag{13.26}$$

Consequently, in order to determine $f(x)$ let us take the equation

$$\int_{-i\infty}^{x} (x - u)^s\,\Omega_1'(u)\,du = f(x) - \frac{\text{v. p.}}{\pi i}\int_{-\infty}^{+\infty}\frac{f(x_0)\,dx_0}{x_0 - x}. \tag{13.27}$$

---

* Evidently, the expression for $\Omega_1(z)$ can be significant in a number of cases when the integral in (13.24) diverges.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Thus, from formulae (13.23) and (13.24) when $s > 0$ we deduce equation (13.27). This equation is valid for any quantity where $s > -1$.

Actually, from (13.23) we have:

$$\Omega'(u) = -\frac{s+1}{\pi i}\int_{-\infty}^{+\infty}\frac{f(x_0)\,dx_0}{(x_0-u)^{s+2}}.$$

Multiplying by $(z-u)^s$ and integrating, we obtain:

$$\frac{{}_{s+1}(n-{}^0x)}{np\,{}_s(n-z)}\int_z^{\infty}{}^{-}\,{}^0xp\,({}^0x)\,f\int_{\infty+}^{\infty-}\frac{\pi i}{1+s}\,-\,=np\,{}_s(n-z)\,(n)\,{}_s\Omega\int_z^{\infty}{}^{-}$$

Let us substitute the variable

$$\frac{1}{1}({}^0x-z)+{}^0x=n$$

in the internal integral; from this

$$x_0-u=\frac{x_0-z}{\lambda},\quad z-u=\frac{x_0-z}{\lambda}(1-\lambda),\quad du=(x_0-z)\frac{d\lambda}{\lambda^2}.$$

Performing this substitution for the internal integral we obtain:

$$\frac{1}{x_0-z}\int_0^1(1-\lambda)^s\,d\lambda=\frac{1}{(s+1)(x_0-z)}.$$

Using this, we find:

$$\int_{-i\infty}^{z}(z-u)^s\,\Omega'(u)\,du=-\frac{1}{\pi i}\int_{-\infty}^{+\infty}\frac{f(x_0)\,dx_0}{x_0-z}.$$

From this, when $z \to x$ formula (13.27) follows.

Formula (13.25) can be generalized for fractions of s, if the fractional derivitive $w^s(z)$ is determined by the formula

$$\omega^s(z)=\frac{1}{\Gamma(1-s)}\int_{-i\infty}^{z}(z-u)^{-s}\,\omega'(u)\,du.$$

Actually,

$$\omega^s(z)=-\frac{1}{\pi i\Gamma(1-s)}\int_{-\infty}^{+\infty}f(x_0)\,dx\int_{-i\infty}^{z}\frac{(z-u)^{-s}\,du}{(x_0-u)^2}.$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

To calculate the internal integral we substitute the variable

$$u = x_0 - (z - x_0)\frac{1}{\lambda},$$

after this we obtain:

$$\int_{-\infty}^{z} \frac{(z-u)^{-s}}{(x_0-u)^{s}}\,du = \frac{1}{(x_0-z)^{s-1}} \int_0^1 \lambda^{s}(1-\lambda)^{-s}\,d\lambda = \frac{B(s+1,\,1-s)}{(x_0-z)^{s-1}};$$

since

$$B(s+1,\,1-s) = \frac{\Gamma(s+1)\,\Gamma(1-s)}{\Gamma(2)} \quad \text{и} \quad \Gamma(2) = 1,$$

thus

$$\omega^{*}(z) = \Gamma(s+1) \int_{-\infty}^{+\infty} \frac{f(x_0)\,dx_0}{(x_0-z)^{s-1}} = \Gamma(s-1)\,\Omega(z).$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

CHAPTER III

APPLICATIONS TO THE THEORY OF THE MOTION OF A VISCOUS FLUID AND TO THE THEORY OF TURBULENCE

Section 1. Diffusion of Vortices in a Viscous Fluid

Considerations of the theory of dimensions can render great assistance in the mathematical solution of certain physical problems. Here and in the following paragraphs we shall present examples of such an application of the theory of dimensions.

Let us consider the problem of the diffusion of vortices in a viscous incompressible fluid under the assumption that the motion of the fluid is plane parallel and the fluid occupies the entire plane*. The motion under consideration is nonsteady. Let, at the initial moment of time $t = 0$, the fluid move potentially everywhere, with the exception of the pole O which represents the trace on the plane of the motion of an infinite rectilinear concentrated vortex with circulation Γ.

Let us assume that the motion possesses axial symmetry. Let us designate by the angular velocity of part of the fluid.

As is well known, the circulation in a circle of radius R with center at O is equal to:

---

* See N.Ye.Kochin, I.A.Kibel' and N.V.Roze, Teoreticheskaya Gidromekhanika (Theoretical Hydrodynamics), Part II, State Technical Press, 1948. This book contains a detailed description and solution of this problem utilizing the considerations concerning dimensional analysis.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\Gamma_R = 2\int_0^R \int_0^{2\pi} \Omega r\, dr\, d\theta = 4\pi \int_0^R r\Omega(r)\, dr. \tag{1.1}$$

At the initial moment of time for any, in particular also for any arbitrarily small, circle we have the following:

$$\Gamma_R = \Gamma. \tag{1.2}$$

The equation describing the propagation of the vortices in the case under consideration possesses the form:

$$\frac{\partial \Omega}{\partial t} = \nu\left(\frac{\partial^2 \Omega}{\partial r^2} + \frac{1}{r}\frac{\partial \Omega}{\partial r}\right), \tag{1.3}$$

where ν is the coefficient of kinematic viscosity ($\nu = \frac{\mu}{\rho}$). The problem consists in the determination of the quantity Ω as a function of the radius r and the time t.

From the statement of the problem it follows that

$$\Omega = f(\Gamma, \nu, r, t).$$

From the linearity of equation (3) and from the initial condition it follows that Ω is proportional to Γ, that is,

$$\Omega = \Gamma f_1(\nu, r, t). \tag{1.4}$$

The dimensionless combination $\frac{\Omega \nu t}{\Gamma}$ must be expressed as a function of the single independent dimensionless quantity $\frac{r^2}{\nu t} = \xi$ which can be composed of the dimensional parameters ν, r, t. Consequently we have,

$$\Omega = \frac{\Gamma}{\nu t}\psi(\xi). \tag{1.5}$$

From formula (1.5) it is evident that the equation in partial derivatives (1.3) for the function of Ω with two independent variables r and t reduces to an ordinary differential equation with one independent variable ξ.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Substituting the expression for Ω from equation (1.5) into equation (1.3), we have the following:

$$\psi(\xi) + \xi\psi'(\xi) + 4[\psi'(\xi) + \xi\psi''(\xi)] = 0.$$

Integrating we obtain the following:

$$\xi\psi + 4\xi\psi' = C.$$

The constant C is equal to zero for the solution in which $\psi(0)$ and $\psi'(0)$ are finite.

Integrating the equation

$$4\frac{d\psi}{d\xi} + \psi = 0,$$

we find the following

$$\psi = Ae^{-\frac{\xi}{4}}.$$

For the magnitude of the vortix Ω this gives:

$$\Omega = \frac{\Gamma}{\nu t} Ae^{-\frac{r^2}{4\nu t}}.$$

We determine the constant A from the initial condition. The circulation around a circle of radius R is equal to the following:

$$\Gamma_R = 4\pi \frac{A\Gamma}{\nu t} \int_0^R re^{-\frac{r^2}{4\nu t}} dr = 8\pi A\Gamma\left(1 - e^{-\frac{R^2}{4\nu t}}\right). \tag{1.6}$$

For t = 0 for any radius R greater than zero (R > 0) we have:

$$\Gamma_R = 8\pi A\Gamma.$$

The initial condition $\Gamma_R = \Gamma$ gives:

$$A = \frac{1}{8\pi}.$$

The final solution of the problem is represented by the following formula

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\Omega = \frac{\Gamma}{8\pi\nu t} e^{-\frac{r^2}{4\nu t}}. \qquad (1.7)$$

Let us designate by $v(r, t)$ the velocity of the particles of the fluid. The motion of the fluid possesses axial symmetry; hence the velocity of the particles of the fluid is directed perpendicularly to the radius vector drawn to the point under consideration from the pole O.

Taking into consideration the direction of the velocity vector, we obtain the following relation between Γ_R and v:

$$\Gamma_R = 2\pi r v.$$

Making use of the eq. (1.6) we find the law governing the distribution of velocity along radius r and in time t:

$$v = \frac{\Gamma}{2\pi r}\left(1 - e^{-\frac{r^2}{4\nu t}}\right).$$

For $t = 0$ there is obtained the law governing the distribution of velocities that corresponds to a point vortex in an ideal fluid. For $r > 0$ and $t = 0$ the motion of the fluid is potential, and vortices are absent; for $r > 0$ and $t > 0$ the motion is of the fluid is vortical at each point of the fluid. The eq. (1.7) gives the law governing the propagation - i.e. diffusion - of the vortices. This formula indicates that the magnitude of the vortex at each point increases in the course of time from zero up to a maximum equal to $\frac{\Gamma}{2\pi r^2 e}$, and thereafter again tends to zero.

Since the eq. (1.3) is linear, then proceeding from the obtained solution for the propagation of a point vortex we can construct by the method of superposition the solution of the problem of the symmetric motion for any initial distribution of velocities.

Section 2. Exact Solutions of the Equations of Motion of a Viscous Incompressible Fluid

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Let us consider the steady motion of a viscous incompressible fluid that fills the entire space.

The Navier-Stokes equation and the equation of continuity can be written in the following form:

$$\left.\begin{aligned} \bar{v}\nabla\bar{v} &= -\operatorname{grad}\left(\frac{p}{\rho}-U\right)+\nu\Delta v, \\ \operatorname{div} v &= 0. \end{aligned}\right\} \qquad (2.1)$$

In what follows we make use of the spherical system of coordinates.

The independent variables and the determining quantities will be:

$$r,\ \theta,\ \lambda,\ \nu$$

where r is the distance of the point under consideration to the pole, θ is the amplitude, λ is the longitude, and ν is the coefficient of kinetic viscosity. The sought for quantities will be the projections of the velocity v_r, v_θ, and v_λ and the dynamic pressure directed on the plane and equal to $\frac{p}{\rho} - U$.

Let us study the solution of the eqs. (2.1), which are completely determined by the parameters r, θ, λ, ν, and by still another dimensional constant A. Let the formula of dimensionality for constant A possess the following form

$$[A] = L^p T^q \qquad (2.1a)$$

where p and q are certain constants.

In the case of this assumption it is evident that all dimensionless combinations consisting of the introduced quantities will be functions of only three abstract parameters :

$$\theta,\ \lambda,\ \pi = \frac{r^{p+2q}\nu^{-q}}{A}.$$

Then the sought for functions can be represented in the following form

$$v_r = \frac{\nu}{r} f(\pi, \lambda, \theta), \qquad v_\theta = \frac{\nu}{r}\varphi(\pi, \lambda, \theta),$$

$$v_\lambda = \frac{\nu}{r}\psi(\pi, \lambda, \theta), \qquad U - \frac{p}{\rho} = \frac{\nu^2}{r^2} F(\pi, \lambda, \theta).$$

In such a form one can represent and consider the most general solution of the

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

eqs. (2.1).

The number of independent variables is reduced if we assume that

$$p + 2q = 0$$

This condition signifies that the dimension of the constant A is represented as a certain power of the dimension of the coefficient of kinematic viscosity ν.

Besides this assumption, we further assume that the motion under study possesses axial symmetry, so that the variable λ is nonessential.

From the assumptions made it follows that for the sought for quantities the following formulas must be correct:

$$v_r = \frac{\nu}{r} f(\theta), \quad v_\theta = \frac{\nu}{r} \varphi(\theta), \quad v_\lambda = \frac{\nu}{r} \psi(\theta), \quad U - \frac{P}{\rho} = \frac{\nu^2}{r} F(\theta). \tag{2.2}$$

These formulas establish a dependence of the velocity field and pressure field upon the variable r.

In this case, we obtain from the eqs. (2.1) for the four functions f, φ, ψ, F a system of ordinary nonlinear differential equations

$$\left.\begin{aligned} f'' + f'(\operatorname{ctg}\theta - \varphi) + f^2 + \varphi^2 + \psi^2 - 2F = 0, \\ \varphi\varphi' - \psi^2 \operatorname{ctg}\theta - f' - F' = 0, \\ \psi'' - \varphi\psi' - \varphi\psi \operatorname{ctg}\theta + \psi' \operatorname{ctg}\theta - \frac{\psi}{\sin^2\theta} = 0, \\ f + \varphi' + \varphi \operatorname{ctg}\theta = 0. \end{aligned}\right\} \tag{2.3}$$

After elimination of the function F and certain simple transformations we obtain the following expressions:

$$\left.\begin{aligned} f''' + 2\psi(\psi' + \psi \operatorname{ctg}\theta) + (f' \operatorname{ctg}\theta)' - (\varphi f')' + 2ff' + 2f' = 0, \\ \varphi(\psi' + \psi \operatorname{ctg}\theta) = (\psi' + \psi \operatorname{ctg}\theta)', \\ f = -(\varphi' + \varphi \operatorname{ctg}\theta). \end{aligned}\right\} \tag{2.4}$$

The general solution of this system of equations depends upon six arbitrary constants.

Before studying the solutions of the system of eqs. (2.4) we note certain general properties of the considered motions of a viscous fluid.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The differential equations for the projections of the lines of flux upon the plane of the meridian can be written in the following form

$$\frac{dr}{v_r} = \frac{r\,d\theta}{v_\theta} \quad \text{with} \quad \frac{dr}{f(\theta)} = \frac{r\,d\theta}{\varphi(\theta)},$$

whence we get

$$\ln \frac{r}{a} = \int \frac{f(\theta)\,d\theta}{\varphi(\theta)},$$

where a is the constant of integration. On the basis of the last equation of system (2.4) we obtain:

$$\frac{r}{a} = \frac{1}{\varphi \sin \theta}. \tag{2.5}$$

From the general considerations of the theory of dimensionality, and also directly from eq. (2.5), it follows that the various lines of flux are similar.

Let us designate by Q the volume discharge of fluid and by $\bar{J}$ the volume flow of momentum through a closed surface S, that is

$$Q = \int_S v_n\,d\sigma, \quad \bar{J} = \int_S \bar{v}\cdot v_n\,d\sigma.$$

The dimensions of Q and J are represented by the following formulas

$$[Q] = L^3T^{-1}, \quad [J] = L^4T^{-2}.$$

In the case of contraction of the surface S toward the pole into a point we obtain the fact that the quantities Q and $\bar{J}$ can depend only upon the coefficient ν and upon the constant A, the dimensions of which is expressed by the dimensions of ν. Since the dimensions of Q and ν are independent, then it is evident that the discharge Q is equal to either zero or infinity. The dimensionality of J is expressed by the dimensionality of the coefficient ν; therefore the quantity J can be finite.

Another situation holds true in the case of plane parallel motions. If for plane parallel motions the velocity field in the entire plane depends only upon the coordinates of the point and upon constants which possess a dimensionality that is

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

dependent upon the dimensionality of the coefficient of kinematic viscosity ν, then in polar coordinates the correct formulas will be ones similar to the eqs. (2.2). (In this case r is the radius vector in the plane of motion.)

For plane parallel motion the discharge and flow of momentum can be determined by the following formulas

$$Q = \int_L v_n \, dS, \quad \bar{J} = \int_L \bar{v} \cdot v_n \, dS,$$

where L is a certain closed contour surrounding the original of coordinates. The dimensions of Q and J in this case are represented by the formulas

$$[Q] = L^2 T^{-1}, \quad [J] = L^3 T^{-1}.$$

Consequently the plane motions of the considered type can be characterized by finite discharge, and the corresponding impulses (momenta) will be equal to zero or infinity.

This circumstance permitted Hamel and a number of other authors in the problem of the motion of a fluid in an angle between two planes to obtain the exact solutions to the Navier-Stokes equations by way of their reduction to ordinary differential equations*.

Let us now pass over to the consideration of the solution to the system of eqs. (2.4). The first of the eqs. (2.4) can be represented in the form

$$\left[\frac{1}{\sin\theta}\left(\frac{\Phi'}{\sin\theta}\right)'\right] = \frac{(\psi^2 \sin^2\theta)'}{\sin^2\theta}, \tag{2.6}$$

where

$$\Phi = \left(\varphi' - \frac{1}{2}\varphi^2\right)\sin^2\theta - \varphi \sin\theta\cos\theta. \tag{2.7}$$

---

* These solutions are expounded in detail in the book: N.Ye.Kochin, I.A.Kibel' and N.V.Roze, Teoreticheskaya Gidromekhanika (Theoretical Hydrodynamics), Part II, State Technical Press, 1948.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

From the eq. (2.6) we can derive the following integral:

$$\psi^2 \sin^2\theta - \sin\theta\left(\frac{\Phi'}{\sin\theta}\right)' + 2\Phi + 2\operatorname{ctg}\theta\cdot\Phi' = D, \tag{2.8}$$

where D is the constant of integration.

The function $\psi(\theta)$ defines the distribution of the components of velocities perpendicular to the plane of the meridian.

It is not difficult to see that for $\psi = 0$ or for $\frac{\text{psi}}{\sin\theta} = \text{const}$. the system of eqs. (2.4) gives for the determination of the functions f and φ one and the same relations.

The condition $\psi = \frac{c}{\sin\theta}$ gives $v_\lambda = \frac{cv}{r\sin\theta}$; the velocity field for v_λ corresponds to a rectilinear vortex which coincides with the axis of symmetry.

Consequently the equations of motion will be satisfied if to any velocity field of the type under consideration we add a field of velocities from the rectilinear vortex.

If we assume that

$$\psi \sin\theta = \text{const} \tag{2.9}$$

then the eq. (2.6) can be integrated three times, after which the solution of the problem is reduced to the integration of the following equation

$$\left(\varphi' - \frac{1}{2}\varphi^2\right)\sin^2\theta - \varphi\sin\theta\cos\theta = M\cos 2\theta + N\cos\theta + R, \tag{2.10}$$

where M, N, R are arbitrary constants of integration*.

1^0. If we set M = N = R = 0, then the eq. (2.10) is easily integrated and gives:

$$\varphi = \frac{2\sin\theta}{A+\cos\theta}, \quad \textit{hence} \quad f = -2 + \frac{2(A^2-1)}{(A+\cos\theta)^2}, \tag{2.11}$$

---

* Eq. (2.10) has been obtained in another way by N.A.Slezkin. See Uchenyye Zapiski MGU (Scientific Notes of Moscow State University), No.2, 1934.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

where A is an abstract constant of integration. The solution (2.11) was obtained by Landau*.

It is not difficult to verify that for this solution in the case of $|A|>1$ the discharge of fluid through a surface enclosing the origin of coordinates is equal to zero, but in the case $|A|<1$ the discharge is equal to infinity.

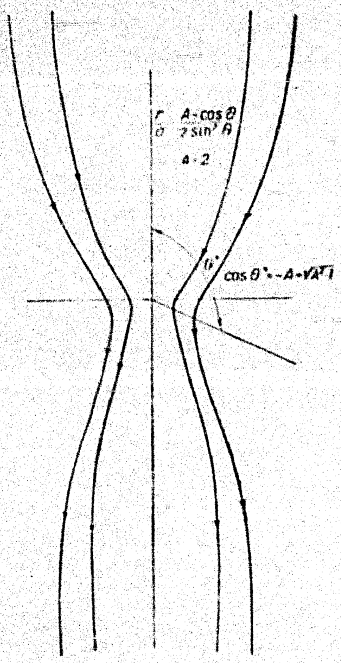


Fig.20. The lines of flux for a source of zero power and for finite impulse (momentum) in a viscous fluid.

For the flow of the projection of the momentum upon the axis of symmetry through any sphere with center at the origin of coordinates the following formula is true:

$$J = \rho\nu^2\left[8(A^2-1)\ln\frac{A-1}{A+1} + 8A - \frac{32A}{3(A^2-1)} + \frac{8A(3A^2-1)}{A^2-1}\right] \tag{2.12}$$

Consequently the magnitude of the impulse (momentum) J does not depend upon the radius of the sphere and gives the mechanical characteristic of the singular point at the origin of coordinates.

For this current the equations of the lines of flux possess the following form:

$$\frac{r}{a} = \frac{1+\cos\theta}{2\sin^2\theta}.$$

* L.Landau and Ye.Lifshits, Mekhanika Sploshnykh Sred (Mechanics of Continuous Media), State Technical Press, page 108, 1953.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

For A>1 the form of the lines of flux is indicated in Fig.20.

In the case of far removal to infinity the lines of flux will acquire a parabolic character. In the case of the motion along a line of flux the radius vector r attains a minimum value for a certain value of $\theta = \theta^*$, determined by the following relation

$$\cos \theta^* = -A + \sqrt{A^2 - 1}$$

The corresponding motion can be considered as the motion of a viscous fluid filling the entire space which is caused by an infinitely fine jet stream at the origin of coordinates, which jet is pulsating from the end of an infinitely fine tube in the direction of the x axis with finite impulse (momentum).

We can immediately point to a whole series of solutions of the Riccati eq. (2.10), which correspond to particular values of the constants M, N, and R. For example, for R = 1, N = 0, and $M = \frac{1}{2}$, we have the following solution

$$\varphi = \left(\operatorname{ctg}\theta + \operatorname{cth}\frac{\theta+\theta_0}{2}\right),$$

where θ_0 is an arbitrary constant. For N = 0, $M = \frac{1}{4} - m^2$, and $R = \frac{9}{4} - (m+1)^2$ we have the following solution

$$\varphi = (2m-1)\operatorname{ctg}\theta - \frac{2\sin^{2m}\theta}{\int \sin^{2m}\theta\, d\theta}$$

and so on.

In the general case the Riccati eq. (2.10) can be solved with the aid of hypergeometric functions*.

By means of the substitutions

$$\varphi = -2\frac{y'(\theta)}{y(\theta)}, \quad \mu = \cos^2\frac{\theta}{2}$$

---

* See V.I.Yatsyev, Zhurnal Eksperimental'noy i Teoreticheskoy Fiziki (Journal of Experimental and Theoretical Physics), No. 11, 1950, page 1031.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the eq. (10) is reduced to the following form:

$$\frac{d^2y}{d\mu^2}+\frac{\frac{M+R-N}{2}+(N-4M)\mu+4\mu^2M}{4\mu^2(\mu-1)^2}y=0. \tag{2.13}$$

This equation is easily integrated by means of hypergeometric functions; its general integral can be written in the following form*

$$Y(\theta)=\left(\cos\frac{\theta}{2}\right)^{\gamma}\left(\sin\frac{\theta}{2}\right)^{1+\alpha+\beta-\gamma}\left\{PF\left(\alpha,\ \beta,\ \gamma,\ \cos^2\frac{\theta}{2}\right)+\right.$$
$$\left.+Q\left(\cos^2\frac{\theta}{2}\right)^{1-\gamma}F\left(\alpha+1-\gamma,\ \beta+1-\gamma,\ 2-\gamma,\ \cos^2\frac{\theta}{2}\right)\right\} \tag{2.14}$$

where the constants α, β, γ are connected with M, N and R by the formulas:

$$M=\frac{1-(\alpha-\beta)^2}{4},$$
$$N=1-(\alpha+\beta)^2+2\gamma(\alpha+\beta-1),$$
$$R=\frac{3[1-(\alpha+\beta)^2]}{4}-\alpha\beta-2\gamma^2+2\gamma(\alpha+\beta+1).$$

In place of the constants M, N, R as arbitrary constants we can take the above mentioned parameters α, β, γ.

The solution obtained for φ depends upon four arbitrary constants, upon three parameters α, β, γ, and upon the ratio $\frac{P}{Q}$

For $M = N = R = 0$ the eq. (2.13) degenerates into the equation $y''(\mu) = 0$, which possesses only one regular singular point for $\mu = \infty$. The corresponding solution has been considered above.

If $N = -4M = -\frac{4}{3}R$, then in the eq. (2.13) the factor $(\mu - 1)^2$ in the denominator is reduced and only two singular regular points remain, namely $\mu = 0$ and $\mu = \infty$.

---

* If γ is a whole number, then the solution also can be written in a somewhat different form. For this purpose one can utilize the representation of the solutions to the hypergeometric equation in the form presented in the book: L.I.Sedov, Ploskiye Zadachi Gidrodinamiki i Aerodinamiki (Two-Dimensional Problems of Hydrodynamics and Aerodynamics), page 432, State Technical Press, 1950.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The eq. (2.13) in this case passes over into the following Euler equation

$$\mu^2 \frac{d^2y}{d\mu^2} + My = 0,$$

which is easily integrated.

Section 3. The Boundary Layer in the Case of the Circulation of a Viscous Fluid Around a Plane Plate

Let us consider the circulation of a plane infinitely thin plate by an incompressible viscous fluid. Far in front of the plate let the fluid be moving progressively with constant velocity U_0. The plate possesses infinite length and is disposed lengthwise along the flow in parallel with the velocity U_0. The problem is two-dimensional (planar); the motion is steady-state; the fluid occupies the entire plane outside the plate. This problem of the motion of a viscous fluid is very simple, but nevertheless it is not amenable to exact solution by means of the Navier-Stokes equations in view of the great mathematical difficulties. We shall analyze this problem with the help of the Prandtl equations, which are obtained from the general equations of motions of a viscous fluid by means of certain approximations*.

The equations of the boundary layer of Prandtl in the case under consideration possess the following form

$$\left.\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= \nu\frac{\partial^2 u}{\partial y^2}, \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \end{aligned}\right\} \qquad (3.1)$$

where u and v are the projections of the velocity of the particles of the fluid on the coordinate axes, and ν is the coefficient of kinematic viscosity. The x axis is directed toward the plate along the flow, and the y axis is perpendicular to the plate. In addition to the eqs. (3.1) for the determination of u (x, y) and v (x, y)

* L.Prandtl, Verhandlungen, der Dritten International. Math. Kongr. in Heidelberg (1904), Leipzig, 1905.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

we have also the boundary conditions

$$\left.\begin{array}{ll} \text{FOR } x > 0,\ y = 0 & u = v = 0; \\ \text{FOR } y = \pm\infty & u = U_0. \end{array}\right\} \tag{3.2}$$

The determining parameters will be the following:

$$U_0,\ \nu,\ \ell,\ y.$$

In view of the fact that the plate is two-dimensional and infinitely long, it is impossible to indicate any characteristic linear dimension. From the general considerations of the theory of dimensions it follows that all the dimensionless quantities are functions of two dimensionless combinations:

$$\frac{y}{x},\quad \frac{y}{\sqrt{\frac{\nu x}{U_0}}}.$$

Therefore formulas of the following form are correct for the problem under consideration

$$u = U_0 f\left(\frac{y}{x},\ \frac{y}{\sqrt{\frac{\nu x}{U_0}}}\right), \tag{3.3}$$

$$v = \sqrt{\frac{\nu U}{x}}\,\Phi\left(\frac{y}{x},\ \frac{y}{\sqrt{\frac{\nu x}{U_0}}}\right). \tag{3.4}$$

We shall now show that in consequence of the peculiarities of the eqs. (3.1) the first parameter y/x in eqs. (3.3) and (3.4) are nonessential. For this purpose we make the following transformation of variables:

$$x = l\xi,\quad y = \sqrt{\frac{\nu l}{U_0}}\cdot\eta,\quad u = U_0 u_1,\quad v = \sqrt{\frac{\nu U_0}{l}}\,v_1, \tag{3.5}$$

where the italic l is a certain constant greater than zero. If we ascribe to the constant l the dimension of length, then the quantities ξ, η, u_1, v_1 can be considered as dimensionless.

After replacement of the variables in accordance with the eqs. (3.5) the eqs.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

(3.1) take the form

$$\left.\begin{aligned} u_1\frac{\partial u_1}{\partial \xi}+v_1\frac{\partial u_1}{\partial \eta}&=\frac{\partial^2 u_1}{\partial \eta^2},\\ \frac{\partial u_1}{\partial \xi}+\frac{\partial v_1}{\partial \eta}&=0. \end{aligned}\right\} \tag{3.6}$$

The boundary conditions (3.2) in the new variables assume the following form

$$\left.\begin{aligned} &\text{FOR}\quad \xi>0,\ \eta=0 \quad u_1=v_1=0;\\ &\text{FOR}\quad \eta=\pm\infty \quad\ u_1=1. \end{aligned}\right\} \tag{3.7}$$

The eqs. (3.6) and the boundary conditions (3.7) can be considered as the formulation of the problem of a boundary layer in dimensionless form. The solution of this problem cannot depend upon the quantity $\frac{U_0 l}{\nu}=R$, which figures in no way in the eq. (3.6) and in the boundary conditions (3.7)*. On the other hand, the general eqs. (3.3) and (3.4) show that

$$u_1=\frac{u}{U_0}=f\left(\frac{\eta}{\xi\sqrt{R}},\ \frac{\eta}{\sqrt{\xi}}\right), \tag{3.8}$$

$$v_1\sqrt{\xi}=\frac{v}{\sqrt{\frac{\nu U_0}{x}}}=\Phi\left(\frac{\eta}{\xi\sqrt{R}},\ \frac{\eta}{\sqrt{\xi}}\right). \tag{3.9}$$

Since the solution must not depend upon R, then it follows from this that the first argument $\frac{y}{x}$ cannot enter into the first part of the eqs. (3.8) and (3.9).

In this manner we have demonstrated that the solution of the problem posed must have the following form**

* This circumstance is the property of the Prandtl eqs. (3.1). If one applies the transformation (3.5) to the Navier-Stokes equations, then as a result we obtain dimensionless equations containing the parameter R, in consequence of which further conclusions lose their correctness in application to the Navier-Stokes equations.

** The demonstration of the correctness of eq. (3.10) was conducted by another method in the book: L.G.Loytsyanskiy, Aerodinamika Pogranichnogo Sloya (Aerodynamics of the Boundary Layer), page 76, State Technical Press, 1941.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$u = U_0 f'\left(\frac{y}{\sqrt{\frac{\nu x}{U_0}}}\right), \tag{3.10}$$

$$v = \sqrt{\frac{\nu U_0}{x}}\,\Phi\left(\frac{y}{\sqrt{\frac{\nu x}{U_0}}}\right). \tag{3.11}$$

Let us now introduce the new variable $\lambda = \frac{\eta}{\sqrt{2}} = \frac{y}{\sqrt{\frac{\nu x}{U_0}}}$ and let us set $f(\lambda) = \varphi'(\lambda)$.

Substituting u and v into the equation of continuity we express the function $\Phi(\lambda)$ by means of $\varphi(\lambda)$:

$$\Phi'(\lambda) = \frac{1}{2}\lambda\varphi''(\lambda) = \frac{1}{2}(\lambda\varphi - \varphi)'.$$

Utilizing this equality we can write the eqs. (3.10) and (3.11) in the form

$$u = U_0\varphi'(\lambda), \tag{3.10'}$$

$$v = \sqrt{\frac{\nu U_0}{x}}\cdot\frac{1}{2}[\lambda\varphi'(\lambda) - \varphi(\lambda)]. \tag{3.11'}$$

Substituting the obtained expressions for u and v into the first of the eqs. (3.1), we obtain for equation $\varphi(\lambda)$ the following ordinary differential equation of the third order

$$2\varphi''' + \varphi\varphi'' = 0 \tag{3.12}$$

From the boundary conditions of the problem (3.2), for the sought for function satisfying the eq. (3.12) we obtain the following boundary conditions:

$$\varphi'(0) = \varphi(0) = 0 \text{ and } \varphi'(\infty) = 1 \tag{3.13}$$

The solution of the nonlinear differential eq. (3.12) in the case of the boundary conditions (3.13) can be obtained approximately*. In the approximate method of solution given by Toepfer, one employs the general property of the solu-

---

* Blasius, Zeitschrift fuer Math. und Physik, Vol. 56 (1908); Toepfer, Zeitschrift fuer Math. und Physik, Vol. 60 (1912).

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

tions of the eq. (3.12), which is included in what follows.

If the function $\varphi_0(\lambda)$ is a certain solution of the eq. (3.12), then the function

$$\varphi(\lambda) = a\varphi_0(a\lambda)$$

where a is any constant, is also the solution of the eq. (3.12). Direct verification easily convinces one of the correctness of this property.

As the original solution $\varphi_0(\lambda)$ we take the solution of the eq. (3.12) satisfying the following boundary conditions

$$\varphi_0(0) = \varphi_0'(0) = 0 \text{ and } \varphi_0''(0) = 1$$

The function $\varphi_0(\lambda)$ can be constructed by ordinary approximation methods. As a result of the approximate solution one can calculate the limit

$$\lim_{\lambda \to +\infty} \varphi_0' \varkappa_0 \lambda = k$$

Numerical computations give $k = 2.0854$. The solution of the eq. (3.12) represented by the following formula

$$\varphi(\lambda) = a\varphi_0(a\lambda),$$

satisfies the following boundary conditions

$$\varphi_0(0) = \varphi_0'(0) = 0 \text{ and } \varphi_0''(0) = a$$

where we have

$$\lim_{k \to +\infty} \varphi'(\lambda) = k \cdot a^{2/3}$$

Hence it is clear that in order to obtain the sought for solution it is sufficient to set:

$$a = \frac{1}{k^{3/2}} = 0{,}332.$$

Having defined $a = \varphi''(0)$, we easily find with the help of the eq. (3.10') the frictional resistance which is experienced by the plate.

We have for the frictional stress τ on the plate the following expression:

$$\tau = \mu \left(\frac{du}{dy}\right)_{y=0} = \mu U_0 \frac{\varphi''(0)}{\sqrt{\frac{\nu x}{U_0}}} = 0{,}332 \sqrt{\frac{\rho \mu U_0^3}{x}}. \qquad (3.14)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Utilizing this result, we calculate the resistance W of the portion of the plate of width b and length l:

$$W = b\int_0^l \tau\, dx = 0{,}644\, b\sqrt{\rho\mu l U_0^3}. \qquad (3.15)$$

The frictional stress τ and the resistance W have turned out to be proportional to the one and a half power of the velocity of circulation U_0.

From the relation (3.15) for the coefficient of friction c_f we obtain the following formula:

$$c_f = \frac{2W}{\rho b l U_0^2} = \frac{1.328}{\sqrt{R}},$$

where

$$R = \frac{U_0 l}{\nu}$$

The data of experiments* with two-dimensional smooth plates is in good agreement with that found by the law governing the distribution of velocity and resistance in the case of laminar regime of circulation, which is characterized by small values of the Reynolds number

$$R = \frac{U_0 l}{\nu} < 3 \times 10^5$$

In the case of large values of the Reynolds number the above considered laminar steady-state motion is unstable. There then arise turbulent motion, which changes in an essential manner the laws governing the resistance and distribution of velocities near the plate.

Section 4. Isotropic Turbulent Motions of an Incompressible Fluid

1. Averaging of turbulent motions. Many motions of fluid which are observed in nature and most motions with which we have to do in engineering are characterized by the presence of orderless nonsteady-state motion of a fluid, which motion is imposed upon the axial motion of a fluid that can be represented as a certain statis-

* Hansen, ZAMM, Vol. 8. No. 3, 1928; Page, ARC, R & M, No. 1580, 1934.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

tical mean motion. Motions of a fluid of such a kind are called turbulent.

In the case of the turbulent motion of a fluid the velocity, pressure and other quantities at each point of the flow undergo irregular pulsating variations around certain average values. Therefore in order to investigate turbulent flows one can expediently utilize the concepts of the theory of probability; in this case the instantaneous values of the mechanical characteristics are considered as random (stochastic) quantities, and the mean values are defined as the mathematical expectations*. More often, however, the mean values are defined as the ordinary averages over the time. The intervals of time over which the averaging is conducted must be sufficiently large in comparison with the time of the individual pulsations and must be small in comparison with the time of a marked variation of the mean values if the averaged motion is nonstationary**.

The mean values of the pressures, of the velocity projections, of the products of the projections of the velocity pulsations (fluctuations), all taken at one and the same point or at various neighboring points (so called moments of connection for the velocity), etc. depend in a strong degree upon the presence of turbulent mixing, which enables one to equalize and to smooth the variation in the mean quantities as a function of the coordinates of the points of the space.

Experience shows that laminar steady-state motions of a fluid in the case of large values of the Reynolds number, i.e. in the case of large velocities and large scales, become unstable and pass over into nonsteady-state turbulent motions, which in the mean in a number of cases can be steady-state motions.

In contemporary hydromechanics and aerodynamics the principal and pressing

---

* See, for example, M.D.Millionshchikov, Doklady Akademii Nauk SSSR (Reports of the Academy of Sciences of the USSR), Vol. 22, No. 5, 1939.

**For the subject of averaging see N.Ye.Kochin, I.A.Kibel' and N.V.Roze, Teoreticheskaya Gidromekhanika (Theoretical Hydromechanics), Part II, State Press, 1948.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

problems on the motion of a fluid in bounded spaces and on the resistance during the motion of bodies in a fluid are closely connected with the investigation of turbulent flows.

All theoretical investigations concerning the motion of a viscous fluid proceed from the postulate concerning the correctness of the Navier-Stokes equations for true nonsteady-state pulsating motion. However, in view of the extreme complication, tortuousness and intricacy of the trajectories of the particles of the fluid in the case of turbulent motion and, apparently, in the case of generally all the principal functional connections, the obtaining of the solution to the Navier-Stokes equations for such motions is an extremely tedious and complex task, which can be compared with the task of describing the motion of the individual molecules of a large volume of gas. Therefore, just as in the kinetic theory of gases, also in hydromechanics the principal tasks in the turbulent motions of a fluid become the problems of finding the functional relations between the mean quantities.

The equations of motion for the mean quantities can be obtained by way of the averaging of the equations of motion for the quantities that describe the instantaneous state of motion. In view of the nonlinearity of the equations of motion after the averaging we obtain a larger number of unknowns than the number of equations, since the mean values of the nonlinear terms, for example the products of two or more quantities, represent new unknowns*. Thus during the averaging of the Navier-Stokes equations for an incompressible fluid**, besides the mean values***, it is

---

* For greater detail on this see L.Keller, Aufstellung eines Systems von Charakteristiken der atmosphaerischen Turbulenz, Izvestiya Glavnoy Fizicheskoy Observatorii (News of the Main Physical Observatory), 1925.

** The assumption of the incompressibility of the fluid is kept in all future says.

*** In accordance with the commonly accepted convention we shall designate the mean values of the quantities by means of letters with a line placed above them.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

necessary to introduce for the projections of the velocity $\bar{u}_1$, $\bar{u}_2$, $\bar{u}_3$ the mean values of the products $\overline{u_i u_k}$ (1, k = 1, 2, 3) also into the considerations.

Consequently some of the equations of hydromechanics which are sufficient for the study of the original motions are insufficient for the mathematical study of the averaged turbulent motions. Therefore a complete theoretical investigation of the averaged turbulent motions is possible only on the basis of certain supplementary hypotheses whose correctness in the final reckoning can be established only by experience*.

The contents of a number of works on the investigation of the turbulent motions of fluids are reduced to a study of the correctness of various more or less probable simple and natural hypotheses which can be verified experimentally and which permit one to state and to solve theoretically the principal problems concerning the turbulent motion of a fluid.

At the present time there still does not exist any general mathematical statement of the problem on arbitrary averaged turbulent motions, and in general there is still no clarification of the possibility itself of giving a mathematical formulation of the problem that is similar to the formulation of the problem on the true motion of a viscous fluid.

The methods of the theory of dimensions and the considerations on the similarity of motions have been utilized many times as the principal methods for the investigation of the turbulent motions of a fluid.

2. The properties of homogeneity and isotropicity. Let us consider the turbulent motion of a viscous fluid which fills the entire space**.

* We shall assume that we are concerned here with the formulation of a mathematical problem with a finite number of unknowns.

** All of what follows can refer to the motion of a fluid in a bounded space or in finite volumes of a fluid if we admit that the influence of the boundaries can be neglected.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The state of the motion at each moment of time t is determined by the initial disturbances (the disturbances of the fluid at the moment t = 0) and by the properties of inertia and viscosity of the fluid, i.e. by the quantities ρ and μ.

Let us select a system of kinematically similar initial disturbances. Each individual disturbed state can be determined by the assignment of the scales for the length and for the time, after we have indicated for this a certain characteristic velocity u_0 and a certain characteristic magnitude l_0 having the dimension of length.

Consequently the turbulent state of the motion of a fluid for a system of kinematically similar initial disturbances is determined by the following parameters

$$\rho,\ \mu,\ l_0,\ t,\ x_1,\ x_2,\ x_3$$

where x_1, x_2, x_3 are the coordinates of the points of the space*.

The thus obtained set of turbulent motions contains motions which are not dynamically similar. For the purpose of the similarity of two turbulent motions it is necessary and sufficient that the Reynolds number for both of the motions possess the same value

$$\frac{u_{01}l_{01}\rho_1}{\mu_1} = \frac{u_{02}l_{02}\rho_2}{\mu_2}.$$

The moment of time and the coordinates of the points corresponding to the similar states are determined from the following relations

$$\frac{u_{01}t_1}{l_{01}} = \frac{u_{02}t_2}{l_{02}},\quad \frac{x_{i1}}{l_{01}} = \frac{x_{i2}}{l_{02}} \quad (i = 1,\ 2,\ 3).$$

The values of u and l are determined by functions of the following form

$$\left.\begin{aligned} \frac{u}{u_0} &= f\left(\frac{u_0 t}{l_0},\ \frac{u_0 l_0 \rho}{\mu},\ \frac{x_1}{l_0},\ \frac{x_2}{l_0},\ \frac{x_3}{l_0}\right), \\ \frac{l}{l_0} &= \varphi\left(\frac{u_0 t}{l_0},\ \frac{u_0 l_0 \rho}{\mu},\ \frac{x_1}{l_0},\ \frac{x_2}{l_0},\ \frac{x_3}{l_0}\right). \end{aligned}\right\} \qquad (4.1)$$

* The systems of coordinates are disposed similarly relative to the distribution of the initial disturbances.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In general, for any dimensionless mechanical characteristic which is connected with one point of the fluid, formulas of similar form will be correct. In addition to the indicated parameters, these functions depend also upon dimensionless parameters that determine the laws governing the distribution of the initial disturbances.

In a study of turbulent motions it is necessary to take into consideration the characteristics that depend upon the state of the motion at two or at several points. For example, the mean values of the products of the projections of the velocity at m points $M_1(x_1', x_2', x_3')$, $M_2(x_1'', x_2'', x_3'')$, ... $M_m(x_1^m, x_2^m, x_3^m)$, namely the values

$$\tau_{k_1 k_2 \ldots k_n}(M_r, M_s, \ldots, M_d) = \overline{u_{k_1}(M_r)\, u_{k_2}(M_s) \ldots u_{k_n}(M_d)} \qquad (k_1, k_2, \ldots, k_n = 1, 2, 3) \tag{4.2}$$

form a tensor whose components for a system of similar initial disturbances depend also upon 3m coordinates x_l^k ($l = 1, 2, 3$, $k = 1, 2, \ldots, m$). The indices r, s, ..., d represent a certain definite sequence of numbers 1, 2, ..., m)($m \leq n$). In the general case the quantities τ depend essentially upon all the coordinates.

Turbulent flow is called homogeneous if all the mean quantities at each point do not depend upon the position of the point, and the mean values for the quantities depending upon several points depend only upon their relative disposition, i.e. only upon the differences of the coordinates $x_i^p - x_i^q$. In a homogeneous turbulent field of velocities the functions (4.1) do not depend upon the ratios $\frac{x_1}{l_0}$ $\frac{x_2}{l_0}$ $\frac{x_3}{l_0}$. The averaged characteristics of the motion of a fluid near any two points are similar (identical). It is obvious that in the case of the homogeneous turbulent flow the initial disturbances must possess certain properties of uniformity of distribution of the disturbances over the volume of the fluid.

Homogeneous turbulent flow is called isotropic if the tensors of the connection of the velocity projections which are defined by the equality (4.2) depend for any n and m upon the orientation in the space of the polyhedron $M_1 M_2 \ldots M_m$ and upon the

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

transition to mirror images of this polyhedron relative to the coordinate planes*.

The components of the tensor of connection can depend upon the position of the axes of the coordinates relative to the polyhedron $M_1M_2...M_m$ and upon the selection of the positive direction along the axes. For isotropic flow the quantities $\tau_{k_1k_2..k_n}$ possess identical value in the systems of coordinates orientated identically relative to the various positions of one and the same polyhedron.

According to definition isotropic turbulent motion possesses the property of symmetry in the mean. It is assumed that in the course of a sufficiently large interval of time over which the averaging is carried out all the directions of the velocity at each point of the space are equally probable; this is in agreement with the assumption that at each instant the motion of the fluid is continuous and the nearby points possess approximately identical velocities.

Homogeneous isotropic turbulent motion can be considered as the simplest form of turbulent motion. A disturbed fluid which is left to itself moves in accordance with inertia; under the action of internal forces of viscosity there occurs a dissipation of the kinetic energy (i.e. the motion is characterized by damping) and degeneration of the turbulent disturbances is brought about. In the study of isotropic turbulence the principal problem consists in the determination of the laws governing the damping**.

In the general case the turbulent motions are characterized by equalization or leveling; that is, by the diffusion of the disturbances. Isotropic turbulent motion can be considered in a number of cases as a special kind of limiting turbulent motion, just as nonsteady-state current frequently can be approximately replaced by

* If this condition is correct only for $2<n<N$, $2<m<M$, where N and M are certain whole numbers, then such a flow can be considered as approximately isotropic.

**At the present time the problem concerning the existence and the possible different aspects of isotropic turbulent motion of a viscous fluid has still not been solved.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

steady-state limiting motion.

As experiments have shown, turbulent motions of air that give rise to rapidly moving checker-pattern grids or lattices behind can be considered as homogeneous isotropic turbulent motions. To the same case one can reduce also turbulent motion of air blown through an immobile lattice or cascade, if one adds to the entire system a translation with constant velocity. We encounter such a problem in the investigation of turbulence in aerodynamic pipes.

3. The properties of symmetry of the tensors of the moments of connection of the velocities. The assumption concerning the isotropicity brings in a number of relations between the components of the tensor $\tau_{k_1 k_2 \ldots k_n}$. For example, if the components of the tensor $\tau_{k_1 k_2 \ldots k_n}$ are formed as the mean values for the projections of the velocity of one and the same point $M_1 = M_2 = \ldots = M_m$, then it is obvious that for n odd we have the following:

$$\tau_{k_1 k_2 \ldots k_n} = 0$$

For the case of n even, only those components are not equal to zero which have each projection of the velocity encountered in the even power. In particular, for n = 1 we have the following:

$$\bar{u}_1 = \bar{u}_2 = \bar{u}_3 = 0$$

and for n = 2 we have the following:

$$\overline{u_1 u_2} = \overline{u_1 u_3} = \overline{u_2 u_3} = 0, \quad \overline{u_1^2} = \overline{u_2^2} = \overline{u_3^2} = \frac{1}{3}\overline{v^2}; \tag{4.3}$$

finally, for n = 3 all the components are equal to zero and so on.

If the points $M_1, M_2, \ldots, M_m$ are different or we have in all two points, then in this case certain conditions of symmetry will hold true, since in the case of only two points the tensor of connection depends upon only the mutual distance r of the points under consideration. The components $\tau_{k_1 k_2 \ldots k_n}$ depend upon r and upon the orientation of the axes of the coordinates relative to the interval $M_1 M_2$.

It is easy to study the correctness of the following relations which proceed

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

It is easy to study the correctness of the following relations which proceed from the isotropicity of the motion:

$$\overline{u_i(M_1)\,u_j(M_2)} = \overline{u_i(M_2)\,u_j(M_1)} \quad (i = 1,\ 2,\ 3,\ \ j = 1,\ 2,\ 3). \tag{4.4}$$

Let us take the point M_1 at the origin of the coordinates, and the point M_2 on the x_1 axis; then we shall have the following:

$$\tau_{11} = b_d^d \neq 0, \quad \tau_{22} = \tau_{33} = b_n^n \neq 0, \quad \tau_{12} = \tau_{13} = \tau_{23} = 0. \tag{4.5}$$

The upper indices correspond to the point M_2, and the lower indices correspond to the point M_1. It is obvious that the quantities b_d^d and b_n^n depend upon x_1 and t and are even functions of x_1.

If the point M_2 is disposed arbitrarily relative to the axes of the coordinates then all the components τ_{ik} are easily expressed in terms of b_d^d and b_n^n. The formulas that express τ_{ik} in terms of b_d^d and b_n^n represent the formulas of the transformation of the components of the tensor in the case of the transition from a special system of coordinates for which the x_1 axis passes through the point M_2 to an arbitrarily given system of coordinates. These formulas possess the form

$$\tau_{ik} = (b_d^d - b_n^n)\, l_{i1} l_{k1} + b_n^n l_{ik} \tag{4.6}$$

where l_{ik} are the direction cosines of the axes of the given system of coordinates relative to the special system.

For r = 0 we have the following:

$$b_d^d = b_n^n = \frac{1}{3}\,\overline{b^2} = b. \tag{4.7}$$

Let us now consider the conditions of symmetry for the tensor of connection of the third order. The formulas of transformation of the components of the tensor of the third order in the case of a transition from one system of coordinates to another possess the form

$$\Pi'_{ijk} = \sum_{\alpha,\ \beta,\ \gamma} \Pi_{\alpha\beta\gamma} l_{\alpha i} l_{\beta j} l_{\gamma k}, \tag{4.8}$$

where l_{sm} are the direction cosines of the new system of coordinates. Let us take

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the transformation

$$x'_1 = -X_1,\ x'_2 = x_2,\ x'_3 = x_3$$

which reduces only to a change in the direction of the x_1 axis to one directly opposite. In this case we then have the following:

$$l_{11} = -1,\ l_{22} = l_{33} = 1,\ l_{sm} = 0\ (s \neq m)$$

therefore if the index of unity is encountered among the indices i, j, k an odd number of times, then it follows from the formula (4.8) that

$$\Pi'_{ijk} = -\Pi_{ijk}.$$

Let the point M_1 coincide with the origin of the coordinates, and let the point M_2 lie on the x_1 axis. It follows from the isotropicity that the components of the tensor of connection of the third order which are formed for the projections of the velocities of the points M_1 and M_2 do not depend upon the direction of the x_2 and x_3 axes.

Hence it follows that the components that contain the indices 2 or 3 an odd number of times are equal to zero, since they cannot change signs in a reversal of the direction of the x_2 or x_3 axes.

Thus if the point M_2 lies on the x_1 axis, then the tensor of connection (which is composed of two projections of the velocity of the point M_1 and one projection of the velocity of the point M_2) possesses the following five components not zero:

$$\tau_{111} = b^d_{dd},$$
$$\tau_{122} = \tau_{133} = b^n_{dn},$$
$$\tau_{221} = \tau_{331} = b^d_{nn}.$$

The components of connection of the third order in the case of the employment of any system of coordinates can be expressed in terms of b^d_{dd}, b^n_{dn}, and b^d_{nn} with the

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

help of the formulas (4.8)*. In the general case, in the place of the variable x_1 it is necessary to take the distance between the points M_1 and M_2, which in what follows is to be designated by the letter r. With increase in the distance between the points their velocities become all the more statistically independent; therefore the components of the tensors of connection of the velocities must tend to zero in the case of unbounded increase in r.

For $r = 0$ the points M_1 and M_2 coincide; in this case we have the following:

$$b^d_{dd} = b^n_{dn} = b^d_{nn} = 0$$

Reversal of the roles of the points M_1 and M_2 are equivalent to a change in the direction of the axes of the coordinates to one directly the opposite; hence it follows that

$$\tau_{ijk}(M_1, M_1, M_2) = -\tau_{ijk}(M_2, M_2, M_1) \tag{4.9}$$

These relations in another form can be written thus:

$$\left.\begin{aligned} b^d_{dd} &= -b^{dd}_d, \\ b^n_{dn} &= -b^{dn}_n, \\ b^d_{nn} &= -b^{nn}_d. \end{aligned}\right\} \tag{4.10}$$

Consequently the moments of connection b^d_{dd}, b^n_{dn} and b^d_{nn} are odd functions of the variable x_1 or of the variable r.

If the projections of the velocities of the pulsation are regular functions which are expanded into Taylor series, then it is obvious that the moments of con-

* Similar properties of the symmetry of the tensors of connection of the projections of velocities can be clarified for tensors of higher orders. Conditions of symmetry for the tensors of connection of the fourth order are given in a paper by M.D.Millionshchikov in Doklady Akademii Nauk SSSR (Reports of the Academy of Sciences of the USSR), Vol. 32, No. 9, 1941.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

nection are also expanded into Taylor series in r. The Taylor series for the moments of the second order b_d^d and b_n^n will contain only even powers of r, and the series for the moments of the third order b_{dd}^d, b_{dn}^n and b_{nn}^d will contain only odd powers of r.

We shall show that the series for b_{dd}^d do not contain any term with the first power of r. In fact we have the following:

$$b_{dd}^d = \overline{u_1^2(0,0,0)\,u_1(r,0,0)} =$$

$$= \overline{u_1^2\left(\frac{\partial u_1}{\partial r}\right)_{r=0}}\cdot r + \frac{1}{6}\,\overline{u_1^2\left(\frac{\partial^2 u_1}{\partial r^2}\right)_{r=0}}\cdot r^3 + \ldots$$

The first term becomes zero, since

$$\overline{u_1^2\left(\frac{\partial u_1}{\partial r}\right)_{r=0}} = \frac{1}{3}\left[\frac{\partial \overline{u_1^3}}{\partial r}\right]_{r=0},$$

and we have $\overline{u_1^3}(r) = 0$ in view of the isotropicity of the turbulent motion. Thus the moments b_{dd}^d expanded in power series can begin with terms of the order at least r^3.

4. The conditions of incompressibility and the dynamic relations. With the help of the operation of averaging of the equation of incompressibility and of the Navier-Stokes equations one can establish the relations between the independent tensors of connection of the velocities*. From the equation of incompressibility we obtain the following relations:

$$b_n^n = b_d^d + \frac{r}{2}\frac{\partial b_d^d}{\partial r}, \tag{4.11}$$

$$b_n^{nd} = -b_d^{nn} - \frac{r}{2}\frac{\partial b_d^{nn}}{\partial r}, \tag{4.12}$$

$$b_d^{dd} = -2v_d^{nn}. \tag{4.13}$$

In experiments the moments of connection b_d^d and b_n^n can be measured directly and independently of each other. Employing the experimental results of Simmons

* Friedman und Keller, Proc. I Intern. Congr. Appl. Mech., Delft, 1924; Karman and Howarth, Proc. Royal Society A, 164, No. 917, 1938.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

which were obtained in an aerodynamic pipe, Taylor showed* that experiments confirm very well the equality (4.11) (see Fig.21).

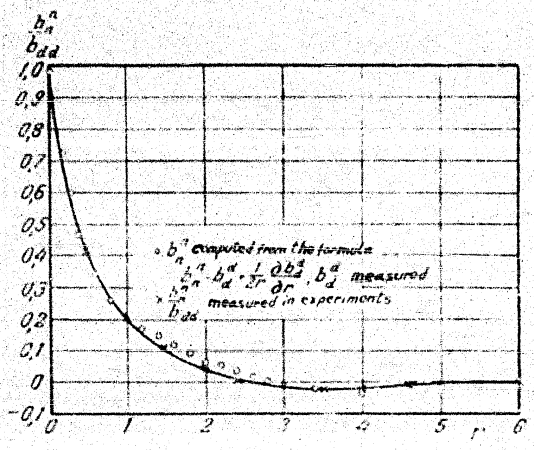


Fig.21. Experimental data confirm well the theoretical formula connecting b_d^d and b_n^n.

The relations (4.12) and (4.13) show that for small r the power series in r for b_d^{nn} and b_n^{nd} must begin with a term of the order at least r^3, since the series for b_d^{dd} possesses this property. The equalities (4.11), (4.12) and (4.13) show that the determination of the tensors of connection of the projections of velocities of the second and third orders reduces to the determination of the two functions $b_d^d)r,t)$ and $b_d^{nn}(r,t)$. From the Navier-Stokes equations for these quantities is obtained only one equation

$$\nu\left(\frac{\partial^2 b_d^d}{\partial r^2} + \frac{4}{r}\frac{\partial b_d^d}{\partial r}\right) - \frac{1}{2}\frac{\partial b_d^d}{\partial t} = \frac{\partial b_d^{nn}}{\partial r} + \frac{4}{r} b_d^{nn}, \qquad (4.14)$$

where $\nu = \frac{\mu}{\rho}$. This equation in somewhat different form was obtained for the functions $f = \frac{b_d^d}{b}$ and $h = \frac{b_d^{nn}}{b^{3/2}}$ by Karman and Howarth. The equation (4.14) in the described form

---

* G.I.Taylor, Journal of the Aeronautical Sciences, No. 8, Vol. 4, June 1937.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

was treated by L.G.Loytsyanskiy* and M.D.Millionshchikov.

Multiplying the eq. (4.14) by r^4 we obtain the following expression:

$$\frac{1}{2}\frac{\partial b_d^d r^4}{\partial t} = \frac{\partial}{\partial r}\left[\nu r^4 \frac{\partial b_d^d}{\partial r} - r^4 b_4^{nn}\right] \qquad (4.14')$$

Assume that for $r \to +\infty$ the order of the cut-off of the functions $\frac{\partial b_d^d}{\partial r}$ and b_d^{nn} is higher than $\frac{1}{r^4}$, and that the expression in the square brackets becomes zero at $r = 0$, we then find:

$$\frac{d}{dt}\int_0^\infty b_d^d r^4\, dr = 0$$

or

$$\Lambda = \int_0^\infty b_d^d r^4\, dr = \text{const.} \qquad (4.15)$$

The existence of the thus determined invariant quantity Λ was shown by L.G. Loytsyanskiy*. We note, however, that the finiteness and invariance of Λ are obtained as a result of the assumption concerning the order of the cut-off of b_d^{nn} and the derivative $\frac{\partial b_d^d}{\partial r}$

For the determination of b_d^d and b_d^{nn} the one equation (4.14) is insufficient. The transition to the moments of connection of the fourth** and higher orders also does not give any closed system of equations. The equation for the moments of the third order contains the moments of the fourth order; and the succeeding equations contain the moments of the fifth order and so on. In addition to the nonclosedness of these equations we also encounter the problem of the assignment of initial conditions; therefore, in order to study theoretically the isotropic turbulence we

---

* Loytsyanskiy L.G., Trudy TsAGI (Works of the Central Aero-Hydrodynamics Institute) No. 440, 1939.

** See M.D.Millionshchikov, Doklady Akademii Nauk SSSR (Reports of the Academy of Sciences of the USSR), No. 9, Vol. 32, 1941.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

also require additional hypotheses of a mechanical nature.

Detailed hypotheses have been posed by Karman and Howarth in the treatment of the cases of small and large values of the Reynolds number.

5. Exceptional stage of degeneration of isotropic turbulence. The turbulent motion of a fluid dampens, and consequently the quantity $b = \overline{u_1^2}$ tends to zero as time t tends to infinity.

For very small magnitudes of the velocities of pulsation the components of the moments of connection of the third order are small in comparison with the components of the moments of connection of the second order. This fact gives the grounds for neglecting in equation (4.14) the moments of connection of the third order. Replacing in equation (4.14) the right hand part by zero we obtain the following:

$$\nu\left(\frac{\partial^2 b_d^d}{\partial r^2} + \frac{4}{r}\frac{\partial b_d^d}{\partial r}\right) - \frac{1}{2}\frac{\partial b_d^d}{\partial t} = 0. \qquad (4.16)$$

In order to find a definite solution of equation (4.16) it is necessary to know the function $b_d^d(r, 0)$, which gives the initial distribution for the moment of connection of the longitudinal velocities of two points.

We have in mind obtaining the solution of equation (4.16) for large values of the time t. For large t the details in the singularities of function $b_d^d(r,0)$ must possess minor significance; therefore during the investigation of the asymptotic behavior of the function $b_d^d(r, t)$ for $t \to +\infty$ the initial conditions can be taken into consideration in a certain simplified form.

The function $b_d^d(r, 0)$, besides on the distance r, depends also upon dimensional constants, which must certainly exist since the dimensions of b_d^d and r are different.

Let us assume now that the influence of the initial distribution of the moments of connection is exerted on the asymptotic behavior of b_d^d for $t \to +\infty$ by means of only one constant factor A having the dimensionality $L^p T^q$. This constant can characterize in a certain sense the total properties of the initial distribution of

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the moment of connection b_d^d. In particular, the value of the constant A can be determined by a formula of the following form

$$A = \left[\int_0^\infty b_d^d (r, t) \frac{r^{p+2q+1}}{\nu^{2+q}} \Phi\left(\frac{r^2}{\nu t}\right) dr \right]_{t=0},$$

where $\Phi\left(\frac{r^2}{\nu t}\right)$ is a certain function

Let us set:

$$b_d^d = A\nu^{1-\frac{p}{2}} \frac{f(r, t, \nu)}{t^{1+q+\frac{p}{2}}}.$$

According to assumptions, the dimensionless quantity f(r, t, ν) depends only upon the three dimensional parameters r, t, and ν; therefore the function f(r, t, ν) depends only upon the combination $\xi = \frac{r^2}{\nu t}$. Consequently the correct formula is one of the following form

$$b_d^d = \frac{A\nu^{1-\frac{p}{2}}}{t^{1+q+\frac{p}{2}}} f\left(\frac{r^2}{\nu t}\right). \tag{4.17}$$

In what follows we shall consider such motion for which we have f(∞) = 0, and f(0) = 1; the first condition corresponds to the statistical independence of the velocities of two points as the distance tends to infinity r → + ∞, and the second condition is easily satisfied by a choice of a numerical value for the constant A.

From the eq. (4.17) for the case r = 0 we have the following:

$$b = \frac{1}{3}\overline{v^2} = \frac{A\nu^{1-\frac{p}{2}}}{t^{1+q+\frac{p}{2}}}. \tag{4.18}$$

In consequence of the damping of the turbulent motion we must have $1 + q + \frac{p}{2} > 0$, therefore, since f(0) = 1, the distribution of the disturbances which are determined by the eq. (4.17) is characterized by nonregularity at t = 0.

It is obvious that the following function

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$f\left(\frac{r^2}{vt}\right)=\frac{\overline{u_1(0,0,0,t)\,u_1(r,0,0,t)}}{\overline{u_1^2}}$$

represents the coefficient of correlation between the projections of the velocities of two points onto the interval which connects these two points. In the case under consideration the coefficient of correlation at $t = 0$ equals zero if $r \neq 0$ and equals unity if $r = 0$.

Utilizing the eq. (4.17) we obtain from eq. (4.16) the following:

$$f'' - \left(\frac{1}{8}+\frac{5}{2\xi}\right)f' + \frac{5\alpha}{4\xi}f = 0, \tag{4.19}$$

where

$$\alpha = \frac{1+q+\frac{p}{2}}{10}.$$

This equation was obtained by Karman and Howarth directly from the assumption that the coefficient of correlation depends only upon the combination $\frac{r^2}{vt}$. The number α is an arbitrary constant introduced by Karman.

The solution of the eq. (4.19) depends only upon the constant α. For the case of α = const. the difference in the values of p and q is important only for the purpose of the indication of the dimension of the constant A. The eq. (4.17) indicates that only the constant $Av^{1-\frac{p}{2}}$, is essential, which possesses the following dimensionality

$$L^2T^{q+\frac{p}{2}-1} = L^2T^{10\alpha-2}.$$

The general solution of the eq. (4.19) is regular for all ξ not equal to zero or infinity $\xi \neq 0, \infty$. The regular solution for $\xi = 0$ which satisfies the condition $f(0) = 1$, is represented by the following formula

$$f(\xi) = M\left(10\alpha, \frac{5}{2}, -\frac{\xi}{8}\right) = \tag{4.20}$$

$$= 1 - \alpha\xi + \frac{\alpha(10\alpha+1)}{4\cdot 7\cdot 2!}\xi^2 - \frac{\alpha(10\alpha+1)(10\alpha+2)}{4^3\cdot 7\cdot 9\cdot 3!}\xi^3 + \ldots,$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

where M (α, λ, x) is the confluent hypergeometric function*. For this solution in the case of very large values of ξ approaching infinity $\xi \to +\infty$ the following asymptotic expansion is correct

$$f(\xi) = \frac{\Gamma\left(\frac{5}{2}\right) 8^{10\alpha}}{\Gamma\left(\frac{5}{2} - 10\alpha\right)} \frac{1}{\xi^{10\alpha}} \left[1 + 8 \frac{10\alpha\left(10\alpha - \frac{3}{2}\right)}{\xi} + \right. \quad (4.21)$$

$$\left. + 8^2 \frac{10\alpha\,(10\alpha+1)\left(10\alpha - \frac{3}{2}\right)\left(10\alpha - \frac{1}{2}\right)}{2!\,\xi^2} + \ldots\right],$$

where Γ is the symbol of the Euler gamma function.

From the eqs. (4.17) and (4.21) it is easy to clarify the behavior of the moment b_d^d for the case of the distance approaching positive infinity ($r \to +\infty$) or for the case of the time approaching zero ($t \to 0$). For all α greater than zero ($\alpha > 0$) the following limiting relation is correct

$$\lim_{\frac{r^2}{\nu t} \to \infty} b_d^d r^{20\alpha} = A \nu^{2+q} \frac{\Gamma\left(\frac{5}{2}\right) 8^{10\alpha}}{\Gamma\left(\frac{5}{2} - 10\alpha\right)}. \quad (4.22)$$

If we have $10\alpha - \frac{5}{2} = k$, i.e. $\alpha = \frac{1}{4} = k \cdot 0{,}1$, where k is a whole positive number, then the studied solution of the eq. (4.19) acquires the following simple final form

$$f = M\left(k + \frac{5}{2}, \frac{5}{2}, -\frac{\xi}{8}\right) = \frac{\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(k + \frac{5}{2}\right)} \left[\frac{\Gamma d^k}{d\lambda^k}\left(\lambda^{k+\frac{3}{2}} e^{-\frac{\xi\lambda}{8}}\right)\right]_{\lambda=1} =$$

$$= \Gamma\left(\frac{5}{2}\right)\left[\frac{1}{\Gamma\left(\frac{5}{2}\right)} - \frac{k}{\Gamma\left(\frac{7}{2}\right)} \frac{\xi}{8} + \frac{k(k-1)}{\Gamma\left(\frac{9}{2}\right) 2!} \frac{\xi^2}{8^2} - \ldots\right.$$

$$\left. \ldots - (-1)^k \frac{\xi^k}{\Gamma\left(k + \frac{5}{2}\right) 8^k}\right] e^{-\frac{\xi}{8}}.$$

---

* See E.Jancke and F.Emde, Tablitsy Funktsiy s Formulami i Krivymi (Tables of Functions with Formulas and Curves), State Technical Press, 1949.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In this case the distribution of the moment b_d^d for $t = 0$ possesses the character of a source. In the case of $r \neq 0$ and $t = 0$ we have: $b_d^d = 0$; and for the case of $r = 0$ and $t \to 0$ we have: $b_d^d \to \infty$.

The eq. (4.16) can be interpreted as the equation of heat conduction in five-dimensional space in the presence of symmetry relative to the origin of the coordinates. The solution corresponding to $\alpha = \frac{1}{4}$ (when $k = 0$) can be considered as the analog of the heat source in five-dimensional space*. In this case the solution possesses the following form

$$b_d^d = A\nu^{1-\frac{p}{2}} \frac{e^{-\frac{r^2}{8\nu t}}}{\sqrt{t^5}}. \qquad (4.23)$$

The constant $A\nu^{1-\frac{p}{2}}$ for $\alpha = \frac{1}{4}$ possesses the dimensionality $L^2T^{0.5}$.

It is easy to verify that the above considered solutions for $\alpha = \frac{1}{4} = 0.1\ k$, where $k > 0$ is a whole number, can be obtained from the solution (4.23), which corresponds to a simple source, by differentiation with respect to time:

$$b_d^d = \frac{A\nu^{1-\frac{p}{2}}}{(-1)^k \cdot \frac{5}{2} \cdot \frac{7}{2} \cdot \frac{9}{2} \cdots \left[\frac{5}{2} + (k-1)\right]} \frac{\partial^k}{\partial t^k} \left(\frac{e^{-\frac{r^2}{8\nu t}}}{t^{5/2}} \right). \qquad (4.24)$$

Hence it is clear that these solutions correspond to time dipoles whose order is determined by the number k. In these cases the character of the variation of the coefficient of correlation $f(r/\sqrt{\nu t})$ is represented in Fig.22.

It is not difficult to see that for $\alpha = \frac{1}{4}$ the parameter Λ possesses a finite value different from zero.

On the basis of the eq. (4.17) we can write Λ as follows:

$$\Lambda = \int_0^\infty b_d^d r^4\, dr = \frac{A\nu^{1-\frac{p}{2}}}{2} \frac{(\nu t)^{5/2}}{t^{10\alpha}} \int_0^\infty f(\xi)\, \xi^{9/2}\, d\xi. \qquad (4.25)$$

* See L.G.Loytsyanskiy, Trudy TsAGI, No. 440, 1939; M.D.Millionshchikov, DAN SSSR, Vol. XXII, No. 5, 1939.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

This equality indicates that the constancy in time of $\Lambda \neq 0, \infty$ is incompatible with the inequality $\alpha \neq \frac{1}{4}$. From the expansion (4.21) it is evident that for $0<\alpha<\frac{1}{4}$ we have: $\Lambda = \infty$. For $\alpha > \frac{1}{4}$ we have: $\Lambda = 0$; in the final case the function $f(\xi)$ certainly changes sign when ξ varies from zero to infinity.

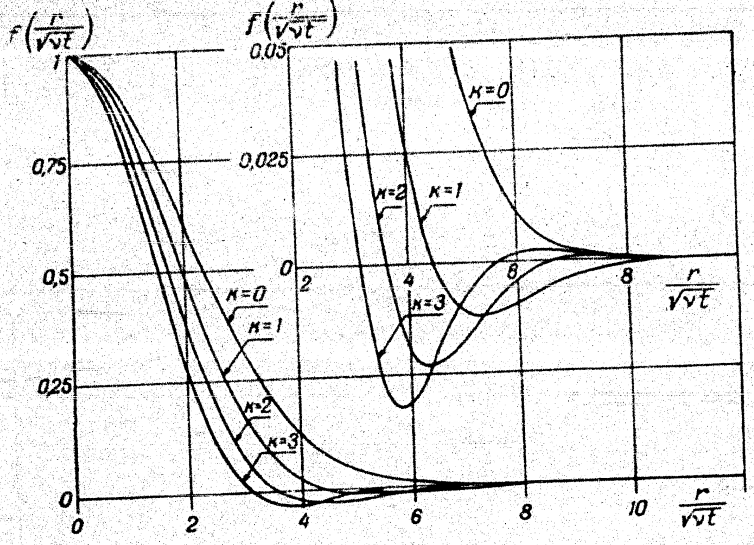


Fig.22. The Coefficient of Correlation for Motions of the Type of a Source for Various Values of k

The solutions determined by the eqs. (4.17) and (4.20) give the continuous laws governing the distribution for $b_d^d(r, t_o)$ for any α and $t_o>0$. It is evident that in these partial cases, and consequently also in the general case, the law governing the damping depends in an essential way upon the properties of the initial disturbances. Therefore, in order to obtain the asymptotic laws governing damping with the aid of the considered solutions, it is still necessary to employ either additional hypotheses of a mechanical character or experimental data.

6. The problem of turbulent motion in an aerodynamic pipe. As we have already indicated, the investigation of isotropic turbulence is connected with the study of

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

turbulence caused by the directing lattices or cascades in aerodynamic pipes.

Let us consider the problem of the development of turbulent motion in an incompressible fluid beyond the lattice moving progressive with constant velocity u along the x axis. For sake of simplicity we assume that the fluid is limitless, and the lattice is formed of a doubly periodic system of congruent cells displaced each relative to the others in a plane perpendicular to the x axis. We take the set of motions of the lattices with geometrically fixed form.

The motion of the fluid in the plane perpendicular to the x axis is determined by the following system of parameters

$$\rho,\ \mu,\ u,\ M,\ x = u\,(t - t_0),$$

where M is a characteristic dimension of the lattice, and x is the coordinate of the plane under consideration; the constant t_0 is determined by the initial reckoning for x.

The dimensionless characteristics of the motion depend upon two parameters

$$\frac{x}{M} \quad \text{и} \quad \frac{\rho u M}{\mu}.$$

We assume that for sufficiently large values of $\frac{x}{p}$ it is possible to take that the turbulent motion is isotropic and that in the different planes perpendicular to the x axis the development of the isotropic turbulence is different only in phase. In this case the characteristics of the turbulent motion of the fluid are determined by the parametersρ, μ, u, M, t.

The coefficients of correlation $f = \frac{b_d^d}{b}$ and $h = \frac{b_d^{nn}}{b^{3/2}}$ depend upon the following dimensionless parameters

$$\frac{\rho u M}{\mu},\ \frac{ut}{M},\ \frac{r^2}{\nu t}.$$

The eq. (4.17) is obtained as a consequence of the assumption that for sufficiently large values of the parameter $\frac{ut}{M}$ this parameter becomes nonessential.

From the eq. (4.18) it follows that

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{b_0}} \left(\frac{t}{t_0}\right)^{5\alpha};$$

setting $t = t_0 + \frac{x}{u}$, we obtain the following:

$$\frac{u}{\sqrt{b}} = \frac{u}{\sqrt{b_0}} \left(1 + \frac{x}{ut_0}\right)^{5\alpha}. \qquad (4.26)$$

The eq. (4.26) gives the law governing the deamping of turbulent pulsations along the axis of the pipe.

On the basis of a number of experimental data obtained in aerodynamic pipes, Taylor* proposed the empirical formula of the following form

$$\frac{u}{\sqrt{b}} = A + B\frac{x}{Mt}, \qquad (4.27)$$

where A and B are constants.

For agreement of the eqs. (4.26) and (4.27) it is necessary to set $\alpha = \frac{1}{5}$. For $\alpha = \frac{1}{5}$ we have: $\left[Av^{1-\frac{p}{2}}\right] = L^2$

7. Turbulent motions with large pulsations. If the parameter $\frac{ut}{M}$ is not essential, then the coefficients of correlation maintain constant value for $X = \frac{r}{vt} = \frac{r}{l} = \text{const}$. The influence of time is reduced to the change of the scale for r.

In the solutions under consideration the change in the scale is determined by the following relation

$$l = \sqrt{\nu t} = \sqrt{\frac{\nu(x + x_0)}{u}}. \qquad (4.28)$$

From the eq. (4.28) it follows that for x = const., i.e. at a fixed point relative to the lattice, the scale l is constant in time. Besides this, the eq. (4.28) indicates that this scale depends upon the velocity u.

In the cited work of Taylor is presented certain experimental data which does

---

* Taylor, Proc. Roy. Soc. A, Vol. 151, 1935 and A, Vol. 156, 1936.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

not confirm the preceeding conclusion. This circumstance has led to the necessity of perfecting and modifying the theory applicable to the case of large pulsations. For large pulsations the principal value undergoes an interchange of momenta between the intermixing masses of the fluid. In these processes the main role is played by the property of inertia of the fluid.

The property of viscosity plays a large role in the development of motions of very small pulsations, in which there is realized the principal process of dissipation of the kinetic energy. Keeping in mind the study of large pulsations and following Karman and Howarth, we assume that for sufficiently large values of r the formulas of the following form are correct

$$\frac{b_d^d}{b} = f\left(\frac{r}{l}\right), \tag{4.29}$$

$$\frac{b_d^{nn}}{b^{3/2}} = h\left(\frac{r}{l}\right), \tag{4.30}$$

where l is a certain linear quantity, which can depend upon time and upon constant parameters that determine the isotropic turbulent motion*. The eqs. (4.29) and (4.30) indicate that the influence of the time upon the coefficients of correlation is reduced to a change in the scale for the distance l. The property of viscosity exerts an influence, implicitly through the quantities b and l, upon the phenomenon of energy dissipation.

To the quantity l we shall not attribute any concrete geometrical or mechanical significance. The determination of l can be considered as an additional hypothesis. In particular, if one assumes that the invariant $\Lambda \neq 0, \infty$, i.e. there exists a corresponding integral and the conditions formulated on page 136 (original Russian-

---

* Further conclusions rely essentially upon the assumptions included in the eqs. (4.29) and (4.30). Applicability of these assumptions for the description of the actual phenomena require additional clarification.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

language work, with formulas (4.14') and (4.15) are satisfied, then this gives the dependence between l and b. In fact we have the following,

$$\Lambda = b\int_0^\infty f\left(\frac{r}{l}\right) r^4\, dr = bl^5 \int_0^\infty f(\chi)\,\chi^5\, d\chi.$$

Hence we find

$$l = a\left(\frac{\Lambda}{b}\right)^{1/5}, \tag{4.31}$$

where a is a constant. If we further assume that $l = \sqrt{\nu t}$, which corresponds to the above considered solution for small pulsations, then we shall obtain at once the law governing damping corresponding to $\alpha = \frac{1}{4}$:

$$b = \frac{a^5\Lambda}{(\nu t)^{5/2}}.$$

On the basis of the eqs. (4.29) and (4.30) the eq. (4.14) gives:

$$h'(\chi) + \frac{4h(\chi)}{\chi} - \frac{1}{2}\chi f'(\chi)\frac{1}{b^{3/2}}\frac{dl}{dt} + \frac{1}{2} f(\chi)\frac{l}{b^{3/2}}\frac{db}{dt} = \frac{\nu}{b^{3/2} l}\left(f''(\chi) + \frac{4f'(\chi)}{\chi}\right). \tag{4.32}$$

This relation can be satisfied with the aid of various assumptions. Following Karman and Howarth, we shall consider the motions corresponding to large values of the Reynolds number $\frac{\sqrt{b}\cdot l}{\nu}$, and therefore we shall disregard the righthand part of eq. (4.32). Then we obtain:

$$h' + \frac{4h'}{\chi} = \frac{1}{2}\chi f'\cdot kc + \frac{1}{2} f\cdot c, \tag{4.33}$$

where

$$\frac{1}{\sqrt{b}}\frac{dl}{dt} = kc, \tag{4.34}$$

$$\frac{1}{\sqrt{b^3}}\frac{dl}{dt} = -c. \tag{4.35}$$

The dimensionless quantities kc and c do not depend upon χ. From the eq. (4.33) and from the general properties of the function $f(\chi)$ it follows that the quantities kc and c are constants.

Integrating the eqs. (4.34) and (4.35) we obtain the following relation:

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\frac{l}{l_0} = \left(\frac{b_0}{b}\right)^k . \qquad (4.36)$$

For $k \geq -\frac{1}{2}$ we have*

$$\frac{b}{b_0} = \left(\frac{t_0 + t}{2t_0}\right)^{-\frac{2}{2k+1}} \text{ and } \frac{l}{l_0} = \left(\frac{t_0 + t}{2t_0}\right)^{\frac{2k}{2k+1}}, \qquad (4.37)$$

where by t_0 we designate the moment of time to which the values l_0 and b_0 correspond.

If to the obtained eqs. (4.37) we add the assumption that there exists the invariant $A \neq 0$, then from the eq. (4.31) it follows that $k = \frac{1}{5}$. Substituting $k = \frac{1}{5}$ into the eqs. (4.37) we obtain the results of A.N.Kolmogorov**

$$\frac{b}{b_0} = \left(\frac{t_0 + t}{2t_0}\right)^{-10/7}, \quad \frac{l}{l_0} = \left(\frac{t_0 + t}{2t_0}\right)^{2/7}, \qquad (4.37')$$

which were found by him with the assistance of a number of assumptions, including the assumptions expressed by the equalities (4.29) and (4.31)***.

In the foregoing derivation the index k is arbitrary. If we assume that for the case of an aerodynamic pipe the scale l possesses a constant value that is proportional to the dimensions of the cells of the lattice M, i.e. l = const. M, then this gives k = 0 and the following

$$\frac{u}{\sqrt{b}} = \frac{u}{2\sqrt{b}}\left(1 + \frac{t}{t_0}\right) = A + B\frac{r}{M} .$$

---

* The condition $k \geq -\frac{1}{2}$ must be satisfied in order that the mean magnitude of the velocity of the turbulent motion does not become zero for a certain finite moment of time.

** A.N.Kolmogorov, Doklady Akademii Nauk SSSR (Reports of the Academy of Sciences USSR), Vol. XXXI, No. 6, 1941.

*** In Paragraph 8 it will be shown that the assumptions (4.29), (4.30) and (4.31) are contradictory if $h \neq 0$; but if $h = 0$, then laws are obtained which are different from (4.37').

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Obtained is a formula that is in agreement with the formula of Taylor (4.27).

8. The laws governing the degeneration of turbulence taking into account the moments of the third order. The laws governing the development of turbulence which are represented by the eqs. (4.37) are obtained with the assistance of the additional assumption of Karman and Howarth that the right-hand side in eq. (4.32) can be replaced by zero. If one equates the right-hand side of the eq. (4.32) to zero, then this gives the following equation for the function $f(\chi)$:

$$f'' + \frac{4f}{\chi} = 0,$$

which does not possess any solutions that satisfy the condition $f(0) = 1$, with the exception of the solution $f(\chi) = 1$, which does not satisfy the physical condition $f(\infty) = 0$.

Consequently the solution of Karman and Howarth is approximate, besides which it does not give any relations for the determination of the functions $f(\chi)$ and $h(\chi)$. Therefore this solution cannot be considered satisfactory by any means.

Further, we find all the physically admissible exact solutions of the eq. (4.14) during preservation of only the principal assumptions of Karman and Howarth, which are included in the following two formulas

$$\frac{b_d^d}{b} = f\left(\frac{r}{l}\right) \tag{4.29}$$

and

$$\frac{b_d^{nn}}{b^{3/2}} = h\left(\frac{r}{l}\right), \tag{4.30}$$

where l and b are certain functions of the time.

In consequence of these assumptions the eq. (4.14) leads to the relation (4.32). Let us now consider in greater detail more exact solutions of this equation. Differentiating the eq. (4.32) with respect to time for constant χ, we obtain the following

$$\frac{1}{2}\chi f'(\chi)\frac{d}{dt}\left(\frac{1}{b^{3/2}}\frac{dl}{dt}\right) - \frac{1}{2}f(\chi)\frac{d}{dt}\left(\frac{l}{b^{3/2}}\frac{db}{dt}\right) + \left(f''(\chi) + \frac{4f'(\chi)}{\chi}\right)\frac{d}{dt}\left(\frac{\nu}{\sqrt{b}\cdot l}\right) = 0. \tag{4.38}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In connection with relation (4.38) we shall analyze the following possible cases:

1°. Functions $f'(\chi)$, $f(\chi)$ and $f''(\chi) + \frac{4f'(\chi)}{\chi}$ are linearly independent.

2°. There exists only one independent linear relation with constant coefficients of the following form

$$c_1 \chi f' + c_2 f + c_3 \left(f'' + \frac{4f'}{\chi} \right) = 0, \qquad (4.39)$$

in which not all the c's c_1, c_2, c_3 are equal to zero.

3°. Among the functions $\chi f'$, f, $f'' + \frac{4f'}{\chi}$ there exist two linearly independent relations with constant coefficients. Since $f \neq 0$, then these relations can always be represented in the following form

$$f'\chi = c_1' f \quad \text{AND} \quad f'' + \frac{4f'}{\chi} = c_2' f.$$

It is not difficult to see that in the last case it must be:

$$c_1' = c_2' = 0 \quad \text{AND} \quad f = \text{const}.$$

This solution is not of physical interest, therefore the third case is dropped.

In the first case all the coefficients in the relation (4.38) are equal to zero, therefore we have

$$\frac{1}{\sqrt{b}} \frac{dl}{dt} = kc, \quad \frac{l}{\sqrt{b^3}} \frac{db}{dt} = -c, \quad \frac{v}{\sqrt{b}\, l} = \frac{1}{mc}, \qquad (4.40)$$

where k, c and m are certain constant numbers. The system of relations (4.40) gives:

$$l = c\sqrt{mv(t + t_0)}, \quad b = \frac{mv}{t + t_0}, \quad k = \frac{1}{2}. \qquad (4.41)$$

Consequently there is obtained a completely definite law for the variation of l and b as functions of the time. This law* coincides with the law (4.37) for the case $k = \frac{1}{2}$. In this case only one equation is obtained for the two functions $f(\chi)$

---

* It is evident that in this case the quantity Λ cannot be finite and an invariant unequal to zero, since in the contrary case the equality (4.31) would contradict the equality (4.41).

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

and $h(\chi)$.

Let us now investigate the second case. It is easy to see that in the relation (4.39) the coefficient c_3 must be different from zero, since otherwise we obtain for the function $f(\chi)$ the following equation

$$c_1 f + c_2 \chi f' = 0,$$

which in the case of $c_1 \neq 0$ does not possess any solution that satisfies the condition $f(0) = 1$, but for the case $c_1 = 0$ we obtain the following: $f' = 0$ or $f = \text{const.} \neq 0$, which is excluded by the condition $f(\infty) = 0$.

Since $c_3 \neq 0$, then the relation (4.39) can be represented in the form

$$f'' + \frac{4}{\chi} f' + \frac{a_1}{2} \chi f' + \frac{a_2}{2} f = 0, \tag{4.42}$$

where a_1 and a_2 are constant coefficients.

From the equations (4.38) and (4.42) we find the following:

$$\frac{1}{2} \chi f' \left[\frac{d}{dt} \frac{1}{\sqrt{b}} \frac{dl}{dt} - a_1 \frac{d}{dt} \frac{\nu}{\sqrt{b} \cdot l} \right] - \\ - \frac{1}{2} f \left[\frac{d}{dt} \frac{l}{\sqrt{b^3}} \frac{db}{dt} + a_2 \frac{d}{dt} \frac{\nu}{\sqrt{b} \cdot l} \right] = 0.$$

Since the functions $\chi f'$ and f are linearly independent, it is evident that the expressions in the square brackets become zero; hence it follows:

$$\frac{1}{\sqrt{b}} \frac{dl}{dt} = a_1 \frac{\nu}{\sqrt{b} \cdot l} + p, \qquad \frac{l}{\sqrt{b^3}} \frac{db}{dt} = -a_2 \frac{\nu}{\sqrt{b} \cdot l} + q, \tag{4.43}$$

where p and q are constants of integration.

Substituting the relations (4.43) into (4.32) and keeping in mind the equation (4.12), we find the equation for the determination of $h(\chi)$ in the following form

$$h' + \frac{4h}{\chi} - \frac{1}{2} \chi f' p + \frac{1}{2} f q = 0. \tag{4.44}$$

Earlier we established that the expansion of the function $\frac{b^{nn}_{d}}{b^{3/2}} = h$ in powers of $\frac{r}{l} = \chi$ must begin with terms of the order of χ^3, therefore it follows from the equa-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

(4.44) that $q = 0$.

Thus we have found in the case under consideration the complete system of eqs. (4.42), (4.43) and (4.44) for the determination of h and f as functions of the variable χ and for the determination of the quantities l and b as functions of the time t.

These equations contain three abstract constants a_1, a_2 and p. It is not difficult to discern that one of these constants is nonessential. In fact, the transformation $\chi = \lambda\chi'$, where λ is a certain constant, leads to the multiplication of the still undetermined scale l by the constant $\frac{1}{\lambda}$ (where $l = \frac{l'}{\lambda}$). Carrying out this transformation in the eqs. (4.42), (4.43) and (4.44) we obtain the same equations, but with transformed values a_1', a_2' and q', which are determined by the following formulas

$$a_1' = a_1\lambda^2, \quad a_2' = a_2\lambda^2, \quad p' = p\lambda.$$

Fixing of a definite value for $a_1 \neq 0$ or for $a_2 \neq 0$ is equivalent to the selection of a definite scale for l.

On the basis of the second eq. (4.43) it follows from the condition for the damping of turbulent pulsations ($b \to 0$ for $t \to \infty$) that $a_2 > 0$.

Further we also assume as a physical condition that $l \to +\infty$ for $t \to \infty$ or for $b \to 0$.

The eqs. (4.43) can be integrated, after which we arrive at the following relation

$$\frac{1}{l} = \frac{2pa_2}{\nu(a_2 - 2a_1)}\sqrt{b} + cb^{a_1/a_2},$$

where c is the constant of integration, here $2a_1 \neq a_2$. From this relation, from the inequality $a_2 > 0$ and from the condition $l \to \infty$ for $b \to 0$ it follows that $a_1 \geq 0$.

Let us consider also the case $a_1 = 0$. For the case $a_1 = 0$ the eq. (4.42) possesses a unique solution satisfying the condition $f(0) = 1$.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

This solution is easily determined and possesses the following form

$$f(\chi) = 3\left[\frac{\sin\sqrt{\frac{a_2}{2}}\chi}{\left(\sqrt{\frac{a_2}{2}}\chi\right)^3} - \frac{\cos\sqrt{\frac{a_2}{2}}\chi}{\frac{a_2}{2}\chi^2}\right].$$

The obtained function fluctuates when $\chi = \frac{r}{l}$ increases, and possesses an infinite number of zeros and slowly decreases like the function $\frac{1}{\chi^2}$. On the basis of this we shall hereafter exclude the case $a_1 = 0$.

Making use of the free choice of the parameter λ, we set $a_1 = \frac{1}{2}$, when eq. (4.42) is represented in the following form

$$f'' + \left(\frac{4}{\chi} + \frac{\chi}{4}\right)f' + \frac{a_2}{2}f = 0. \qquad (4.45)$$

Eq. (4.45) after use of the designation $a_2 = 10\alpha$ and the following substitution of variables

$$\frac{r^2}{l^2} = \chi^2 = \xi$$

passes over into eq. (4.19), which we investigated earlier in the case of the study of the damping of very small pulsations. Consequently we obtain for $f(\xi)$ the same solution as in the case of small pulsations:

$$f(\xi) = M\left(10\alpha, \frac{5}{2}, -\frac{\xi}{8}\right),$$

but now we have $l \neq \sqrt{vt}$ and $h \neq 0$. Thus taking into account the moments of the third order exerts an influence upon the coefficient of correlation $f = \frac{b_d^d}{b}$ only through the variation in the dependence of the linear scale l upon time.

The function $h(\chi)$ on the basis of the eq. (4.44) ($q = 0$) is easily expressed through the function f. Integrating the eq. (4.44) and keeping in mind the condition $h = 0$ for $\chi = 0$, we obtain the following

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$h = \frac{p}{2}\,\frac{1}{\chi^4}\int_0^{\chi}\chi^5 f'(\chi)\,d\chi. \qquad (4.46)$$

If $\alpha = 1/4$ and $h \neq 0$, then it is evident that the function $h(\chi)$ vanishes for $\chi \to \infty$ like the function $\frac{1}{\chi^4}$; therefore the moment of the fourth order b_d^{nn} possesses the order $\frac{1}{r^4}$ for $r \to +\infty$, and consequently the conditions necessary for the invariance of Λ (see page 136 of the original, the page with eq. (4.14') and (4.15)) are not satisfied.

Thus, for $\alpha < 1/4$ we have $\Lambda = \infty$, for $\alpha > 1/4$ we have $\Lambda = 0$, and for $\alpha = 1/4$ and $p \neq 0$ (the moments of the third order are different from zero) the quantity Λ is finite and different from zero, but is not an invariant. In fact, we obtain from eqs. (4.14') and (4.46) for $\alpha = 1/4$ the following derivative

$$\frac{d\Lambda}{dt} = -p\int_0^{\infty}\chi^5 f'(\chi)\,d\chi \cdot l^4 b^{3/2} \neq 0.$$

If $p = 0$, then the integral Λ is finite and invariant, but in this case the moments of the third order are equal to zero.

Thus, if the moments of the third order are different from zero and fulfill the hypotheses that are expressed by the equalities (4.29) and (4.30), then (i) the integral Λ equals zero or infinity and are consequently inconvenient for the determination of the scales in correspondence with the equality (4.31), or (ii) for $\alpha = 1/4$ the quantity Λ varies with the course of time and therefore cannot be considered as the characteristic constant.

Multiplying the eq. (4.45) by $\chi^4 d\chi$ and integrating we find:

$$\int_0^{\chi}\chi^5 f'\,d\chi = -4\chi^4 f' - 20\alpha\int_0^{\chi}\chi^4 f\,d\chi = -4\chi^4 f' - 4\alpha\chi^5 f + 4\alpha\int_0^{\chi}\chi^5 f'\,d\chi.$$

Employing this relation for $\alpha \neq \frac{1}{4}$ we obtain the following:

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$h=\frac{2p}{4\alpha-1}[f'(\chi)+\alpha\chi f(\chi)]. \tag{4.47}$$

For the determination of l and b as functions of the time we have the eq. (4.43), which for $a_1=\frac{1}{2}$, $a_2=10\alpha$ and $q=0$ assume the following form

$$\left.\begin{aligned}\frac{1}{\sqrt{b}}\frac{dl}{dt}&=\frac{1}{2}\frac{\nu}{\sqrt{b}\cdot l}+p,\\ \frac{l}{\sqrt{b^3}}\frac{db}{dt}&=-10\alpha\frac{\nu}{\sqrt{b}\cdot l}.\end{aligned}\right\} \tag{4.48}$$

It is not difficult to see that in the particular case where $p=0$ the system of eqs. (4.48) and (4.45) possesses the following solution

$$l=\sqrt{\nu t},\quad b=\frac{A'}{(\nu t)^{10\alpha}},\quad f=M\left(10\alpha,\ \frac{5}{2},\ -\frac{\xi}{8}\right)\ \text{AND}\ h\equiv 0,$$

which corresponds to the solution of Karman and Howarth for small pulsations.

For any p and $\alpha\neq 0.1$ we obtain from the eq. (4.48) the following:

$$\frac{1}{l}=Cb^{\frac{1}{20\alpha}}+\frac{2p\sqrt{b}}{(10\alpha-1)\nu}, \tag{4.49}$$

where C is the constant of integration.

For $p\neq 0$ and $\alpha\neq 0.1$ the general solution of the eq. (4.48) can be represented in the following form

$$\frac{l}{l^*}=\frac{1}{w^{\frac{1}{20\alpha}}+\sqrt{w}} \tag{4.50}$$

and

$$\frac{\nu(t+t^*)}{l^{*2}}=\frac{1}{10\alpha}\int_w^\infty\frac{dw}{w^{\frac{1}{2}+\frac{1}{10\alpha}}\left(1+w^{\frac{10\alpha-1}{20\alpha}}\right)^2}, \tag{4.51}$$

where we employ the following designations:

$$w=\frac{b}{b^*},\quad l^*=\frac{(10\alpha-1)\nu}{2p\sqrt{b^*}}; \tag{4.52}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the letters b* and t* designate the constants of integration, which possess the dimension of the square of velocity and time (l* possesses the dimension of length).

It is not difficult to see that the law governing the degeneration of the turbulent motion is determined essentially only by one constant α, since b* and l* play the role of scale constants, and the constant t* depends only upon the origin of time reckoning.

Let us define the Reynolds number R according to the following formula

$$R = \frac{\sqrt{b} \cdot l}{\nu}.$$

From eq. (4.50) it follows that

$$R = \frac{\sqrt{b^*} \cdot l^*}{\nu} \frac{\sqrt{w}}{\sqrt{w} + w^{\frac{1}{20\alpha}}} = \frac{\sqrt{b^*} \cdot l^*}{\nu} \frac{1}{1 + w^{\frac{1}{2}\left(\frac{1}{10\alpha} - 1\right)}}.$$

Hence, of $\alpha > \frac{1}{10}$ then $R \to 0$ for $w \to 0$ or $t \to +\infty$; if $\alpha < \frac{1}{10}$, then for $w \to 0$ we have: $R = \frac{\sqrt{b^*} \cdot l^*}{\nu}$, i.e. the number R possesses a finite limiting value. It is evident that if we disregard the moments of the third order the limiting behavior of the $R = \frac{b\sqrt{vt}}{\nu}$ (for $\alpha > 0.1$) possesses the same character.

The laws obtained by us for the degeneration of isotropic turbulence (4.50) and (4.51) are the consequences of the assumptions (4.29) and (4.30). For the determination of the constant α we need additional hypotheses or experimental data.

From the eqs. (4.50), (4.51) and (4.52) it follows that for $\alpha > 0.1$ and when $t \to +\infty$ the following asymptotic relations hold true

$$\frac{b}{b^*} \approx \left(\frac{l^{*2}}{\nu t}\right)^{10\alpha}, \quad l \approx \sqrt{\nu t} \text{ AND } h \to 0,$$

which indicate that the laws found by us for the variation of b and l as functions of the time, taking into consideration the moments of the third order, tend asymptotically toward the laws obtained by Karman and Howarth when the moments of the

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

third order are disregarded. For small α, however, the limiting motion will be reached slowly.

Section 5. Steady-State Turbulent Motions

In a number of cases, in particular in the case of the motion of a fluid in a pipe and in channels, we encounter turbulent motions for which the averaged motion is steady; such flows are called steady turbulent flows.

Let us consider the problem of the steady turbulent motion of an incompressible fluid in an immobile smooth infinitely long cylindrical circular pipe.

We assume that the mean motion of the fluid possesses axial symmetry and the mean velocities are directed along the axis of the pipe. From the equation of incompressibility it follows that the magnitudes of the mean velocities do not depend upon the coordinate of x along the axis of the pipe; over the cross section of the pipe the mean velocity is variable and depends upon the distance r of the point under consideration to the center of the pipe.

It is not difficult to see that the characteristics of the averaged motion are determined by the following system of parameters:

$$\rho,\ \mu,\ a,\ \tau_0 = -\frac{1}{2}\frac{\partial \bar{p}}{\partial x}a, \quad r = a - y, \tag{5.1}$$

where a is the radius of the pipe $\frac{\partial \bar{P}}{\partial x}$ is the mean value of the gradient of pressure along the pipe, and τ_0 is the stress of friction on the walls of the pipe. All the dimensionless quantities are functions of the following two parameters:

$$R = \frac{v_* a \rho}{\mu}, \quad \frac{r}{a} = 1 - \frac{y}{a}, \tag{5.2}$$

where $v_* = \sqrt{\frac{\tau_0}{\rho}}$ represents the so called velocity of tangential stress on the walls of the pipe. The dimensionless quantities characterizing the properties of the motion in the large do not depend upon the variable r and consequently are determined only by some Reynolds number.

Let us designate the magnitude of the velocity of the averaged motion by the

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

letter u and the velocity at the center of the pipe by u_{max}. From the theory of dimensions it follows that

$$\frac{u_{max}}{v_*} = f(R) \tag{5.3}$$

and

$$\frac{u_{max} - u}{v_*} = F\left(R, \frac{r - y}{a}\right). \tag{5.4}$$

The quantity $u_{max} - u$, called the velocity defect, characterizes the distribution of velocity over the cross section of the pipe relative to the motion at the center of the pipe.

In what follows we shall consider the sharply expressed turbulent motions, which corresponds to large values of the Reynolds numbers.

The distribution of velocities in the pipe is closely connected with the phenomenon of turbulent mixing, thanks to which there occurs exchange of momentum between neighboring layers of the fluid. Smoothing or leveling of the velocities which is due to the transfer of the momenta is determined by the property of the inertia of the fluid.

From the point of view of the kinetic theory of matter the property of viscosity is explained by the presence of the chaotic molecular motion which enables the smoothing or leveling of the velocities of the observed motion and leads to the transformation of the kinetic energy of the observed motion into the energy of heat motion.

The law governing the conservation of energy is expressed in the constancy of the sum of mechanical energy of the observed motion and energy of molecular motion. Both forms of energy can be considered as components of different forms of mechanical energy. If one disregards the intra-molecular forces, then the property of viscosity is determined by the mean kinematic characteristics of the state of the molecular motion and by the property of the inertia of the molecules of the fluid.

Orderless turbulent mixing in relation to the averaged motion is analogous to

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

molecular motion in relation to true turbulent motion. The turbulent pulsations are analogous to the pulsations of the molecular chaotic motion. The difference consists in the different order of mean magnitudes that characterize the pulsational motions. In place of the motion of the individual molecules in the thermal process during turbulent mixing we have pulsational motions of the "mols" - volumes of the fluid which are very large in comparison with the dimensions and mass of the molecules. In addition to this, the magnitudes of the mean velocities of the pulsations in the turbulent motion are very small in comparison with the magnitude of the mean velocity of the thermal motion.

During the turbulent motion one can consider the transformation of the energy of the averaged motion into the energy of the turbulent molar motion. In this sense one can speak about the dissipation of the energy of mean motion; here, the dissipation is not immediately connected with the transition of the mechanical energy into thermal energy, and consequently this process can be independent of the property of the viscosity of the fluid. The redistribution of the kinetic energy between the observed mean motion and the motion of pulsations can be considered also even for the ideal fluid. As is well known, for the ideal incompressible fluid the transition of the mechanical energy into thermal energy is impossible. Consequently the transition of the energy from the mean motion into the molar turbulent motion of pulsations in a number of cases can be determined in the main only by the property of inertia.

In the case of the motion of a fluid in a pipe there occurs the loss of mechanical energy; consequently there must be regions in which the influence of the viscosity is essential and considerable. In consequence of the adhesion of the fluid to the walls of the pipe the instantaneous and mean velocities of the fluid at the walls of are equal to zero. Therefore in the immediate neighborhood of the walls of the pipe there cannot be any intensive intermixing of the fluid. This serves as the basis for the conclusion that immediately around the walls the sharp

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

change in the velocity must be determined by the property of the viscosity of the fluid and that around the walls there must exist a layer with laminary motion. Experimental data confirms well this conclusion.

Let us assume that in the main cell of the turbulent flow close to the axis of the pipe the smoothing or leveling of the velocities is determined by the molar intermixing of the fluid, in which the property of the viscosity possesses a secondary nonessential significance. Let us designate by the letter δ the thickness of the layer of the fluid close to the walls, in which it is inadmissible to disregard the property of the viscosity of the fluid. Approximately the quantity δ can be equated to the thickness of the laminar layer near the walls of the pipe. In accordance with assumption for the case of $y > \delta$ the viscosity is nonessential and consequently in the eq. (5.4) for the case $y > \delta$ The Reynolds number is non-essential; that is, we have the following

$$\frac{u_{max} - u}{v_*} = F\left(\frac{a-y}{a}\right). \tag{5.5}$$

The equality (5.5) indicates the existence of the universal law governing the distribution of velocities in the pipes.

Experiments on the measurement of the distribution of velocities in pipes in the case of turbulent motion, which were carried out for all possible values of the Reynolds number, confirm very well the correctness of the existence of a similar universal law governing the distribution of velocities, which is independent of the Reynolds number (see Fig. 23). Already as early as 1858 Darcy* proposed the following empirical formula

$$\frac{u_{max} - u}{v_*} = 5{,}08\left(1 - \frac{y}{a}\right)^{1/8}. \tag{5.6}$$

The graphical representation of the results of the experiments for the quantity

* Darcy, Memoires de div. savants etrangers, t. 15, 1858.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

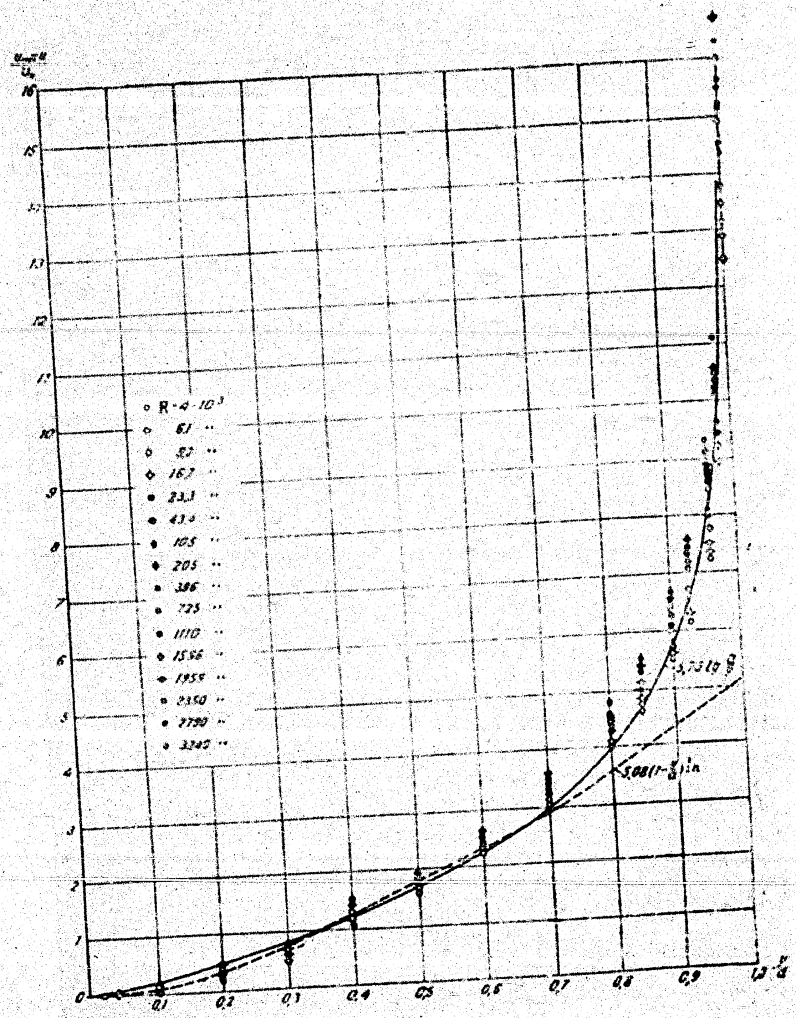


Fig.23. Experimental Confirmation of the Universal Law Governing the Distribution of Velocities Close to the Axis of the Pipe

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$\frac{u_{max}-u}{v_*}$ as a function of the ratio $\frac{y}{a}$ is given also in the works of Stanton* Fritsch** and Nikuradze***. These authors noted the existence of the indicated universal dependence, which turned out to be correct in the central part of smooth and rough pipes independently of the roughness, in spite of the fact that the resistance, and also the ratio $\frac{u_{max}}{v_*}$ etc. essentially depend upon the Reynolds number and upon the roughness.

The presented interpretation of the correctness of the law (5.5) was obtained from an analysis of the dimensions as a consequence of the disregard of the property of viscosity for relative motion at the center of the pipe. This explanation was given in the works of Prandtl and Karman.

Let us consider now the problem concerning the determination of the form of the function which gives the law governing the distribution of the velocities in the cross section of the pipe in the case of turbulent motion. Having in mind the large values of the Reynolds number, which for given v_* and $\frac{\mu}{\rho}$ is equivalent to large values of the radius a, we shall consider the limiting case $a \to +\infty$. In this case is obtained the problem of the turbulent motion in the half space $y>0$ bounded by the plane $y = 0$.

The system of the determining parameters will be the following:

$$\rho,\ \mu,\ \tau_0,\ y.$$

For the distribution of the averaged velocities we obtain the following formula

$$\frac{u}{v_*} = \varphi(\eta), \tag{5.7}$$

where η is given by

$$\eta = \frac{v_* \rho y}{\mu}.$$

---

* Stanton, Proc. Roy. Soc. (A), vol. 85, 1911.

** Fritsch, ZAMM, t. 8, 1928 and Aachener Abhand., VIII, 1928.

*** Nikuradze, Proceedings of the Third International Congress, 1930.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In the case of laminar motion the form of the function $\varphi(\eta)$ is easily determined theoretically. In fact, in the case of laminar motion in a cylindrical pipe all the particles of the fluid move uniformly and rectilinearly; therefore the property of inertia must be nonessential, and consequently the distribution of the velocities must not depend upon the quantity ρ . Since the defining parameters are τ_0, μ and y, then we obtain from the theory of dimensions the following:

$$u = k\tau_0 \frac{y}{\mu} = k \cdot \sqrt{\frac{\tau_0}{\rho}} \frac{\sqrt{\frac{\tau_0}{\rho}} \cdot \rho y}{\mu},$$

where k is a dimensionless constant. Composing the following relation

$$\tau_0 = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0},$$

we obtain $k = 1$. Consequently, the following formula is correct for laminar motion

$$\varphi(\eta_1) = \eta_1. \tag{5.8}$$

If we assume that in the general case of turbulent motion near the wall there exists a laminar layer, then the eq. (5.8) determines the form of the function $\varphi(\eta)$ immediately in the neighborhood of the walls.

For turbulent motion the function $\varphi(\eta)$ can be determined with the help of experiments. But in experiments in a pipe with finite radius there is added also the Reynolds number besides the parameter η

$$R = \frac{v_* \rho a}{\mu}.$$

In investigation of the distribution of the velocities in the case of turbulent motion a great role has been played by the power empirical formulas of the form

$$\frac{u}{v_*} = A\eta^n, \tag{5.9}$$

where A and n are constants. These constants can be determined either by way of direct measurement of the velocity distribution or indirectly with the assistance of an experimental determination of the law governing the resistance of the pipe. For

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

For the clarification of the latter method we express the resistance of the pipe in terms of the law governing the velocity distribution with respect to the radius of the pipe.

The coefficient of resistance of a circular pipe is determined by the equality

$$\psi = \frac{(p_1 - p_2)\, a}{l \cdot \rho \frac{\bar{u}^2}{2}} = \frac{4\tau_0}{\rho \bar{u}^2} = 4\left(\frac{v_*}{\bar{u}}\right)^2, \tag{5.10}$$

where

$$\bar{u} = \frac{Q}{\pi a^2};$$

$\bar{u}$ is the mean velocity over the cross section of the pipe, here, Q is the volume discharge of the fluid. In the general case for the distribution of the velocities with respect to the radius of the pipe the formula of the following form is correct

$$\frac{u}{v_*} = \varphi\left(\mathrm{R}_1 = \frac{\rho a \bar{u}}{\mu},\ \frac{\rho v_* y}{\mu}\right).$$

Averaging over the cross section of the pipe we obtain:

$$\frac{u}{v_*} = \frac{1}{\pi a^2}\int_0^a \varphi \cdot 2\pi (a - y)\, dy = 2\int_0^1 \varphi\left(\mathrm{R}_1, \frac{v_*}{\bar{u}}\mathrm{R}_1\lambda\right)(1 - \lambda)\, d\lambda. \tag{5.11}$$

Solving this equation relative to the ratio $\frac{v_*}{\bar{u}}$ we find ψ as a function of the Reynolds number R_1.

It is not difficult to discern that for the power law governing the velocity distribution which is defined by the eq. (5.9), in which A and n do not depend upon the Reynolds number R_1, we obtain for the coefficient ψ a formula of the following form

$$\psi = \frac{a}{\mathrm{R}_1^m}, \tag{5.12}$$

where a and m are constants. In fact, substituting

$$\varphi = A\left(\frac{\rho v_* y}{\mu}\right)^n = A \cdot \mathrm{R}_1^n \left(\frac{v_*}{\bar{u}}\right)^n \lambda^n$$

into the relations (5.11) we obtain:

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\frac{u}{v_*} = \frac{2A}{(n+1)(n+2)} R_1^n \left(\frac{v_*}{u}\right)^n;$$

from which we determine the ratio $\frac{v_*}{u}$, after which we find on the basis of the equality (5.10) the following expression:

$$\psi = 4\left[\frac{(n+1)(n+2)}{2A}\right]^{\frac{2}{n+1}} \frac{1}{R_1^{\frac{2n}{n+1}}}. \qquad (5.13)$$

Comparing the eqs. (5.12) and (5.13), we obtain simple relations between the constants m and n and the constants a, A and n.

The empirical formula of Blasius for the resistance of smooth cylindrical pipes possesses the form

$$\psi = \frac{0{,}132}{R_1^{1/4}}. \qquad (5.14)$$

If we assume the power law for the velocity distribution, then the formula of Blasius (5.14) leads to the so called one seventh law:

$$\frac{u}{v_*} = 8{,}6\left(\frac{v_* 2y}{\mu}\right)^{1/7}. \qquad (5.15)$$

The eqs. (5.14) and (5.15) are in good agreement with experience* for the Reynolds numbers $2R_1$ in the interval of from 10^4 to 10^5; for less values a better formula of the form (5.9) with index $n = \frac{1}{6}$ is in agreement with experiments. For $2R_1 > 10^5$ the index must decrease.

The results of experiments (Fig.24) show that the best coincidence with experience is obtained for the empirical formula of the following form

$$\frac{u}{v_*} = \varphi(\eta_1) = 5{,}75 \lg \eta_1 + 5{,}5. \qquad (5.16)$$

---

* A formula with coefficient 8.7 instead of 8.6 is in still better agreement with experiments.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The eqs. (5.9) and (5.16) lose correctness in the immediate neighborhood of the walls, where $\eta = 0$. Near the walls there is a laminar layer for which $\varphi(\eta) = \eta$. If we assume that the laminar layer borders on the turbulent flow and we require that the velocities of the particles of the fluid at the boundary of the laminar layer pass over continuously into turbulent distribution of velocities, which are

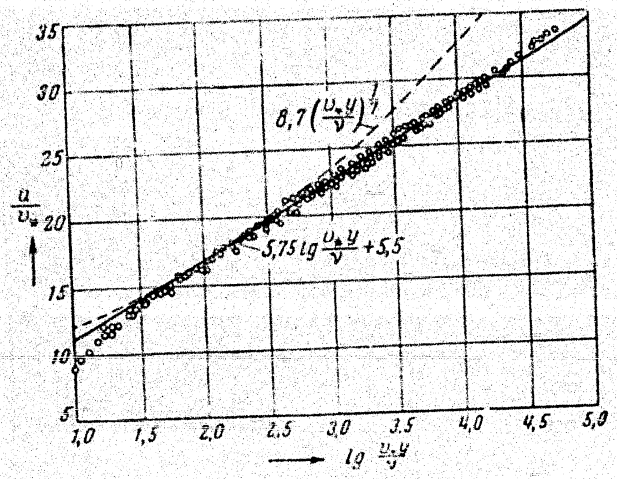


Fig.24. The Distribution of Velocities in the Turbulent Boundary Layer

determined according to the eqs. (5.15) and (5.16), then this makes it possible to determine the thickness of the laminar layer either from the following equation

$$\eta = 5{,}75 \lg \eta + 5{,}5,$$

or from the equation

$$\eta = 8{,}7\eta^{1/7}.$$

The solution of these equations in both cases gives for η a value that is close to $\eta \approx 12$ approximately. Setting $\eta = 12$, we find the formula for the thickness of the laminar layer to be the following

$$\frac{\delta}{a} = 12\frac{\mu}{\rho v_* a} = \frac{24}{R_1\sqrt{\psi}}. \qquad (5.17)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Setting Reynolds number $R_1 = 40\,000$ and employing the formula of Blasius, we obtain:

$$\frac{\delta}{a} = \frac{68}{R_1^{7/8}} = 0{,}0065.$$

Hence it can be concluded that the thickness of the laminar layer is small in comparison with the radius of the pipe a.

Let is now consider the theoretical considerations of Prandtl and Karman concerning the determination of the form of the function $\varphi(\eta)$. Let us designate by u' and v' the projections upon the x and y axes the velocities of the turbulent pulsations. The mean value of the transfer of momentum of the fluid along the y axis referred to unit time and area is represented in the following form

$$\tau = \rho\overline{u'v'}.$$

Applying the theorem concerning the variation of the momentum to the volume of the fluid included within the circular cylinder with radius r and co-axial with the pipe (see Fig.25), we obtain after averaging the following expression:

$$(p_1 - p_2)\pi r^2 = \tau \cdot 2\pi rL + \mu \frac{d\bar{u}}{dy} 2\pi rL.$$

Hence with the assistance of the relation

$$(p_1 - p_2)a = 2\tau_0 L$$

we find the following

$$\tau + \mu \frac{d\bar{u}}{dy} = \tau_0 \left(1 - \frac{y}{a}\right). \tag{5.18}$$

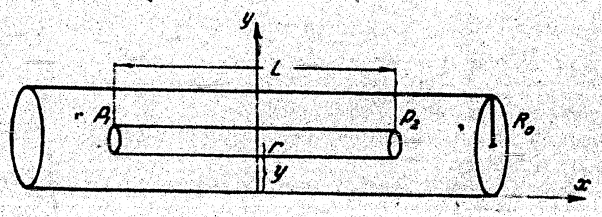


Fig.25. The Scheme for the Calculation of the Turbulent Motion in a Circular Cylindrical Pipe

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In the case of laminar motion we have $\tau = 0$, and we then have Poiseuille flow. In this case there is obtained from the eq. (5.18) the parabolic law governing the velocity distribution. In the case of turbulent motion immediately near the walls for $y = 0$ there is a laminar layer in which $\tau = 0$ and approximately $\frac{y}{a} \approx 0$, in consequence of which the eq. (5.18) leads to the following relation

$$\mu \frac{d\bar{u}}{dy} = \tau_0,$$

whence we get $\bar{u} = \frac{\tau_0 y}{\mu}$ or $\frac{u}{v_*} = \frac{\rho v_* y}{\mu}$, which coincides with the eq. (5.8).

If we turn our attention to the kinetic theory of gases, then the tangential stress of viscous flow $\mu \frac{d\bar{u}}{dy}$ can be considered as the mean value of the transfer of the momentum referred to unit time and area and due to the chaotic thermal motion of the individual molecules. In this sense both of the terms of the left-hand side of the eq. (5.18) possess the same nature.

In the region of a sharply expressed turbulent flow we have $\tau \neq 0$ and large in comparison with $\mu \frac{d\bar{u}}{dy}$, therefore it is admissible to disregard the term $\mu \frac{d\bar{u}}{dy}$ in comparison with τ.

The determination of the dependence of τ upon the characteristics of the averaged motion can be reduced to the determination of the quantity l having the dimension of length and connected with τ by the following relation

$$\tau = \rho l^2 \left|\frac{d\bar{u}}{dy}\right| \frac{d\bar{u}}{dy}. \qquad (5.19)$$

For a more detailed consideration of the mechanism governing turbulent intermixing, a number of intuitive consideration permit one to interpret the length l as a quantity similar to the path of free flight of molecules in the thermal motion of gases*. Therefore the quantity l is called the path of intermixing.

---

* For more details on this matter see the article of Prandtl entitled "The Mechanics of Viscous Fluids" in the book: Dyurend, Aerodinamika, Defense Press, 1939.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The principal significance of the transition from τ to l is connected with the great graphic concreteness of the quantity l. In the case of the absence of the influence of the viscosity, the quantity τ depends upon the square of the velocity and therefore l does not depend upon the velocity, which permits one to rely, for the establishment of the connection with the characteristic dimensions, upon certain intuitive considerations. For a more detailed consideration it is sufficient to show that l decreases as one approaches the walls, and to connect the quantity l at the walls with the characteristics of the roughness.

For $a = \infty$ the path of intermixing is determined by the parameters ρ, μ, v_* and y; therefore in this case the following formula is correct

$$l = y \cdot \Phi\left(\frac{\rho v_* y}{\mu}\right).$$

Let us assume that for a certain special selection of the origin of reckoning for the y coordinate the property of viscosity is nonessential. From this assumption the following relation follows:

$$l = ky,$$

where k is a certain dimensionless constant. Disregarding in eq. (5.18) the terms $\mu \frac{du}{dy}$ and $\frac{Y}{a} \approx 0$, we obtain

$$\tau = \tau_0 = \rho k^2 y^2 \left(\frac{du}{dy}\right)^2$$

Integrating this equation, we find the following:

$$u = \frac{v_*}{k}[\ln y - \ln y_0]. \qquad (5.20)$$

On the axis of similarity for $y = 0$ we have $u = -\infty$. The constant of integration y_0 gives the distance to the axis of similarity of the point at which $u = 0$. Immediately around the walls there will exist a laminar layer to which the turbulent flow is adjacent; if we continue the turbulent flow to the walls at which the condition $u = 0$ is satisfied, then we obtain the fact that y_0 is equal to the distance of the axis of similarity to the wall. Since y_0 must be determined by the quantities

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

ρ, μ and v_*, this gives:

$$y_0 = \beta \frac{\mu}{\rho v_*},$$

where β is a dimensionless constant.

For sharply expressed turbulent motion the quantity y_0 is small. Substituting the found value for y_0 into eq. (5.20), we find the following:

$$u = \frac{v_*}{k} (\ln \eta - \ln \beta). \qquad (5.21)$$

In the region of turbulent motion the eq. (5.21) can be considered as the theoretical basis for the empirical eq. (5.16). The constants k and β must be taken from experiments. With increase in the Reynolds number the assumptions made during the derivation of eq. (5.21) become more exact. This permits one to make the conclusion that the eq. (5.16) must correspond will with reality in the case of increasing Reynolds number.

If one assumes that the logarithmic law governing the velocity distribution is correct for the turbulent motion in a circular pipe up to the axis itself of the pipe, then we obtain the following formula

$$\frac{u_{max} - u}{v_*} = F\left(\frac{y}{a}\right) = 5{,}75 \lg \frac{a}{y},$$

which agrees well with the experimental data both for smooth pipes as well as for rough pipes; the latter is explained by the fact that the influence of the roughness can be reduced to the variation in the quantity y_0, which quantity is included in the derivation of this formula*.

We are still left with a number of considerations of the theory of dimensions and similitude, which can be applied to the problem under consideration on turbulent motion.

---

* For more on the influence of the roughness see the article by Prandtl entitled "The Mechanics of Viscous Fluids" in the book: Dyurend, Aerodinamika, v.III,1939

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Let us designate by v_x, v_y the projections of the velocity of instantaneous motion, and by u', v' the projections of the velocity of pulsation. In the studied problem on the rectilinear averaged motion we have the following relations:

$$v_x = u + u',$$
$$v_y = v'.$$

Let us consider the field of velocities relative to the motion determined by the projections $u + u' - u_M$, v', where u_M is the mean velocity at a certain point M.

The main hypothesis of Karman is included in the assumption that the turbulent fields of velocities relative to the motion at different points of the flow are kinematically similar. Employing the operation of averaging, we obtain the fact that the field of averaged relative velocities $[u(y) - u_M, 0]$ is also kinematically similar at the different points of the flow.

Recomputation of the values of all the kinematic quantities in the case of the transition from one point to another can be carried out with the aid of transitional scales for two independent kinematic quantities. The magnitudes of these scales can be obtained from a consideration of the law governing the distribution of mean velocities.

The law governing the distribution of mean relative velocities close to a given point can be characterized by the following sequence of derivatives

$$u', u'', u''', u^{IV}, \ldots \qquad (5.22)$$

Any two derivatives possess independent dimensions, and from any three derivatives one can form a dimensionless combination. From the first three derivatives one can form the following dimensionless combination

$$\frac{u''^2}{u'u'''} = k.$$

Since all the derivatives are functions of one and the same variable, it is then evident that between any two dimensionless combinations which are composed from the sequence (5.22) there holds a functional relation which does not contain dimensional constants. For example we have

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\left(\frac{\varepsilon_{,,,}^{n}{}_{,}^{n}}{\varepsilon_{,,}^{n}}\right)\Phi = \frac{\Delta I^{n}{}_{,,}^{n}}{\varepsilon_{,,,}^{n}}$$

Consequently the necessary and sufficient condition for the similarity of the relative motions for all values of y is the following

$$\frac{u''^2}{u'u'''} = k = \text{const.} \qquad (5.23)$$

The condition (5.23) represents a differential equation for u(y). Integrating this equation, we find for $k \neq 1$ and $k \neq 2$ the following:

$$u = A(y+B)^{\frac{2-k}{1-k}} + C, \qquad (5.24)$$

where A, B, C are certain dimensional constants of integration. For k = 1 the solution possesses the form

$$u = Me^{y/d} + N, \qquad (5.25)$$

and finally for k - 2 it possesses the form:

$$u = P\ln(y+Q) + S, \qquad (5.26)$$

where M, d, N, P, Q, S are constants of integration.

Thus it is evident that the assumption concerning exact similarity leads to completely definite special forms of the laws governing the distribution of mean velocity.

The eq. (5.26) corresponds to the logarthmic law governing the velocity distribution.

From the condition concerning the similarity for the path of mixing l and for the turbulent transfer of momentum τ follows the following formula

$$l = k_1 \frac{u'}{u''} \qquad (5.27)$$

and

$$\tau = \rho l^2 u'^2 = \rho k_1^2 \frac{u'^4}{u''^2}, \qquad (5.28)$$

where k_1 is a dimensionless constant.

For the laws governing velocity distributions (5.24) and (5.26) the eq. (5.27)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

leads in the case of a suitable choice of the origin of reckoning for y to the following relation

$$l = k_1 y,$$

which we obtained earlier as a consequence of the hypothesis concerning the nonexistence of the property of viscosity.

The law governing velocity distribution (5.26) corresponds to $\tau = \tau_0 = \text{const.}$ τ is obtained as a variable for the laws governing velocity distribution (5.25) and (5.24).

The condition (5.23) strongly restricts the possible laws governing the distribution of mean velocities. The original exact condition for similarity can be weakened and one can assume the presence of similarity of the relative fields of velocity in the neighborhood of any points with accuracy only up to magnitudes of the second order of smallness. In the case of such an assumption in the sequence (5.22) only u' and u" are preserved as the characteristic quantities, and therefore the eq. (5.23) falls away whereas the eqs. (5.27) and (5.28) are kept.

With the aid of the eq. (5.28) and the equality (5.18) one can obtain the theoretical formulas for the law governing the distribution of the mean velocities*.

---

* See the article by Karman in the symposium entitled "Problemy Turbulentnosti" (Problems of Turbulence), Publishing House 'ONTI', 1939.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

CHAPTER IV

ONE-DIMENSIONAL NON-STEADY STATE MOTIONS OF A GAS

1. Self-Modeling Motions of a Gas with Spherical, Cylindrical and Plane Waves

One-dimensional motions of a gas or liquid are defined as motions, all characteristics of which depend on only a single geometric coordinate, and on time. It can be shown that one-dimensional motions are possible only with spherical cylindrical and plane waves*. Methods of the theory of dimensions permit one to find accurate solutions for certain problems on the one-dimensional non-steady state motion of a compressible fluid**. These problems represent, in many cases, considerable theoretical and practical interest. But even in those cases where the statement of the problem does not in itself represent interest, the accurate solutions obtained can be used as examples for checking the accuracy of the different approximation methods of solving the problems of gas dynamics.

---

* Cf. Lipschitz, Z.f. reine und angew. Math., (Journal of Pure and Applied Mathematics) Vol.100 (1887), p.89.

** This class of solutions was demonstrated in the work of L.I.Sedov, "On Certain Non-Steady State Motions of a Compressible Fluid", Prikladnaya Mathematika i Mekhanika (Applied Mathematics and Mechanics), Vol.IX, No.4, 1945.

Cf. also Doklady AN SSSR (Reports of the Academy of Sciences USSR), Vol.47, No.2, 1945, p.91.

Analogous solutions, without the use of the considerations (cont'd. next page)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In order to characterize those problems, the solution of which can be obtained by methods of dimension theory, let us examine the functions sought and the defining parameters of one-dimensional motion. The basic functions sought are the velocity v, density ρ and pressure p, and the defining parameters are the linear coordinate r, the time t, and the constants which enter into the equations and the limit and initial conditions of the problem.

Since the dimensions of the quantities ρ and p contain symbols of mass, there must be then among the defining parameters, even if it is alone, a constant a, the dimension of which also contains the symbol of mass. While not limiting commonness it is possible to assume that its dimension has the form

$$[a] = ML^{k_T s}$$

Hereby, for the desired functions of velocity, density and pressure it is always possible to write the formulas

---

** (cont'd) of dimension theory or the theory of groups, and without connection with the statements of the problems under discussion below, are examined in: Bechert, K., "Differential Equations of Wave Propagation in Gases". Annalen der Physik (Annals of Physics), Vol.39 (1941), p.357, (Bechert examines only polytropic motions) and in: Stanyukovich, K.P. "Concerning Self-Modeling Solutions of Hydrodynamics Equations Having Central Symmetry", Doklady AN SSSR, Vol.48, 1945, No.5, p.331.

The application of the methods which have developed to the theory of the filtration of a liquid in a porous medium, and the generalization of these methods for the purpose of finding motions which are the threshold of self-modeling are given in the works of Barenblatt, G.I.: "Concerning Self-Modeling Motions of a Compressible Liquid in a Porous Medium", PMM, Vol.XVI, 1952, No.6: "Concerning Threshold Self-Modeling Motions in the Theory of the Non-Stationary Filtration of Gas in a Porous Medium and the Theory of the Boundary Layer", PMM, Vol.XVIII, 1954, No.4

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$v = \frac{r}{t} V, \quad \rho = \frac{a}{r^{k+3} t^{s}} R, \quad p = \frac{a}{r^{k+1} t^{s+2}} P, \tag{1.1}$$

where V, R, and P are abstract quantities and, consequently can depend only on dimensionless combinations containing r, t and the other parameters of the problem.

In the general case, they are functions of two dimensionless variables. But if, among the defining parameters, there is, besides a, just one more constant b with a dimension independent of a* then only one variable abstract combination can be formed from r, t, a, and b.

Since the dimension of the constant a contains the symbol of mass, then, not limiting commonness, the constant b can always be inserted so that its dimension does not contain the symbol of the unit of measurement of mass, i.e.,

$$[b] = L^{m} T^{n}$$

In this case the single dimensionless variable combination will be:

$$\lambda = b r^{-m} t^{-n}$$

Besides the variable parameter λ, the solution may still depend on a series of constant abstract parameters.

Thus, when there are among the defining parameters of the problem, in addition to r and t, in all, two constants with independent dimensions, the partial equations which velocity, density and pressure must satisfy in the case of the non-steady state one-dimensional motion of a compressible fluid, may be replaced by the ordinary differential equations for the quantities V, R, and P. Too, the solution of ordinary differential equations can sometimes be obtained in closed form accurately; in other cases the solution can be obtained approximately with the help of numerical

* Generally, there may be many defining constants, but their dimensions must be dependent on a and b.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

integration.

Motions of such a form are called self-modeling. Let us dwell on the statement of some problems, the solution of which is easily obtained by the method described above.

For the sake of definition, we shall assume that the gas is ideal, not viscous and not heat conducting, whereby the motion is not accompanied by any physical or chemical transformations (in what measure it is possible in one problem or another to avoid these propositions will be examined later). In this case, the equations of motion, continuity and energy may be taken in the following form:

$$\left.\begin{aligned} &\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \\ &\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial r} + (\nu - 1)\frac{\rho v}{r} = 0, \\ &\frac{\partial}{\partial t}\left(\frac{p}{\rho^{\gamma}}\right) + v\frac{\partial}{\partial r}\left(\frac{p}{\rho^{\gamma}}\right) = 0. \end{aligned}\right\} \qquad (1.2)$$

where γ is the adiabatic curve index, $\nu = 1$ for plane waves, $\nu = 2$ for cylindrical waves and $\nu = 3$ for spherical waves.

The equations do not contain any dimensional constants. Hence, the question of self-modeling of motion reduces to the proposition that not more than two parameters with independent dimensions enter into the supplementary conditions of the problem. Let us examine examples of self-modeling problems.

1. The Problem of the Piston

In a long cylindrical tube, closed at one end by a piston, there is a gas. At the initial moment of time the gas is at rest ($u_1 = 0$), and the piston begins to move at a constant velocity U (Fig.26).

The defining parameters in this problem, besides r and t will be the initial density of the gas ρ_1, the initial pressure p_1, and the velocity of the piston U. Since there is between the dimensions ρ_1, p_1 and U, the relationship

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$[U^2] = \frac{[p_1]}{[\rho_1]}.$$

there are in all, actually two constants with independent dimensions.

An analogous problem can be stated for motions with cylindrical and spherical symmetry: at the initial moment of time the gas, filling the space, begins to move apart correspondingly to a cylinder or sphere the radius of which increases from zero in proportion to time (Fig.27).

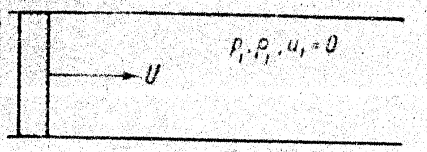


Fig.26 - The Piston begins to Move at a Constant Velocity U; in Front of the Piston the Gas is Initially at Rest and has a Uniform Density ρ_1 and Pressure p_1

If the velocity of the motion of the piston is not constant but is proportional, for example to some power of time

$$U = ct^n$$

then there will emerge a third real constant with a dimension not dependent (at $n \neq 0$) upon the dimensions ρ_1 and p_1:

$$[c] = LT^{-n-1}$$

hence, the corresponding turbulent motion of the gas will not be self modeling.

Motion in the case of such a law of the change of velocity (at $n \neq 0$) of the piston will be self modeling in the boundary case when $p_1 = 0$; and, consequently, two independent dimensional constants ρ_1 and c will enter into the conditions of the problem.

2. The Problem of Focusing on a Point and of the Dispersion of Gas from the Point

At the initial moment of time all the particles of a homogeneous gas filling a space have a uniform velocity directed toward the center (focusing) or away from the center (dispersion)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In the case of cylindrical symmetry all the particles have a uniform velocity directed toward the axis of symmetry or away from it. It is obvious that the analogous problem for plane waves, after the addition of progressive velocity equal to but opposite to the directed initial velocity of the gas, leads to the problem of the motion of a piston with constant velocity.

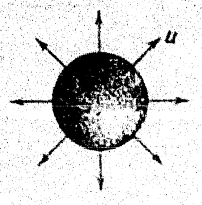


Fig.27 - Expansion at Constant Velocity U of a Sphere or Circular Cylinder in a Gas which, at the Initial Moment of Time is at Rest, and the Radius of the Sphere is Equal to Zero. The initial density of the gas ρ_1 and the initial pressure p_1 are constant.

In the case of spherical or cylindrical symmetry, the problems examined and the problems of the spherical and cylindrical piston are different problems.

As in the preceding problem, among the dimensional constants entering into the initial and boundary conditions, there are in all only two with independent dimensions.

3. The Propagation of a Flame Front or Detonation

A fuel mixture which fills a space is ignited at a moment t = 0 along a plane (plane case), in a straight line (cylindrical symmetry), or in a point (spherical symmetry). Through the mixture there will propagate a plane, a cylindrical or a spherical flame front or detonation.

As is well known, the depth of the combustion zone is very small (of the order of fractions of a millimeter).

If interest is not in the process occurring directly in the zone of combustion, then it is possible to consider that its depth is equal to zero; i.e., that the gas burns momentarily on some geometric surface. The defining parameters in this case will be: the initial density of the mixture ρ_1, the initial pressure p_1, the quantity of heat Q given off during the combustion of a unit mass of the gas and, in the case of propagation of a flame front - its velocity through the particles U, which

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

is a known physicochemical constant for a given mixture.

The dimension Q in mechanical units can be expressed using the dimensions ρ_1 and p_1:

$$[Q] = \frac{[p_1]}{[\rho_1]}.$$

Consequently, of the four defining parameters, again there are but two independent dimensions.

4. Problem of the Resolution of an Arbitrary Discontinuity in a Fuel Mixture

At a moment t = 0, to the left of a plane r = 0 there is a gas having a velocity v_1, density ρ_1, and pressure p_1 and to the right - a fuel mixture with the velocity v_2, density ρ_2 and pressure p_2. Since in the course of such a discontinuity the conditions of the conservation of mass, quantity of motion and energy, generally speaking, will not be fulfilled, then at a subsequent moment of time it (the discontinuity) can not exist isolatedly; and motion of the gas must occur with one or several surfaces of the discontinuity, on each of which surfaces there will already be fulfilled the conditions of conservation (whereby flame front or detonation can be propagated through the fuel mixture). Among the parameters of the problem (v_1, ρ_1, p_1, v_2, ρ_2, p_2, Q is the quantity of heat given off during the combustion of a unit mass of the gas, and U is the velocity of the flame front) there are in all two parameters with independent dimensions. Therefore, the motion arising will be self-modeling.

5. The Problem of the Strong Explosion

At a moment t = 0 in a gas at rest, at the center of symmetry (in a point) there occurs an explosion; i.e., a terminal energy E_0 is released instantaneously. In this instance we shall disregard the mass and dimensions of the substance which is releasing the energy. The problem concerning a strong explosion thus reflects the characteristics of the phenomenon occurring in an atomic bomb explosion. In Sec-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

tion 7, below, we shall point out experimental data for this case.

Into the conditions of the problem there enter three constants with independent dimensions: the initial density of the gas ρ_1, the initial pressure p_1, and the blast energy E_0.

The system of definitive parameters for adiabatic turbulent motions of a gas after an explosion is represented by the quantities

$$\rho_1,\ p_1,\ E_0,\ r,\ t,\ \gamma$$

Hence, from the general considerations of the theory of dimensions, it follows that all the dependent dimensionless quantities can depend only on the three dimensionless parameters:

$$\gamma,\ \frac{E_0 t^2}{\rho_1 r^5} = \lambda,\quad \frac{p_1^{2/5} t}{E_0^{1/3} \rho_1^{1/2}} = \tau, \tag{1.3}$$

of which λ and τ are variable quantities. Experiment and theory show that during an explosion at the boundary of the region of turbulent motion of the gas, there is a sharp jump of the characteristics of motion; the so-called shock wave is formed. In the statement of the problem under examination, this will be a sphere, the radius of which increases with time. The effect of the initial pressure p_1, and consequently the parameter τ also, rises only on account of the dynamic conditions at the shock wave.

However, if the explosion is strong (E_0 is large), then the pressure behind the shock wave formed as a result of the explosion will be many times greater than the initial pressure in the gas; and the motion of the gas behind the shock wave at short distances from the center of the explosion will practically not depend on the initial pressure p_1. Thus, only two dimensional quantities are real: ρ_1 and E_0.

Mathematically, the possibility of disregarding the initial pressure is expressed in the circumstance that under the conditions at the shock wave, the undisturbed pressure p_1 is assumed equal to zero; owing to this fact the parameter p_1

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

drops out and, hence, also the independent variable τ, as a result of which, it is obvious that the turbulent motion of the gas can be viewed as self modeling.

With further weakening of the shock wave, disregarding of the initial pressure - "counter pressure" p_1 - becomes illegal. Consequently, the problem of the turbulent motion of a gas, in the instance of point explosion, at long distances from the explosion center ceases to be self modeling.

Let us note that in the numerical solution of the problem of a point explosion, taking into account the counter-pressure p_1, it is sufficient to carry out computation of only one concrete case; in the same way it is possible to obtain the relationship of all the desired quantities to the dimensionless quantities λ, τ, after which, for the same value γ, it is possible to determine easily the characteristics of the turbulent region of the explosion for any values of E_0, any initial density ρ_1 and any initial pressure p_1.

In the case of cylindrical symmetry, the explosion proceeds along a straight line*, and in the case of plane waves, along a plane. In this case the quantity E_0 denotes the energy released, respectively, per unit length or area.

Section 2. Ordinary Differential Equations and the Conditions for Jumps in the Case of Self-Modeling Motions

For solution of the indicated problems we shall introduce equations which must satisfy V, R, and P.

By inserting into equations (1.2), in place of v, ρ and p, their expressions in terms of V, R and P, we obtain:

$$\lambda\left[(n+mV)V'+m\frac{P'}{R}\right]=V^2-V-(k+1)\frac{P}{R},$$

$$\lambda\left[mV'+(n+mV)\frac{R'}{R}\right]=-s-(k-\nu+3)V,$$

$$\lambda(n+mV)\left[\frac{P'}{P}-\gamma\frac{R'}{R}\right]=-s(1-\gamma)-2-[k(1-\gamma)+1-3\gamma]V.$$

* A strong explosion along a straight line in certain cases can be (cont'd next page)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

By introducing in place of P a new variable $z = \frac{\gamma P}{R}$ we convert these equations to the following form:

$$\frac{dz}{dV} = \frac{z([2(V-1)+\varkappa(\gamma-1)V](V-q)^2-(\gamma-1)V(V-1)(V-q)-[2(V-1)+\varkappa(\gamma-1)]z)}{(V-q)[V(V-1)(V-q)+(\varkappa-\gamma V)z]}, \quad (2.1)$$

$$\frac{d\ln\lambda^{1/m}}{dV} = \frac{(V-q)^2-z}{V(V-1)(V-q)+(\varkappa-\gamma V)z}, \quad (2.2)$$

$$(V-q)\frac{d\ln R}{d\ln\lambda^{1/m}} = \frac{V(V-1)(V-q)+(\varkappa-\gamma V)z}{z-(V-q)^2} - [s+(k-\gamma+3)V], \quad (2.3)$$

where $$q = -\frac{n}{m} \text{ and } \varkappa = \frac{s+2+q(k+1)}{\gamma}.$$

It is easy to see that the basic problem consists in the integration of equation (2.1). If equation (2.1) is integrated, then the relationship of V and R to λ is determined from the equations (2.2) and (2.3) by quadrature.

Since, in most of the problems indicated above, shock waves arise in the flow, then before proceeding to an analysis of the solutions of equations (2.1), (2.2) and (2.3) in concrete cases, we shall examine in general form the relationships between the values of V, R, and z on both sides of the surface of a strong discontinuity.

During passage through a surface of strong discontinuity, the conditions of conservation of mass, quantity of motion and energy must be fulfilled. By denoting with the index 1 the quantities on one side of the discontinuity, and with the index 2 the conditions on the other side, we can write:

$$\rho_1(v_1-c) = \rho_2(v_2-c), \quad (2.4)$$

$$\rho_1(v_1-c)^2 + p_1 = \rho_2(v_2-c)^2 + p_2, \quad (2.5)$$

$$\frac{1}{2}(v_1-c)^2 + \frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} = \frac{1}{2}(v_2-c)^2 + \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2}, \quad (2.6)$$

* viewed as a high intensity electric discharge in a gas.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

where c is the velocity of the surface of the discontinuity.

We shall replace in these relationships the quantities v, ρ and p with their expressions in V, R and P according to formulas (1.1) and we shall introduce the variable $z = \frac{\gamma P}{R}$. We note in addition that for a jump, r is a function of time t. Hence, t, a and b will be the dimensional defining parameters for the jump. From them it is impossible to form a dimensionless combination, and therefore we have for the surface of the discontinuity

$$\lambda = \lambda_0 = \text{const.} \quad \text{and} \quad r = \lambda_0 a^{-l/m} b^{l/m} t^{-n/m}$$

From this we obtain for the rate of jump c:

$$c = \frac{dr}{dt} = -\frac{n}{m}\frac{r}{t}$$

By considering this, the equations (2.4), (2.5) and (2.6) can easily be transformed into the following form:

$$R_1\left(V_1 + \frac{n}{m}\right) = R_2\left(V_2 + \frac{n}{m}\right),$$

$$V_1 + \frac{n}{m} + \frac{z_1}{\gamma\left(V_1 + \frac{n}{m}\right)} = V_2 + \frac{n}{m} + \frac{z_2}{\gamma\left(V_2 + \frac{n}{m}\right)},$$

$$\frac{\left(V_1 + \frac{n}{m}\right)^2}{2} + \frac{z_1}{\gamma - 1} = \frac{\left(V_2 + \frac{n}{m}\right)^2}{2} + \frac{z_2}{\gamma - 1}.$$

From these equalities, the quantities V_2 and z_2 are expressed through V_1 and z_1 according to the formulas:

$$\left.\begin{aligned} V_2 + \frac{n}{m} &= \left(V_1 + \frac{n}{m}\right)\left[1 - \frac{2}{\gamma + 1}\frac{z_1 - \left(V_1 + \frac{n}{m}\right)^2}{\left(V_1 + \frac{n}{m}\right)^2}\right], \\ z_2 &= \left(\frac{\gamma - 1}{\gamma + 1}\right)^2 \frac{1}{\left(V_1 + \frac{n}{m}\right)^2}\left[\left(V_1 + \frac{n}{m}\right)^2 + \frac{2z_1}{\gamma - 1}\right] \times \\ &\quad \times \left[\frac{2\gamma}{\gamma - 1}\left(V_1 + \frac{n}{m}\right)^2 - z_1\right]. \end{aligned}\right\} \qquad (2.7)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

By setting up in the plane (V, z) the point (V_1, z_1) from the relationships in (2.7), we find the point (V_2, z_2) into which it passes after the jump. We note that the particles of the gas pass through the jump from state 1 to state 2. From the symmetry of equations (2.4), (2.5) and (2.6) it is clear that in the formulas (2.7), the indexes 1 and 2 can be replaced by points.

The points of the parabola

$$z = \left(V + \frac{n}{m}\right)^2 \tag{2.8}$$

transpose mutually. Weak discontinuities, i.e., the surfaces of discontinuity of the derivatives, correspound to this parabola. Actually, the equation (2.8) written in dimensional quantities, gives:

$$\frac{\gamma P}{\rho} = (v - c)^2$$

i.e., the square of the jump velocity through the particles is equal to the square of the speed of sound. The points lying under the parabola (2.8) convert to points lying above it, and vice-versa.

Since z in its physical concept is always positive, then there is a physical concept only for those cases when the points of the upper semi-plane convert to points of the same upper semi-plane.

Points on the axis V which correspond to the limiting case, when z = 0, transform into points of the parabola

$$z = \frac{2\gamma}{\gamma - 1}\left(V + \frac{n}{m}\right)^2. \tag{2.9}$$

Thus, we find that if a point which corresponds to the state of the gas before the jump, is located between the axis V and the parabola (2.8), then a point which corresponds to the state of the gas after the jump will be located between the parabola (2.8) and parabola (2.9), and vice-versa (Fig.28). Points lying within the parabola (2.9) convert to points of the lower semi-plane and hence, cannot

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

correspond to the state of the gas either before or after the jump. Still another limitation on the possible transitions is imposed by the Tsemplen theorem - resulting from the second law of thermodynamics on the increase of entropy. From this theorem it follows that the following inequality must be fulfilled*:

$$V_1 - V_2 < 0 \qquad (2.10)$$

But from the first equation of (2.7) we find:

$$V_1 - V_2 = -\left(V_1 + \frac{n}{m}\right)\frac{2}{\gamma+1}\,\frac{z_1 - \left(V_1 + \frac{n}{m}\right)^2}{\left(V_1 + \frac{n}{m}\right)^2}.$$

From this it can be seen that the inequality (2.10) is fulfilled if at $V_1 < -\frac{n}{m}$, the point (V_1, z_1) lies in the area outside the parabola (2.8) and at $V_1 > -\frac{n}{m}$, inside this parabola.

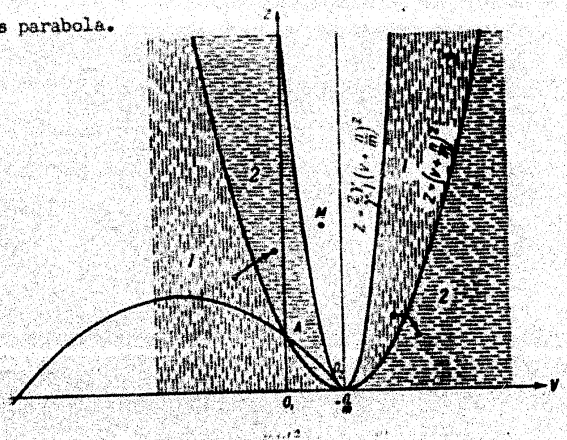


Fig.28 - In the Plane $z = \frac{\gamma p t^2}{\rho r^2}$, $V = \frac{vt}{r}$, the Transition of Points from the Vertically-Hatched Area to the Horizontally Hatched Area may Correspond to Jumps. Possible transitions are shown by arrows.

* Derivation of this condition is contained, for example, in the book: Sedov, L.I., Plane problems of hydrodynamics and aerodynamics, Moscow-Leningrad, Gost., 1950

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In Fig.28, the areas whose points can express the state of the gas before the jump are hatched vertically; and the areas into which the characterizing point passes after the jump are hatched horizontally.

The directions of possible transitions from the point $(V_1 z_1)$ to point (V_2, z_2) are shown by arrows.

We note again that if the shock wave is propagated through a gas at rest, i.e., if the point (V_1, z_1) is located on the axis $z(V = 0)$, then the point (V_2, z_2) must lie on the parabola

$$z_2 = \left(\frac{n}{m}\right)^2 \left(1 + \frac{m}{n} V_2\right)\left(1 - \frac{n}{m}\frac{\gamma - 1}{2} V_2\right). \qquad (2.11)$$

This parabola is shown in the lower left part of Fig.28.

After these preliminary observations, it is simple to give a solution for the problems stated above.

In the problems concerning the piston, focusing and scattering, propagation of flame and detonation, it is possible to take for the characteristic constants a and b, through the dimensions of which there can be expressed the dimensions of all of the constants entering into the problem

$$a = \rho_1 \quad \text{and} \quad b = \frac{\gamma p_1}{\rho_1}$$

In this instance the constants entering into the formulas (1.1) and the equations (2.1), (2.2) and (2.3) will be equal to

$$k = -3,\ s = 0,\ m = 2,\ n = -2,\ q = 1,\ \varkappa = 0$$

and

$$\lambda = \frac{\gamma p_1}{\rho_1}\frac{t^2}{r^2}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The equation (2.1) and (2.2) take the form (for the sake of definition we shall examine in the following instance, equations of motion with spherical symmetry):

$$\frac{dz}{dV} = 2\,\frac{z}{V}\cdot\frac{z-(V-1)(\gamma V-1)}{3z-(V-1)^2}, \qquad (2.12)$$

$$\frac{1}{2}\,\frac{d\ln\lambda}{dV} = \frac{z-(V-1)^2}{V\,[3z-(V-1)^2]}. \qquad (2.13)$$

The last equation (2.3) is integrated and gives the connection between P, R and λ:

$$P = \frac{R^{\gamma}\lambda}{\gamma\beta^{\gamma-1}},$$

where β is the constant of integration.

The main difficulty consists in the integration of equation (2.12).

We shall examine the behavior of the integral curves of equation (2.12) in the plane (V, z).

In the points of the parabola $z = (V-1)(\gamma V-1)$ the integral curves have tangents parallel to the axis V; in the points of the parabola $z = \frac{1}{3}(V-1)^2$ they have tangents parallel to the axis z. The points O(0,0), A(0,1), $B\left(\frac{2}{3\gamma-1}, \frac{3(\gamma-1)^2}{(3\gamma-1)^2}\right)$ C(1,0), D($\pm\infty$, ∞) are specific points of the equation.

In the points O, A, and C there is tangency, and in the points B and D, intersection. The integral curves have the form shown in Fig.29 (where only the semi-plane $z > 0$ is shown since for $z < 0$ there is no physical concept).

The arrows indicate the direction of increase of the parameter λ along the integral curve. On the parabola $z = (V-1)^2$ the parameter λ reaches, at $V < 0$ and $V > 1$, a maximum; and at $0 < V < 1$, a minimum. For this reason continuous passage along the integral curve through this parabola can not be achieved since this would lead to non-identity.

The points of the integral curve $V = 0$ correspond to a state of rest. In the case of motion in physical space from ∞ to a center of symmetry, the parameter λ

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

varies from 0 (at $r = \infty$) to ∞ (at $r = 0$).

In the plane (V, z) the motion of a fluid can be continued to the center $r = 0$ ($\lambda = \infty$) along the integral straight line $V = 0$, $z \to +\infty$ responding to a state of

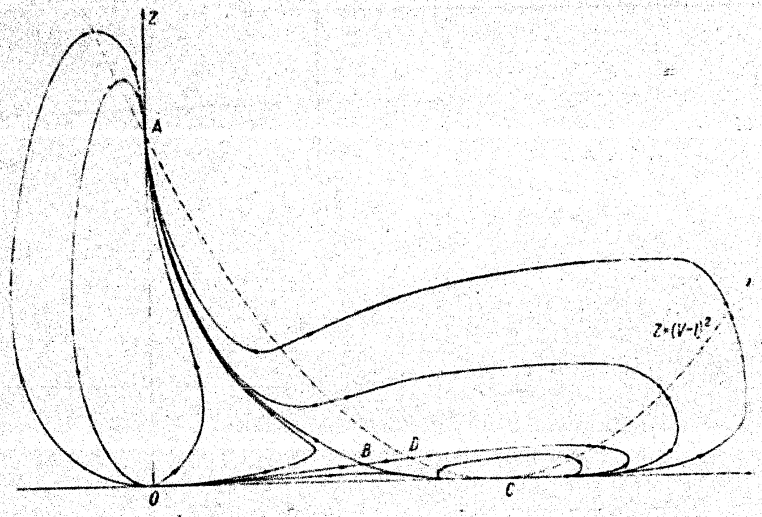


Fig.29 - General Form of the Integral Curves for Equation (2.12) in the Spherical Case. The arrows indicate the direction of increase of the parameter $\lambda = \frac{\gamma p_1}{\rho_1} \frac{t^2}{r^2}$, which corresponds to motion toward the center of symmetry.

rest, along the straight line $z = 0$, $V \to \pm\infty$ for which either the density is infinite or the pressure is equal to zero, and along the integral curves OB and DB, abutting point B.

To the infinitely remote point of flow $r = \infty$ ($\lambda = 0$) there may correspond only the specific points O and B. In this case, for solutions starting from the point O, the velocity of a fluid in infinity can have any value; and for point B the velocity is found to be infinitely large.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Section 3. The Problem of the Piston*

Let us examine the picture of a motion at some moment of time t. Since for the particles of a fluid contiguous to a piston $v = U$ and $r = Ut$, then $V = \frac{v}{\frac{r}{t}} = 1$. Hence, in the plane (V, z), the characteristic point corresponding to the piston will lie on the straight line $V = 1$. The motion along the integral curve in the direction of decreasing λ corresponds to the motion along the radius from the piston towards infinity. But the integral curve intersects the parabola $z = (V - 1)^2$, continuous passage through which is impossible. Hence a continuation of the motion as far as the point O, which corresponds to an infinitely remote point is possible only by means of a jump.

On the other hand, in the region which has not yet been reached by turbulence, the gas is at rest, i.e., the point of the axis z corresponds to the outer side of the jump.

The points of the axis z, as was shown above, pass by jumps into points of the parabola

$$z = (1 - V)\left(1 + \frac{\gamma - 1}{2}\right) \qquad (3.1)$$

---

* Solution of the problem concerning the motion of a gas displaced by a sphere which is expanding at a constant rate, with numerical calculations was first published in 1945 in the work cited above (Prikladnaya matematika i mekhanika, Vol.IX, No.4, 1945.

Solution of the problem concerning the motion of a gas displaced by a sphere which is expanding at a rate equal to $U = ct^n$ with a comparative analysis for various n (wherein for the case $n = 1/2$ viscosity and heat conductivity are taken into consideration) is given in the dissertation of N.L.Krasheninnikova, defended in the spring of 1954 at Moscow University (MGU) (see Referativnny Zhurnal-Mekhanika, No.8, 1954, Abstract No.4400).

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Thus, in the case of motion in physical space from a piston to infinity the characteristic point moves along the integral curve from some point by the straight line V = 1 to intersection with the parabola (3.1), after which, by means of a jump, it passes on to the straight line V = 0 (Fig.30).

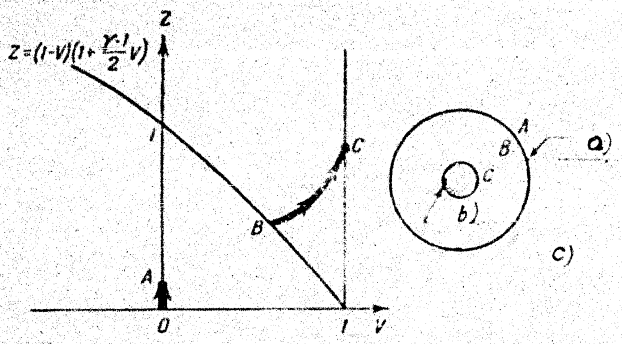


Fig.30 - Integral Curve in the Plane z, V for Solution of the Spherical Piston Problem. Transition from Point A to point B proceeds by a jump through the shock wave. Point C corresponds to the piston. The curve BC corresponds to adiabatic compression between piston and shock wave.

a) Shock wave; b) Piston; c) Gas at rest

In physical space the following picture of motion corresponds to this case: a shock wave is propagated through a gas at rest; adiabatic compression occurs between the shock wave and the piston.

The case of a cylindrical piston does not differ qualitatively from the case of the spherical piston. For a flat piston, in the case of motion of the piston in a tube, in contrast to the spherical case, it is obvious that between the shock wave and the piston there arises a region where the gas moves with constant velocity, density and pressure.

Figures 31 and 32 show curves which give the relationship of the pressure and density of gas particles contiguous to a piston, to the pressure and density in a

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

gas at rest as a function of the ratio of piston velocity to the primary speed of sound. Figure 33 shows the relationship of the shock wave velocity to the speed of sound as a function of the ratio of piston velocity to the speed of sound. It is seen from the graphs that at one and the same piston velocity in the plane case, the compression of the gas is greater than in the spherical case. The shock wave in the plane case is also more intensive than in the spherical case (particularly for low piston velocities).

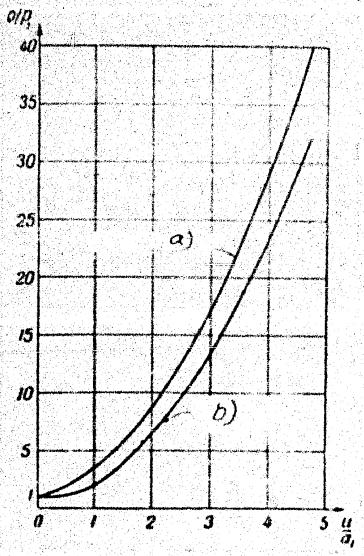


Fig.31 - Pressure on the Piston p as a Function of Piston Velocity U; p_1 is the Pressure and a_1 is the Speed of Sound in a Non-Turbulent Gas

a) Plane piston; b) Spherical piston

In the plane case, in contrast to the spherical and cylindrical cases, besides the problem of gas compression by the piston there are also problems concerning the movement of the piston out of the gas. Corresponding to this problem there is a particular solution of the system of equations (1.2) at $\nu = 1$ having the form

$$z = (V-1)^2, \quad V - \frac{2}{\gamma - 1}(1 - V) = C\sqrt{\lambda} \qquad (3.2)$$

where C is the integration constant.

By converting to dimensional variables we obtain formulas for the nonlinear solution for a progressive depression wave in a gas (the simplest case of the Riemann solution)

$$x = (v + a)t, \quad v = \pm \frac{2}{\gamma - 1} a + \text{const}, \qquad (3.3)$$

where a is the speed of sound.

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

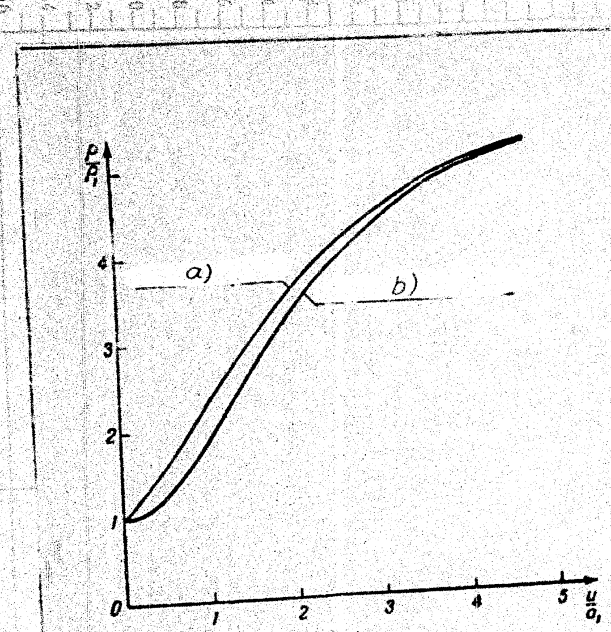


Fig.32 - Density ρ of Particles Contiguous to a Piston as a Function of the Piston Velocity U; ρ_1 is the Density: a_1 is the Speed of Sound in a Non-Turbulent Gas

a) Plane piston; b) Spherical piston

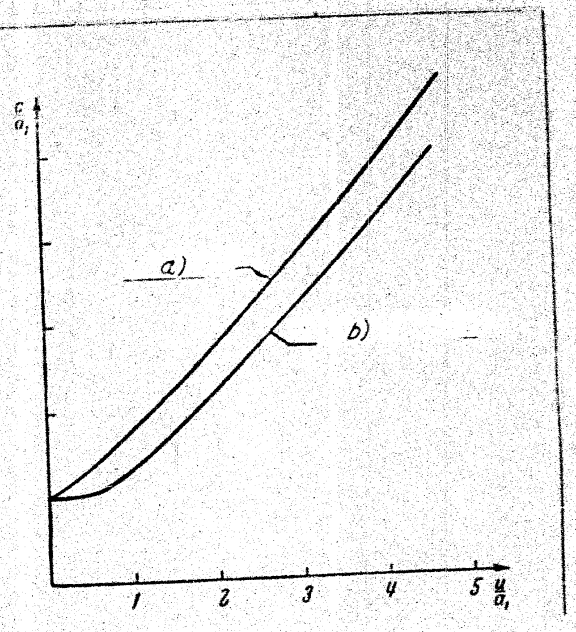


Fig.33 - Velocity c of the Shock Wave Arising in Front of a Piston as a Function of Piston Velocity U; a_1 is the Speed of Sound in a Non-Turbulent Gas

a) Plane piston; b) Spherical piston

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Section 4. The Problem of the Focusing of a Gas in a Point and Dispersion from a Point

If the initial velocity is directed toward a center (focusing), i.e., $v_1 < 0$, then the motion along the integral curve originating from point 0 in the direction of negative V corresponds to the motion from infinity toward the center during a fixed time t.

Since the velocity of the gas in the center at $t \neq 0$ is equal to zero, the center can be reached either during motion along the integral curve running into a particular point B, or during motion along the integral straight line $V = 0$. But it is generally impossible to pass from the region $V < 0$ where the characteristic point is

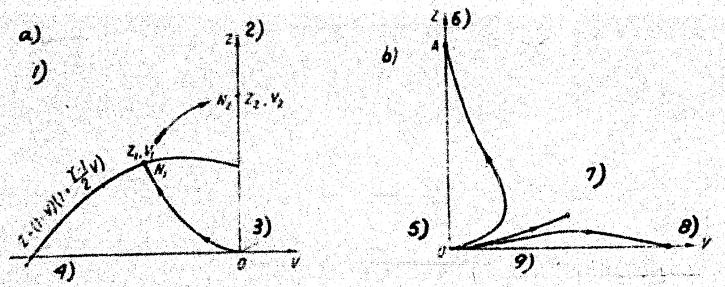


Fig.34 - Integral Curves in the Plane (z,V) Corresponding to:
a) Collision in a Point and b) Dispersion from a Point
1) Shock wave; 2) Nucleus of rest; 3) Infinitely remote point;
4) Collision in a point; 5) Infinitely remote point; 6) Nucleus of rest; 7) Center of symmetry; 8) Vacuum; 9) Dispersion

located, to the integral curve running into point B; and it is possible to pass to the axis z only by means of a jump.

Consequently the characteristic point moves along the integral curve to intersection with the parabola $z = (1 - V) \times \left(1 + \frac{\gamma - 1}{2} V\right)$, into which the points of the axis z pass in the case of discontinuity. After intersection with the parabola the

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

characteristic point passes by means of a jump to some point of the axis z. The form of the integral curve is shown in Fig.34a.

In physical space, in such a case, the gas, moving from infinity toward the center is first compressed adiabatically, after which it passes by means of a jump to a state of rest (Fig.35a).

In the case of dispersion ($v_1 > 0$ - the velocity of the particles of the gas is directed away from the center) at small values of the initial velocity, the characteristic point moves along the integral curve from the point O to the point A and

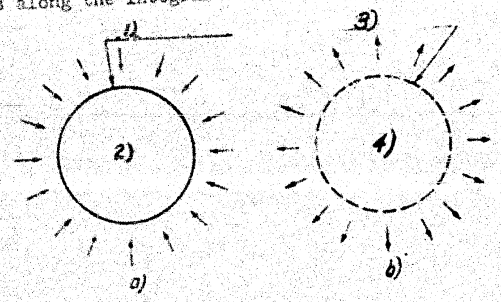


Fig.35 - Diagrams of Motion: a) For Collision in a Point and b) For Dispersion from a Point. Adiabatic compression or depression occurs in front of the nucleus.

1) Shock wave; 2) Gas at rest; 3) Weak discontinuity; 4) Gas at rest

then along the integral straight line V = 0 (Fig.34b). During motion from infinity to a center at a fixed moment of time, the drop in density and pressure of a gas to a definite quantity, after which the gas passes by means of a weak jump to a state of rest (Fig.35b), corresponds, in physical space, to the above process. The particular point A corresponds to the boundary of the spherical nucleus of the gas at rest.

At some initial velocity $v = v_1^*$, the integral curve OB is obtained in the plane (V, z). The point B corresponds to the center of symmetry. In physical space, in this case, the velocity is equal to zero only in the center of symmetry.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

If $v_1 > v_1^*$, then along the integral curve it is possible to reach only the specific point C, in which the parameter λ has some finite - different from zero - constant value λ^*; but the density and pressure are equal to zero.

Consequently, in the gas a vacuum is formed which expands with a constant velocity determined by the value λ^*.

Section 5. Detonation and Combustion

During transition through a front of detonation or flame, the relationships (2.4) and (2.5), which express the law of conservation of mass and quantity of motion, remain valid. But in equation (2.6) which expresses the law of conservation of energy, a term is added which defines the amount of energy given off during combustion of a unit-mass of gas. It takes the form

$$\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1}+\frac{1}{2}(v_1-c)^2=\frac{\gamma'}{\gamma'-1}\frac{p_2}{\rho_2}+\frac{1}{2}(v_2-c)^2-Q, \qquad (5.1)$$

where γ is the relationship of the heat capacities of the fuel mixture, and γ' the relationship of the products of combustion.

The velocity of a flame front through the particles before it - $U = v_1 - c$ -

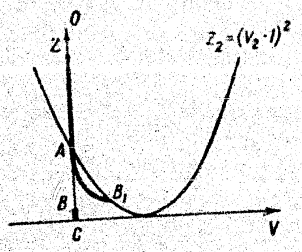


Fig.36 - Integral Curves in the Plane (z,V) Corresponding to Spherical Detonation

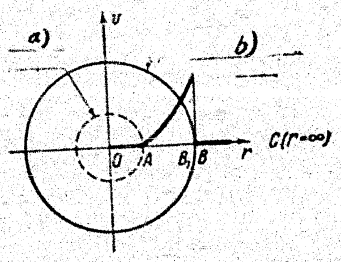


Fig.37 - Picture of Flow during Spherical Detonation

a) Weak discontinuity; b) Detonation-front jump

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The velocity of detonation in real conditions is almost always such that the particles of gas behind the detonation front move, relative to the front, at the speed of sound*.

$$(v_2 - c)^2 = \frac{\gamma p_2}{\rho_2}. \qquad (5.2)$$

Thus, during the propagation of a flame or detonation, the velocity of the front is known; and, consequently, all of the quantities behind the front can be identically expressed through the quantities before the front.

To determine the picture flow at any moment of time, we revert to the integral curves in the plane (V, z).

a) Let a detonation occur in a gas**. In that part of the space which the detonation wave has not yet reached, the gas is at rest; i.e., a point located on the axis z corresponds to the outer side of the detonation wave in the plane (V, z). The condition (5.2), written in dimensionless form shows that the point by a jump, passes to the parabola

$$z_2 = (V_2 - 1)^2 \qquad (5.3)$$

From the point (V_2, z_2) of the parabola (5.3) it is possible to reach the point V = 0, z = ∞ corresponding to a center, along the integral curve, which goes to specific point A, and thence along the integral straight line V = 0, z → ∞ (Fig.36). In physical space we find that behind a detonation front there follows an exhaustion wave in which the velocity falls from the value v_2 behind the detona-

---

* For more detail on this point see, for example: Landau, L. and Lifshits, Ye., Mechanics of Continuous Media, Gostekhizdat, 1953; or Sedov, L.I., Plane Problems of Hydrodynamics and Aerodynamics, Gostekhizdat, 1950.

** A theoretical solution for the problem of spherical detonation has been given by Zel'dovich, Ya.B. (Zhetf, 1941).

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

tion front to zero in the expanding nucleus of the gas at rest (Fig.37).

The cylindrical and plane cases do not differ qualitatively from the spherical case.

Figures 38 and 39 show the distribution of velocity and pressure in the detonation products behind a plane (unbroken line) and a spherical (broken line) detonation wave in trinitrotoluene with an initial density $\rho = 1.51$ g/cm^3.

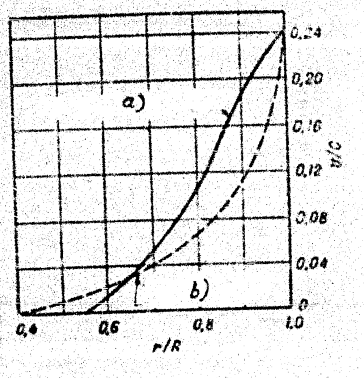


Fig.38 - Results of Calculation for Distribution of the Velocity v of the Products of Detonation of Trinitrotoluene Behind the Detonation Front in Cases of Plane and Spherical Detonation; c is the Velocity of Propagation of the Detonation Front.

a) Plane detonation; b) Spherical detonation

The velocity of the detonation wave in this example is equal to 6380 m/sec. The density of the gas behind the detonation wave is equal to 2 g/cm^3, and the ratio of the velocity of the particles behind the wave to the velocity of the wave is equal to 0.242.

b) A flame front is propagated through a fuel mixture after ignition*.

We shall examine again the state of the gas at some moment of time t. In points sufficiently remote from the center, the gas is still at rest, i.e., V = 0. Hence, during motion in a space from infinity to a center in the plane (V, z), the characteristic point will move from the origin of coordinates along the positive semiaxis z to some point z_1 - a point which the disturbance has already reached. Passage from an arbitrary point on the axis z to any integral curve can be accomplished only by means of a jump.

* This problem has been studied in detail by G.M.Bam-Zelikovich.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

After the jump, the characteristic point appears in some point (V_2, z_2) of the parabola (3.1) $z = (1 - V)\left(1 + \frac{\gamma - 1}{2} V\right)$ and motion along the integral curve passing through this point in the direction toward which the parameter λ increases, i.e., motion in the region above the parabola (3.1), corresponds to further motion toward the center. But in order to reach the center it is necessary to fall again onto an integral straight line $V = 0$ ($z = +\infty$). From the region above the parabola (3.1), where the characteristic point is located, it is possible to strike the straight line $V = 0$ only by means of a new jump. This jump is accomplished at the flame front. Thus, behind the flame front $V = 0$; i.e., the gas is at rest. By substituting V, R and P in equations (2.4), (2.5) and (5.1) for v, ρ and p, by introducing $z = \frac{\gamma P}{R}$ and assuming $V_2 = 0$, with the help of simple transformations we find the connection between V and z before the flame front for the case when the gas is at rest behind the flame front:

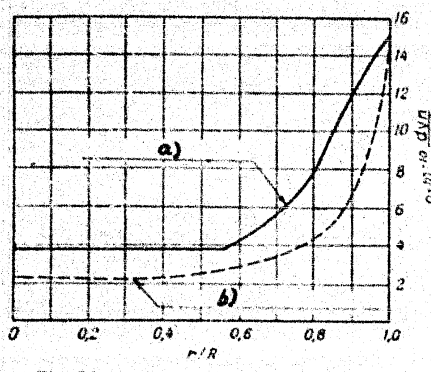


Fig.39 - Distribution of Pressure behind the Detonation Front for Trinitrotoluene
a) Plane detonation; b) Spherical detonation

$$z_3 = \frac{V_3(1-V_3)\left(1+\frac{\gamma'-1}{2}V_3\right)+(\gamma'-1)\frac{Q}{U^2}(1-V_3)^2}{\frac{\gamma'}{\gamma}-\frac{\gamma'-1}{\gamma-1}(1-V_3)}. \qquad (5.4)$$

The characteristic point moves along the integral curve to its intersection with the curve (5.4), after which it passes to axis z by means of a jump (Fig.40).

It is easily seen that z, along the curve (5.4), diminishes monotonnically from ∞ (at $V = 0$) to zero (at $V = 1$) with any Q and U.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Consequently, the integral curve along which the characteristic point moves must intersect the curve (5.4); i.e., a solution of the problem exists for any

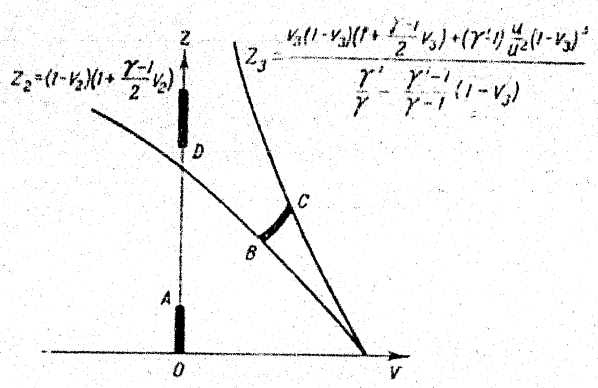


Fig.40 - Integral Curves in the Plane (z, V) Corresponding to Spherical Combustion. The points A and B correspond to the shock wave before a flame front. The points D and C correspond to the flame front.

value of heat liberation Q and flame front velocity U.

Thus, in the case of propagation of a flame front we find that a shock wave is propagated through the gas: between the shock wave and the flame front the gas undergoes additional adiabatic compression, after which by way of combustion at the flame front it passes into a state of rest. Examples of calculations are given in Figs.41, 42 and 43.

If it is possible to disregard the effect of viscosity and heat conductivity and if there is no liberation or absorption of heat owing to phase transformations and chemical reactions, related to new dimensional constants, it is then obvious that the parameters of mechanical motion of the gas can be sought independently of the parameters of molecular motion (temperature, etc.)

In other words, in the case of any form of equation of state of the gas and with any relationship of internal energy to density and temperature, the temperature

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

can be excluded from the system of equations which describe the mechanical motion of a gas. With this exclusion there will be no new dimensional constants in the

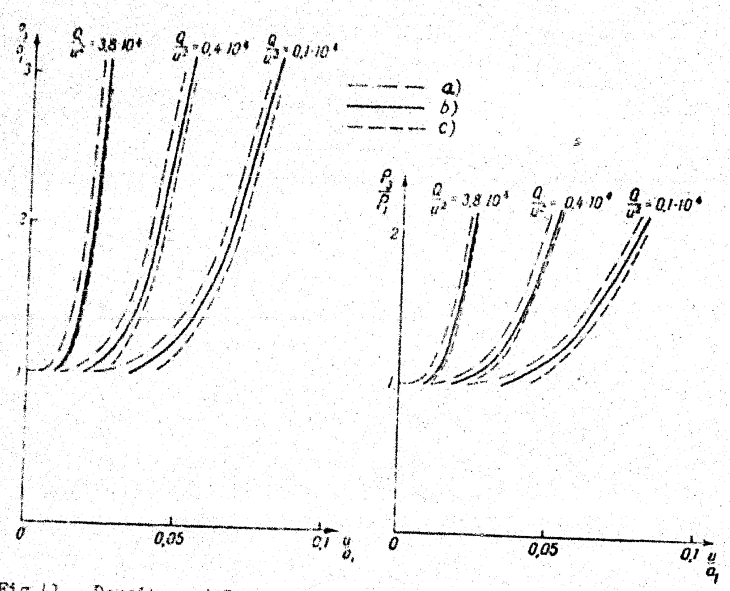


Fig.41 - Density and Pressure before a Flame Front for Various Quantities of Heat Liberation $\frac{Q}{U^2}$ (Q is the Energy Given Off by a Unit-Mass; U is the Velocity of the Flame Front through the Particles; p_1 is the Pressure; ρ_1, Density; and a_1 is the Speed of Sound in the Fuel Mixture).

a) Plane flame; b) Cylindrical flame c) Spherical flame

equations; i.e., the motion in the problems examined above with the real constants p_1 and ρ_1 remains self modeling.

This conclusion can be obtained from the equations in the following manner. We have

$$ds(\rho, T) + pd\frac{1}{\rho} = dQ(p, \rho, T), \qquad (5.5)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

where ε is the internal energy and dQ is the influx of heat from the outside.

Excluding temperature with the help of the equation of state $T = f(p, \rho)$ and

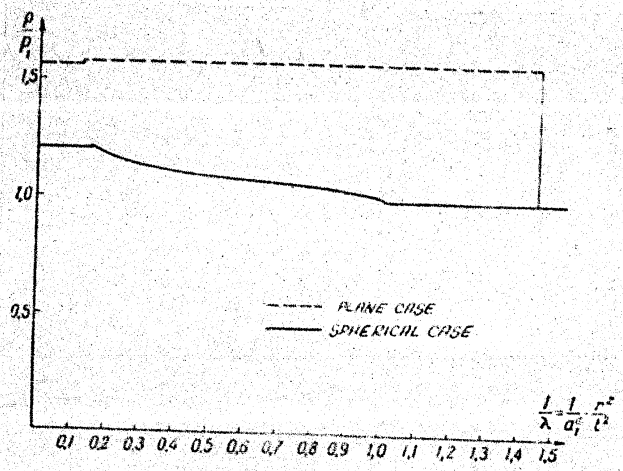


Fig.42 - Distribution of Pressure during Propagation of Combustion from a Plane Wall (Plane Flame Front) and from a Point (Spherical Flame Front):

$$\frac{Q}{a_1^2} = 60 \text{ and } \frac{U}{a_1} = 0.016.$$

reducing (5.5) to dimensionless form, we obtain

$$d\bar{\varepsilon}\left(\frac{p}{p_1}; \frac{\rho}{\rho_1}\right) + \frac{p}{p_1} d\frac{\rho_1}{\rho} = \overline{dQ}\left(\frac{p}{p_1}; \frac{\rho}{\rho_1}\right),$$

where $\bar{\varepsilon}$ and $\overline{dQ}$ are dimensionless functions of their arguments*, and p_1 and ρ_1 are certain constants with the dimension of pressure p and density ρ.

Consequently, in the energy equation after exclusion of temperature, there are no constants with a dimension which does not depend on the dimensions p and ρ.

* The quantity dQ in the general case is not a complete differential.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Section 6. Resolution of an Arbitrary Discontinuity in a Fuel Mixture

Omitting here a detailed analysis of the problem of the resolution of an arbitrary discontinuity (formulated in Section 1), we shall point out only the general

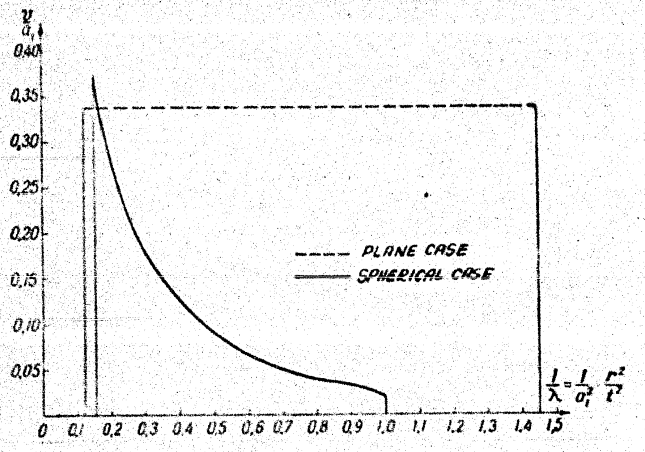


Fig.43 - Distribution of Velocity during the Propagation of Combustion from a Plane Wall (Plane Flame Front) and from a Point (Spherical Flame Front):

$$\frac{Q}{a_1^2} = 60 \text{ and } \frac{U}{a_1} = 0.016$$

character of the motion which occurs.

We assume first that on both sides of the surface of the discontinuity there are two inert gases and that the pressure in the left (2nd) gas is greater than in the right (1st) (the opposite case is completely analogous to this one). Then if the axis x is directed from the second gas to the first, and if the difference $v_1 - v_2$ in the initial velocities of the gases is negative and large in module (such will be the case, for example, if the initial velocities of the gases are directed toward the surface of the discontinuity so that their difference is large in absolute quantity, but negative), then shock waves will run on both sides. At

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the boundary which separates the gases there can still be a stationary discontinuity at which the pressure and normal velocity are continuous, and the density can tolerate the discontinuity. Figure 44a is a graph of the pressure for this case.

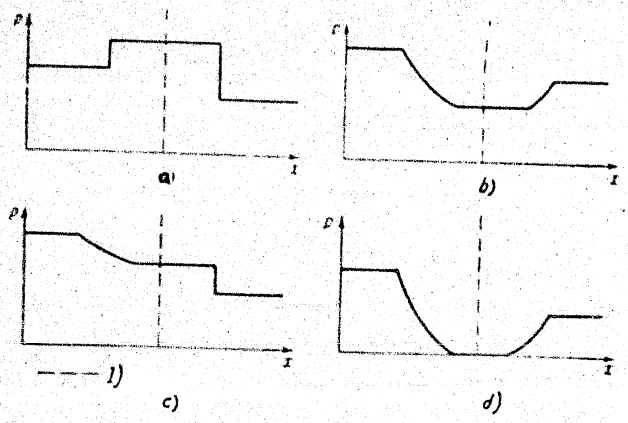


Fig.44 - Various Cases of Resolution of an Arbitrary Discontinuity in an Inert Gas

1) Stationary discontinuity

With an increase in the difference of the initial velocities the shock wave in the second gas will be replaced by a depression wave (Fig.44b); subsequently, in the first gas too, in place of a shock wave there will be a depression wave (Fig.44c). When the difference in the initial velocities becomes a very large positive quantity, a vacuum is formed between the depression waves running to both sides (Fig.44d).

A more complex case occurs when there is to the right of the surface of the explosion a fuel mixture so that during the resolution of an arbitrary discontinuity a flame front may run through it. In this case, in overall form, the motion which occurs will be analogous to that examined above*.

* This problem has been analyzed quantitatively in detail in the (cont'd. next page)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In the case of a small difference in initial velocities a shock wave propagates through the inert gas; a shock wave also runs through the fuel mixture and behind

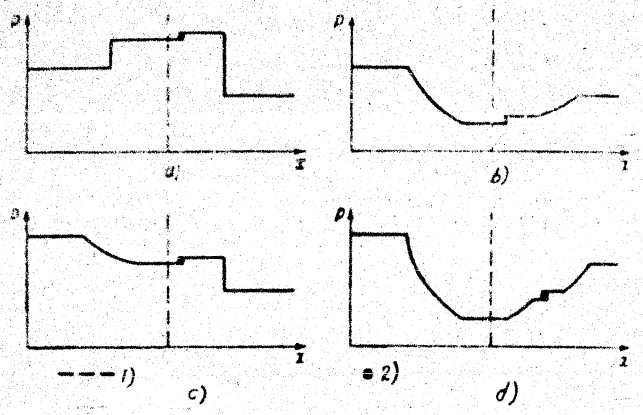


Fig.45 - Various Cases of the Resolution of an Arbitrary Discontinuity in a Fuel Mixture

1) Stationary discontinuity; 2) Flame front

it a flame front. There can be a stationary discontinuity between the inert gas and the products of combustion.

Figure 45a shows a graph of pressure for this case (the wide vertical line represents the flame front, the broken line represents the stationary discontinuity. In the case of an increase in the difference of the initial velocities, at the beginning the shock wave in the inert gas is replaced by a depression wave (Fig.45b); subsequently, a depression wave runs before the flame front in place of a shock wave (Fig.45c). In this case the velocity of the combustion products relative to

*(cont'd.) work: Bam-Zelikovich, G.M., The Resolution of an Arbitrary Explosion in a Fuel Mixture. Symposium No.4, "Theoretical Hydromechanics", edited by L.I.Sedov, Oborongiz, 1949.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the flame front increases until it reaches the speed of sound.

With a further increase in the difference of initial velocities, the flow before the flame front does not vary; but immediately behind the front there appears another depression wave (Fig.45d). In the case of a very large difference of initial velocities a vacuum can arise between the inert gas and the combustion products.

If the pressure in the inert gas is less than that in the fuel mixture, the possibility arises of a situation where a depression wave runs through the fuel mixture before the flame front, and a shock wave runs through the inert gas.

Analogously, if a detonation wave passes through the fuel mixture, then, by increasing the difference of the initial velocities from $-\infty$ to $+\infty$ we find that first, a shock wave passes through the inert gas, and through the fuel mixture a detonation wave of compatibly great velocity. The shock wave in the inert gas is then replaced by a depression wave and the velocity of the detonation wave decreases to some definite quantity. In this case the velocity of the products of detonation relative to the front increases until it reaches the speed of sound. The velocity of the detonation wave does not vary subsequently, and a depression wave arises behind it.

We shall present several examples of the formation and resolution of an arbitrary discontinuity.

1. Assume a shock wave propagated through a gas, and coming up from behind it, a second shock wave. At the moment when the second wave overtakes the first a surface of discontinuity is formed on which the conditions for conservation of mass, quantity of motion and energy will not be fulfilled; i.e., an arbitrary discontinuity is formed.

Calculations show that in this case, after resolution of the discontinuity shock waves will run on both sides.

2. A shock wave approaches the boundary of separation between two media of different density. In the moments of passage of the shock wave from one medium into

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the other an arbitrary discontinuity is formed. In the case of the resolution of this discontinuity two types of motion are possible.

During passage of the wave from the less dense medium into the denser medium (e.g., from air into water) shock waves will run into both sides. But if the wave passes from the denser medium into the less dense medium (e.g., from water into air) a shock wave will run in front (into the air), and a depression wave will run behind (into the water).

3. A shock wave of low intensity overtakes a flame front. (This case is encountered during pulsation burning in closed vessels.) After the wave overtakes the flame shock waves will pass to both sides of the flame front. But if a shock wave of low intensity collides with a flame front, then after resolution of the arbitrary discontinuity before the flame front a depression wave will run through the fuel mixture and a shock wave will run through the combustion products.

Section 7. The Problem of a Strong Explosion*

From the considerations below, it follows that in the case of a strong explosion the region of turbulent motion of the air is separated from the non-turbulent states by a shock wave.

As already pointed out, in the case of strong explosion, it is possible to disregard the pressure in front of the shock wave in comparison with the pressure behind the shock wave. First of all, we shall evaluate with what degree of accuracy and for which shock waves this proposition is valid.

For this purpose, considering that $v_1 = 0$ we shall rewrite the conditions at the shock wave (2.4), (2.5) and (2.6) in the following form:

---

* Below, we set forth the accurate theoretical and numerical solution of the problems of a strong explosion both for spherical and for cylindrical and plane waves, first published by us in DAN, Vol.42, No.1, 1946 and in Prikladnaya Matematika i Mekhanika, Vol.10, No.2, 1946.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\left.\begin{aligned} v_2 &= \frac{2}{\gamma+1} c\left[1-\frac{a_1^2}{c^2}\right] = \frac{2c}{\gamma+1} f_1, \\ \rho_2 &= \frac{\gamma+1}{\gamma-1}\rho_1\left[1+\frac{2}{\gamma-1}\frac{a_1^2}{c^2}\right]^{-1} = \frac{\gamma+1}{\gamma-1}\rho_1 f_2, \\ p_2 &= \frac{2}{\gamma+1}\rho_1 c^2\left[1-\frac{\gamma-1}{2\gamma}\frac{a_1^2}{c^2}\right] = \frac{2}{\gamma+1}\rho_1 c^2 f_3, \end{aligned}\right\} \qquad (7.1)$$

where c is the velocity of propagation of the shock wave.

The more intensive the shock wave, the smaller the ratio $\frac{a_1}{c}$.

The relationship of f_1, f_2 and f_3 to $\frac{a_1}{c}$ is shown in Fig.46. The relationship of $\frac{\rho_2}{\rho_1}$ to the ratio $\frac{a_1}{c}$ for $\gamma = 1.4$ is shown in Fig.47. It is easily seen that even with $\frac{a_1}{c} < 0.1$ the quantities f_1, f_2 and f_3 vary from unity by less than 0.05.

If, in the relationships (7.1), we place $\frac{a_1}{c} = 0$ and $f_1 = f_2 = f_3 = 1$ (or, amounting to the same thing, we place $p_1 = 0$, then at $\frac{a_1}{c} < 0.1$ we permit an error in the values v_2, ρ_2 and p_2 of less than 5%.

The conditions at the shock wave in this case assume the form

$$\left.\begin{aligned} v_2 &= \frac{2}{\gamma+1} c, \\ \rho_2 &= \frac{\gamma+1}{\gamma-1}\rho, \\ p_2 &= \frac{2}{\gamma+1}\rho_1 c^2. \end{aligned}\right\} \qquad (7.2)$$

The velocity of propagation of the shock wave c is the quantity being determined.

If we make use of the equations of motion in the form (1.2) and the above indicated statement of the problem of strong explosion, then the following quantities can be taken as basic dimensional constants:

$$a = \rho_1 \text{ and } b = \frac{E}{\rho_1}$$

where E is some constant which we shall determine later, having the same dimension

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

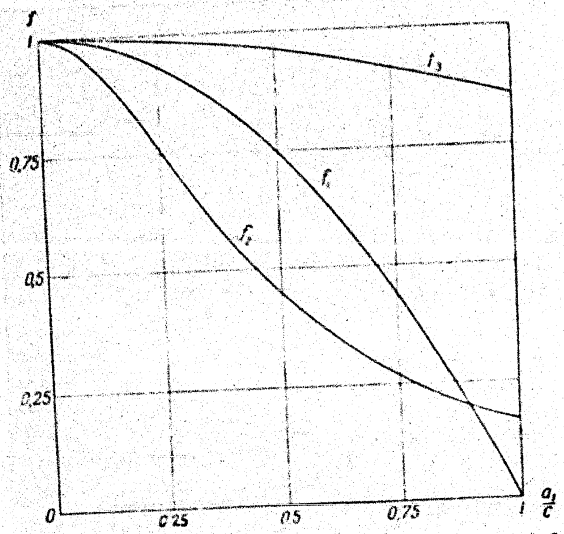


Fig.46 - Relationship of the Quantities f_1, f_2 and f_3 to the Ratio $\frac{a_1}{c}$, where a_1 is the Speed of Sound in a Non-Turbulent Medium and c is the Velocity of the Shock Wave

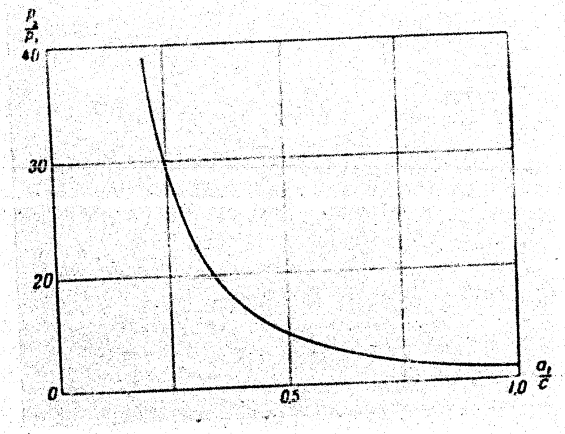


Fig.47 - Pressure Drop in a Shock Wave in Relation to the Ratio $\frac{a_1}{c}$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

as the energy E_0 released during the explosion; the dimension E is equal to:

$[E] = ML^2T^{-2}$ in the spherical case,

$[E] = MLT^{-2}$ in the cylindrical case,

$[E] = MT^{-2}$ in the plane case.

It is possible to combine all three cases using one formula

$$[E] = ML^{\nu-1}T^{-2}$$

It is obvious that the constant E is simply proportional to E_0:

$$E = \alpha E_0$$

where α is a constant.

The single dimensionless variable parameter λ in this case is equal to:

$$\lambda = \frac{E}{\rho_1} \frac{t^2}{r^{2+\nu}}$$

The law for the motion of a shock wave is easily established without solving the equations of motion of a gas.

The equations of motion can be varied; however, these equations must not contain any new real physical constants with dimensions which do not depend on ρ_1 and E. In particular, in the equations (1.2) there is no need to assume that the coefficient $\gamma = \frac{c_p}{c_v}$ has a constant value.

For a shock wave, the coordinate r_2 is a function of the time t; and since it is impossible to form a dimensionless combination from the dimensional quantities t, ρ_1 and E, then

$$r_2 = \left(\frac{E}{\rho_1 \lambda^*}\right)^{\frac{1}{2+\nu}} t^{\frac{2}{2+\nu}}, \qquad (7.3)$$

where λ^* is a constant; λ^* can be made equal to any number but 0, and the value of E can be calculated in relationship to the value of the charge E_0. In our further treatment, for simplicity and definition we will assume $\lambda^* = 1$. By this means,

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

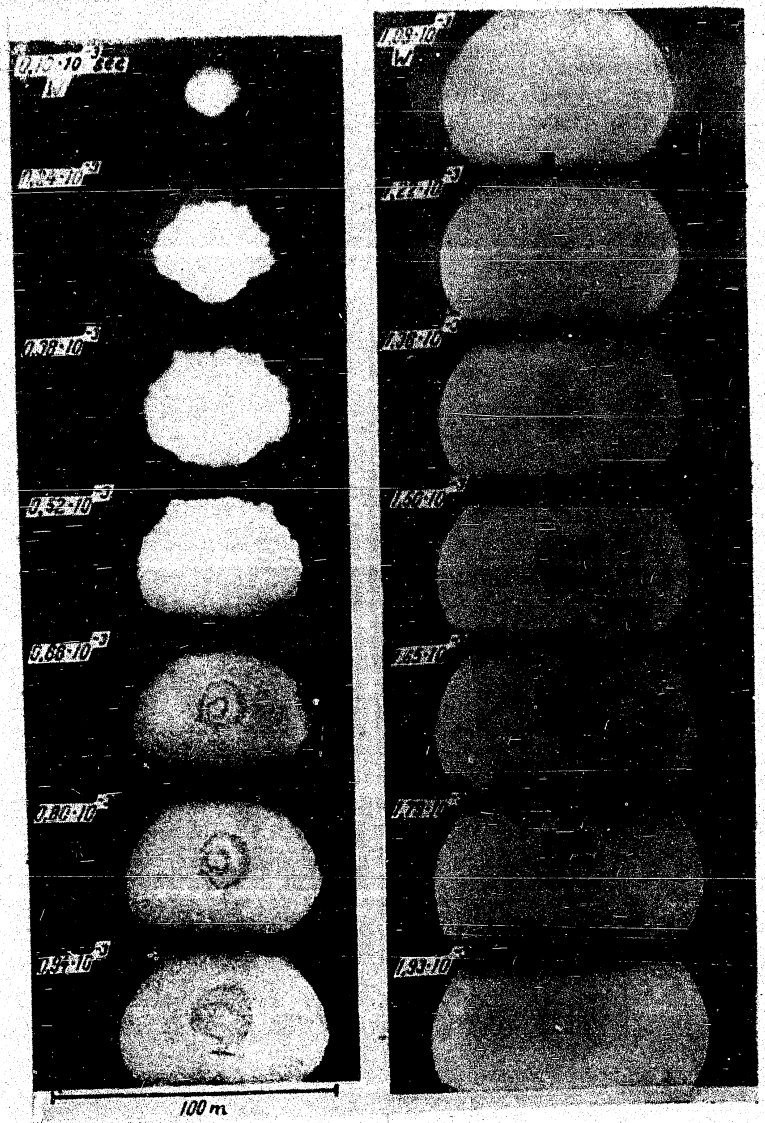


Fig.48 - Sequence Photograph of a Fire Ball from t = $0.1\text{-}10^{-3}$ sec to 1.93×10^{-3} sec during the Explosion of an Atomic Bomb in New Mexico

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

on the basis of the solution of the equations of motion the constant α in the formula $E_0 = \alpha E$ is determined.

Thus, for the law of motion and for the velocity of a shock wave we obtain, in the case of spherical symmetry:

$$r_2 = \left(\frac{E}{\rho_1}\right)^{1/5} t^{2/5}, \quad c = \frac{2}{5}\left(\frac{E}{\rho_1}\right)^{1/5} t^{-3/5} = \frac{2}{5}\sqrt{\frac{E}{\rho_1}}\frac{1}{\sqrt{r_2^3}}; \qquad (7.4)$$

in the case of cylindrical symmetry:

$$r_2 = \left(\frac{E}{\rho_1}\right)^{1/4}\sqrt{t}, \quad c = \frac{1}{2}\left(\frac{E}{\rho_1}\right)^{1/4}\frac{1}{\sqrt{t}} = \frac{1}{2}\sqrt{\frac{E}{\rho_1}}\frac{1}{r_2}; \qquad (7.5)$$

for plane waves:

$$r_2 = \left(\frac{E}{\rho_1}\right)^{1/3} t^{2/3}, \quad c = \frac{2}{3}\left(\frac{E}{\rho_1}\right)^{1/3} t^{-1/3} = \frac{2}{3}\sqrt{\frac{E}{\rho_1}}\frac{1}{\sqrt{r_2}}. \qquad (7.6)$$

These formulas show that the law of damping of a shock wave depends on the form of the charge.

The formula obtained above in the case of spherical symmetry is well confirmed by published experimental data - photographs of an atomic bomb explosion in New Mexico in 1945.

Figures 48, 49 and 50 show photographs of an atomic bomb explosion published in 1950 in the work of G.Taylor*.

Within the region of turbulent motion of the air at rather considerable distances from the center of an atomic bomb explosion, the temperature of the air is very high. Hence, this region is represented on the photographs as a light spot. In the upper part of the spot the boundary has a spherical form, is sharply out-

---

* Taylor, G., Proceedings of the Royal Society, No.8, 1065, Vol.201, p.175, 1950.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

lined and corresponds with the shock wave. With increase in time, the shock wave weakens, the temperature behind its front decreases and, for this reason, the wave ceases to be visible. On the basis of photographs, Taylor's work demonstrates the relationship between the radius r_2 of the expanding spherical shock wave for values from 11 to 185 meters and the corresponding moments of time t from the moment of initiation of the explosion in the interval from 0.1×10^{-3} sec to 62×10^{-3} sec.

In Fig.51 experimental data are represented by x's. The straight line answers the formula

$$\frac{5}{2} \log r_2 \text{ cm} - \log t \text{ sec} = 11{,}915$$

which coincides with the theoretical formula (7.4) after its transposition to logarithmic form, if we assume, additionally, that

$$\frac{1}{2} \log \frac{E}{\rho_1} = 11.915, \text{ whence at } \rho_1 = 0.00125 \text{ g/cm}^3$$

$$E = 6.76 \times 10^{23} \rho_1 = 8.45 \times 10^{20} \text{ erg} \qquad (7.7)$$

Experimental data confirm well the law for shock wave propagation (7.4) established earlier with the help of general considerations of the theory of dimensions, and permit determination of the value of the constant E according to the equality (7.7).

The formulas for velocity of a shock wave can be written in the form

$$c = \frac{2}{\nu + 2} \frac{r}{t}$$

By placing this expression for c in the conditions at the shock wave (7.2) and going over, according to the formulas (1.1) to the dimensional variables V, R, P and $z = \frac{\gamma P}{R}$, we shall find the values of V_2, R_2 and z_2 behind the shock wave:

$$V_2 = \frac{4}{(\gamma+1)(\nu+2)}, \quad R_2 = \frac{\gamma+1}{\gamma-1}, \quad z_2 = \frac{8\gamma(\gamma-1)}{(\gamma+1)^2(\nu+2)^2}. \qquad (7.8)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

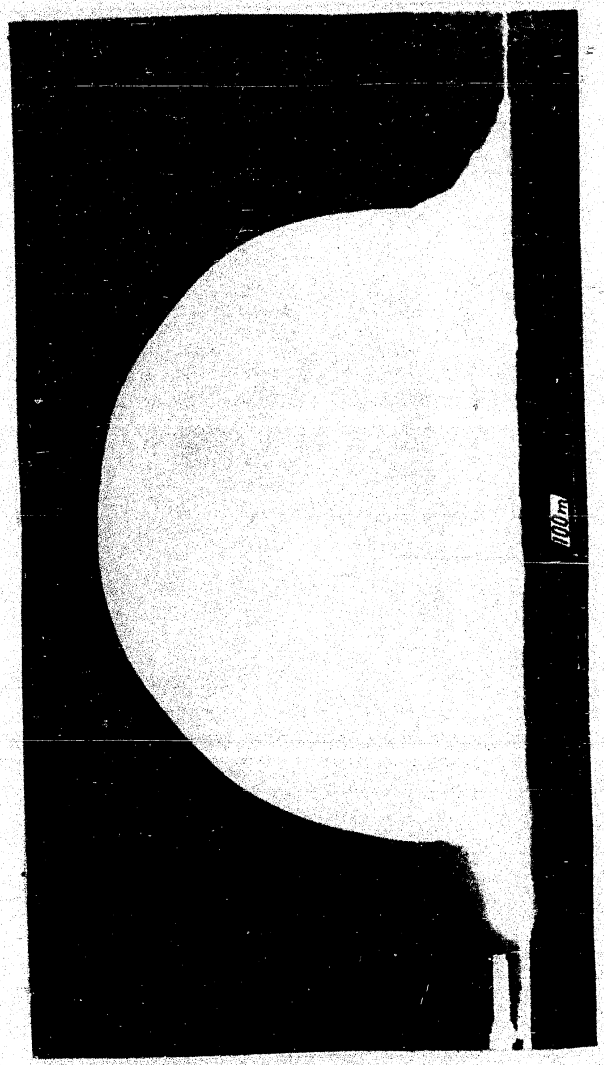


Fig.49 - Fire Ball at t = 1.5 × 10^{-3} sec

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

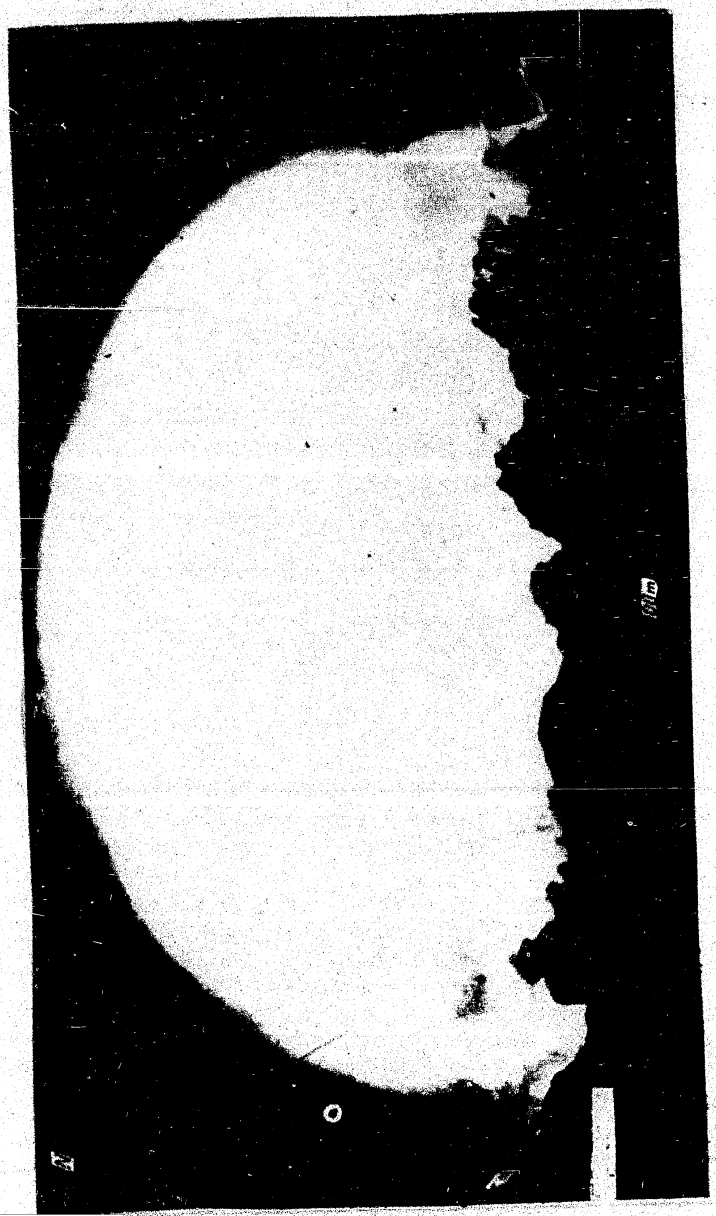


Fig.50 - Photograph of Explosion at t = 127 x 10^{-3} sec

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

We shall proceed now to seek an accurate solution to the problem with the help of equations (1.2). We note that the constants entering into the basic equations

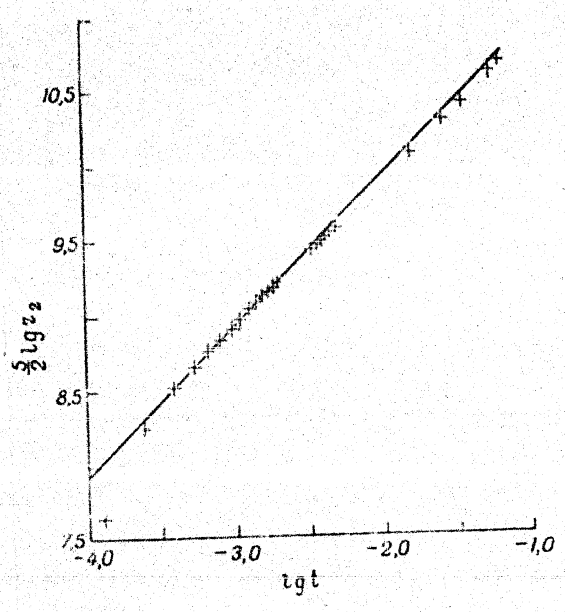


Fig.51 - Experimental Data, Indicated by × , Lie on a Straight Line Inclined at an Angle of 45°, to the Coordinate Axis , a Good Substantiation of the Theoretical Formula

$$\left(r^2 = \frac{E}{\rho_1}\right)^{1/5} t^{2/5}$$

are

$$k = -3,\ s = 0,\ m = \nu + 2,\ n = -2,$$

$$q = -\frac{n}{m} = \frac{2}{\nu + 2},\quad x = \frac{2\nu}{\gamma(\nu + 2)}$$

and the equations themselves take the form

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

$$\frac{dz}{dV} = z\left\{[2(V-1)+\nu(\gamma-1)V]\left(V-\frac{2}{\nu+2}\right)^2 - \right.$$
$$\left. -(\gamma-1)V(V-1)\left(V-\frac{2}{\nu+2}\right)-2\left[(V-1)+\frac{\nu(\gamma-1)}{\gamma(\nu+2)}\right]z\right\} \times \tag{7.9}$$
$$\times\left(V-\frac{2}{\nu+2}\right)^{-1}\left[V(V-1)\left(V-\frac{2}{\nu+2}\right)+\nu\left(\frac{2}{\gamma(\nu+2)}-V\right)z\right]^{-1},$$

$$\frac{d\ln\lambda^{\frac{1}{\nu+2}}}{dV} = \frac{\left(V-\frac{2}{\nu+2}\right)^2 - z}{V(V-1)\left(V-\frac{2}{\nu+2}\right)+\nu\left(\frac{2}{\gamma(\nu+2)}-V\right)z}, \tag{7.10}$$

$$\left(V-\frac{2}{\nu+2}\right)\frac{d\ln R}{d\lambda^{\frac{1}{\nu+2}}} =$$
$$= \frac{V(V-1)\left(V-\frac{2}{\nu+2}\right)+\nu\left(\frac{2}{\gamma(\nu+2)}-V\right)z}{z-\left(V-\frac{2}{\nu+2}\right)^2} + \nu V. \tag{7.11}$$

Solution of the problem depends on the integration of equation (7.9). The general picture of the field of the integral curves for the spherical case $\nu = 3$ in the semiplane V ($z > 0$) at $\gamma = 1.4$ is shown in Fig.52, in which can be seen the character of the specific points

$$O(0, 0), \quad B\left(\frac{2}{5}, 0\right), \quad F(1, 0), \quad D\left(\frac{2}{5}, \infty\right) \text{ и } C\left(\frac{2}{7}, \infty\right).$$

The direction of increase of the parameter $\lambda = \left(\frac{E}{\rho_1}\right)\frac{t^2}{r^5}$ along the integral curves is indicated by arrows. In the points of the parabola $z = (V - 0.4)^2$, the parameter λ has a maximum; hence, according to the condition of identity it is impossible to extend the motion continuously through the points of this parabola.

In the case of continuous motion along the integral curve, the parameter λ can tend toward zero only during approach to specific points O and F. Consequently, in the case of continuous motion only the points O and F can correspond in the plane z, V to an infinitely remote point in the gas. At $t \neq 0$ the value $\lambda = \infty$ corresponds to the center of symmetry. Along the integral curves the parameter λ tends toward ∞ only on the integral curves $z = 0$ when $V = \pm\infty$ and during approach to the specific

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

point C against which abuts a single integral curve which, with its other end abuts against point B and intersects the parabola $z = (V - 0.4)^2$. Finite values of the parameter $\lambda \neq 0$ correspond to the specific points B and D.

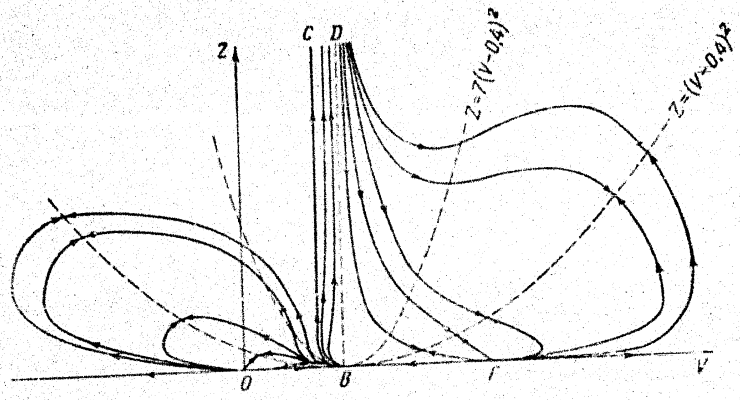


Fig.52 - Field of the Integral Curves of Equation (7.9)

It is not difficult to see that a single integral curve abutting the specific point C, which corresponds to the center of symmetry* can satisfy the solution of the problem of a strong explosion. However, this solution can not be extended continuously to $r = \infty$; hence, continuous motion of gas in the case of a strong explosion is impossible. In order that this solution can be extended and joined with the rest state through a strong shock wave, it is necessary and sufficient that, in accordance with (7.8), a point M with the coordinates

$$V_2 = \frac{4}{(\gamma+1)(\gamma+2)}, \qquad z_2 = \frac{8\gamma(\gamma-1)}{(\gamma+1)^2(\gamma+2)^2}$$

belong to the integral curve which strikes infinity at specific point C.

An important theoretical question on the existence of a solution for the

* A detailed analysis of this question has been given in our work, dating back to 1945; see Sedov, L.I., The Propagation of Strong Explosion Waves, Prikl. Mat. i Mekh., Vol.10, No.2, 1946.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

above-stated problem concerning a strong explosion is connected with a demonstration of the pertinence of the two points M and C to one and the same integral curve for the ordinary first-order differential equation (7.9). Obviously, this question can not be resolved with the help of an approximate numerical solution.

Equation (7.9) can not be integrated normally in closed form.

With the help of expressions of dimension theory in the work cited above, we showed that the stated problem of a strong explosion represents a very interesting example when solution of the problem not only leads to the integration of ordinary differential equations, but succeeds in obtaining, entirely accurately, the required solution in the form of simple finite formulas. By relying on expressions of dimension theory, we established the presence of the algebraic integral equation (7.9), which corresponds expressly to the sought-after solution. It is notable that other solutions corresponding to the close integral curves of equation (7.9) can not be represented in a similar elementary form.

To find the needed integral we shall examine the variation of the absolute energy E' in a certain volume Q bounded by a movable surface Σ:

$$\frac{dE'}{dt} = \frac{d}{dt}\int_{Q}\left(\frac{\rho v^2}{2} + \rho\varepsilon\right)d\tau,$$

where ε is the internal energy.

If Q* is a fluid volume coincident with Q at the moment of time under examination, then the following relationship is valid

$$\frac{dE'}{dt} = \frac{d}{dt}\int_{Q^*}\left(\frac{\rho v^2}{2} + \rho\varepsilon\right)d\tau + \int_{\Sigma}(v_{n_0} - v_n)\left(\frac{\rho v^2}{2} + \rho\varepsilon\right)d\sigma,$$

where v_{n_0} is the normal velocity of points in the surface Σ and v_n is the component, along a normal to the surface, of the velocity of particles of the fluid on Σ. But

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

$$\frac{d}{dt}\int_{\Omega'}\left(\frac{\rho v^2}{2}+\rho\varepsilon\right)d\tau = -\int_{\Sigma} pv_n\,d\sigma$$

and hence

$$\frac{dE'}{dt} = \int_{\Sigma}\left[(v_{n\bullet}-v_n)\left(\frac{\rho v^2}{2}+\rho\varepsilon\right)-pv_n\right]d\sigma. \qquad (7.12)$$

In our case, for the volume Ω we assume a mobile volume bounded by two spheres (cylinders or planes), on which λ = const. ($\lambda = \lambda'$ and $\lambda = \lambda''$). On these spheres r is a function of t and, consequently, in the isolated volume, the energy E' depends only on ρ_1, E, t, λ' and λ''. For this reason, it must be necessary to have

$$E' = kE = \text{const.}$$

where k is some dimensionless constant.

Since the quantity E' is constant, then the relationship (7.12) in the case of spherical symmetry yields:

$$\left[(v_{\lambda'}-v')\left(\frac{\rho' v'^2}{2}+\rho'\varepsilon'\right)-p'v'\right]4\pi r'^2 = \left[(v_{\lambda''}-v'')\left(\frac{\rho'' v''^2}{2}+\rho''\varepsilon''\right)-p''v''\right]4\pi r''^2. \qquad (7.13)$$

In the case of cylindrical symmetry the factor $4\pi r^2$ is replaced by $2\pi r$, and for plane waves, it is a constant. The relationship (7.13) yields the integral being sought; it is valid for equations of motion of a gas which are more general than the equations (1.2).

For an ideal gas $\varepsilon = \frac{p}{(\gamma-1)\rho}$. By placing this expression in formula (7.13), replacing v, ρ and p in it with V, R and P according to formulas (1.1) and $v_{\lambda'}$ and $v_{\lambda''}$ according to the formula

$$v_\lambda = \frac{2}{2+\nu}\frac{r}{t}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

we obtain the equality

$$V^2\left(V-\frac{2}{2+\gamma}\right)R+\frac{2}{\gamma-1}P\left(V-\frac{2}{2+\gamma}\right)+2PV=A.$$

The value of the constant A is determined, if we substitute in this equation $=$ $= 1$ and $V = V_2$, $R = R_2$ and $P = P_2$ from (7.8). Having made this substitution we find that $A = 0$. Thus, we obtain the sought integral of equation (7.9), which satisfies the boundary conditions at the shock wave:

$$z=\frac{(\gamma-1)V^2\left(V-\frac{2}{2+\gamma}\right)}{2\left(\frac{2}{(2+\gamma)\gamma}-V\right)}. \qquad (7.14)$$

From formula (7.14) it is thus easily seen that this solution defines precisely a single integral curve striking the specific point C; consequently, for this solution the conditions are satisfied in the center of symmetry (in the center of the explosion).

In this manner, the existence and uniqueness of the solution to the problem of strong explosion have been rigorously proven.

We have obtained a solution (7.14) simultaneously in the cases of spherical, cylindrical and plane symmetry.

By using the solution of equation (7.9) according to formula (7.14), from equations (7.10) and (7.11) we easily find and R as functions of V. In the case of spherical symmetry

$$\lambda=\frac{(\gamma+1)^2}{0,64}V^2\left[\frac{5(\gamma+1)}{7-\gamma}\left(1-\frac{3\gamma-1}{2}V\right)\right]^{\alpha_1}\times \times\left[\frac{\gamma+1}{\gamma-1}\left(\frac{5\gamma}{2}V-1\right)\right]^{\alpha_2}, \qquad (7.15)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

$$R = \frac{\gamma+1}{\gamma-1}\left[\left(\frac{5\gamma}{2}V-1\right)\frac{\gamma+1}{\gamma-1}\right]^{\alpha_3}\times \\ \times\left[\frac{5(\gamma+1)}{7-\gamma}\left(1-\frac{3\gamma-1}{2}V\right)\right]^{\alpha_4}\left[\frac{\gamma+1}{\gamma-1}\left(1-\frac{5}{2}V\right)\right]^{\alpha_5},$$

where

$$\alpha_1 = \frac{13\gamma^2-7\gamma+12}{(3\gamma-1)(2\gamma+1)}, \quad \alpha_2 = \frac{5(1-\gamma)}{2\gamma+1}, \quad \alpha_3 = \frac{3}{2\gamma+1},$$
$$\alpha_4 = \frac{13\gamma^2-7\gamma+12}{(2-\gamma)(3\gamma-1)(2\gamma+1)} = \frac{\alpha_1}{2-\gamma}, \quad \alpha_5 = \frac{2}{\gamma-2}.$$

In the case of cylindrical symmetry

$$\left.\begin{aligned} \lambda &= (1+\gamma)^4\left(\frac{1-\gamma}{1+\gamma}\right)^{\frac{2(\gamma-1)}{\gamma}}\times \\ &\quad\times V^2(1-\gamma V)^2(1-2\gamma V)^{-\frac{2(\gamma-1)}{\gamma}}, \\ R &= \frac{\gamma+1}{\gamma-1}\left[\frac{\gamma+1}{\gamma-1}(2\gamma V-1)\right]^{1/\gamma}\times \\ &\quad\times[(\gamma-1)(1-\gamma V)]^{\frac{2}{2-\gamma}}(1-2V)^{-\frac{2}{2-\gamma}} \end{aligned}\right\} \quad (7.16)$$

For plane waves

$$\left.\begin{aligned} \lambda &= \frac{9(1+\gamma)^2}{16}V^2\left[\frac{\gamma+1}{\gamma-1}\left(\frac{3\gamma}{2}V-1\right)\right]^{\frac{3(1-\gamma)}{2\gamma-1}}\times \\ &\quad\times\left[3\left(1-\frac{\gamma+1}{2}V\right)\right]^{\frac{5\gamma-4}{2\gamma-1}}, \\ \dot{R} &= \frac{\gamma+1}{\gamma-1}\left[\frac{\gamma+1}{\gamma-1}\left(\frac{3\gamma}{2}V-1\right)\right]^{\frac{1}{2\gamma-1}}\times \\ &\quad\times\left[3\left(1-\frac{\gamma+1}{2}V\right)\right]^{\frac{5\gamma-4}{(2\gamma-1)(2-\gamma)}}\left[\frac{\gamma+1}{\gamma-1}\left(1-\frac{3}{2}V\right)\right]^{\frac{2}{\gamma-2}} \end{aligned}\right\} \quad (7.17)$$

Formulas (7.14), (7.15), (7.16) and (7.17) give an accurate solution for the stated problem.

At $\gamma = 1.4$, Fig.53 shows the distribution of velocity behind the shock wave;

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

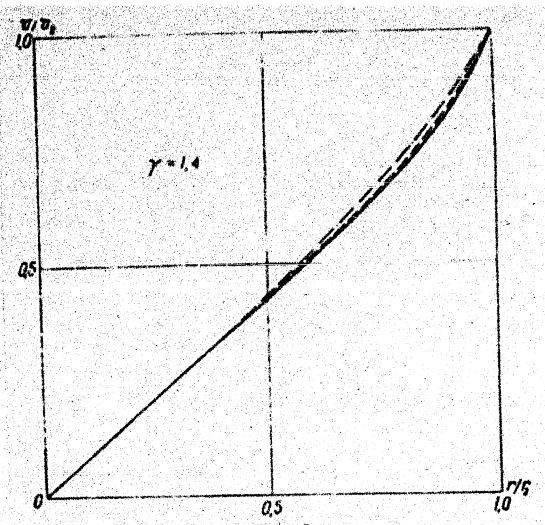


Fig.53 - Velocity Distribution behind the Shock Wave:

———— Spherical case

– – – – – – – – Cylindrical case

–·–·–·–·–·–·– Plane case

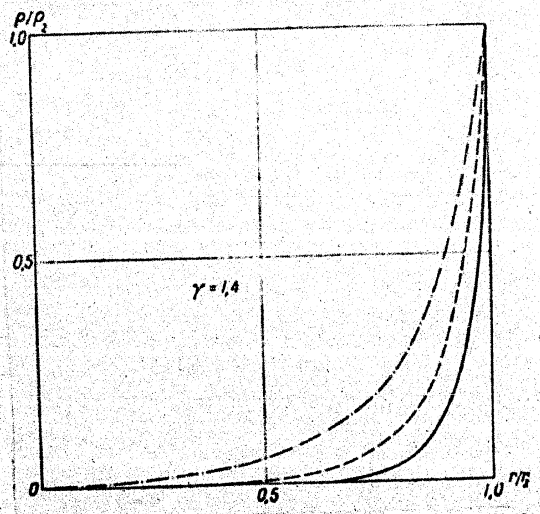


Fig.54 - Density Distribution behind the Shock Wave

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

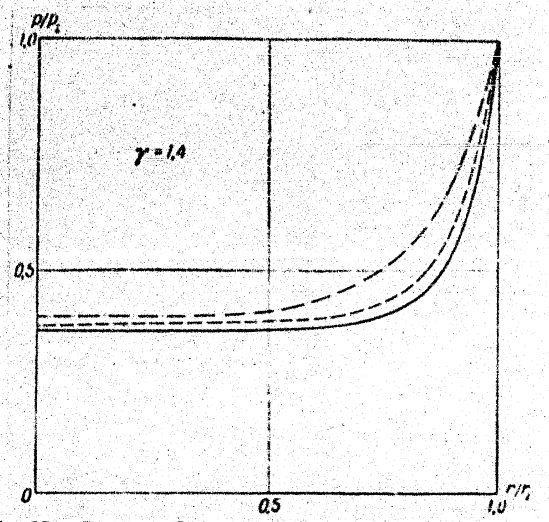


Fig.55 - Pressure Distribution behind the Shock Wave:

———— Spherical case

- - - - - - - - Cylindrical case

-.-.-.-.-.-.-.- Plane case

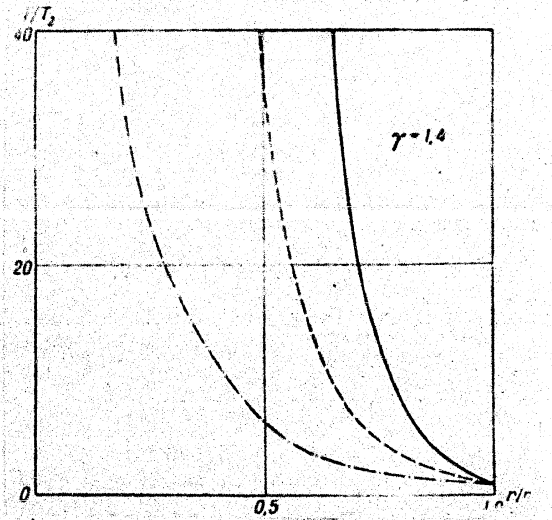


Fig.56 - Temperature distribution behind the Shock Wave

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

Fig.54 shows the density distribution, Fig.55 shows pressure distribution, and Fig.56 shows temperature distribution.

The unbroken line gives the spherical case, the broken line gives the cylindrical case and the dot-and-dash line gives the plane case. Velocity, density and pressure are related to the corresponding quantities behind the shock wave. The variation of v_2, ρ_2 and p_2 behind the shock wave with the passage of time is given by the formulas (7.2) where, in place of the velocity of the shock wave its expression in terms of t must be substituted. Making the substitution and considering the connection between r_2 and t, we obtain:

$$\left.\begin{aligned} v_2 &= \frac{4}{(\nu+2)(\gamma+1)}\cdot\left(\frac{E}{\rho_1}\right)^{\frac{1}{2+\nu}} t^{-\frac{\nu}{2+\nu}} = \frac{4}{(\nu+2)(\gamma+1)}\cdot\left(\frac{E}{\rho_1}\right)^{1/2} r_2^{-\frac{\nu}{2}}\cdot \\ p_2 &= \frac{8}{(2+\nu)^2(\gamma+1)}\rho_1\cdot\left(\frac{E}{\rho_1}\right)^{\frac{2}{2+\nu}} t^{-\frac{2\nu}{2+\nu}} = \frac{8E}{(\nu+2)^2(\gamma+1)} r_2^{-\nu}, \\ \rho_2 &= \frac{\gamma+1}{\gamma-1}\rho_1. \end{aligned}\right\} \qquad (7.18)$$

For the temperature behind the shock wave, we have:

$$T_2 = \frac{p_2}{R\rho_2}$$

From the formulas which give the solution, it is a simple matter to derive asymptotic formulas for v, ρ, and p and temperature T close to the center of the explosion. In the case of r → 0 we have:

$$\left.\begin{aligned} v &= \frac{2}{(2+\nu)\gamma}\frac{r}{t}, \\ \rho &= k_1\rho_1\left(\frac{E}{\rho_1}\right)^{-\frac{\nu}{(\nu+2)(\gamma-1)}} t^{-\frac{2\nu}{(\nu+2)(\gamma-1)}} r^{\frac{\nu}{\gamma-1}}, \\ p &= k_2\rho_1\left(\frac{E}{\rho_1}\right)^{\frac{2}{\nu+2}} t^{-\frac{2\nu}{\nu+2}}, \\ T &= k_3\frac{1}{c_v}\left(\frac{E}{\rho_1}\right)^{\frac{2(\gamma-1)+\nu}{(\gamma-1)(\nu+2)}} t^{\frac{2\nu(2-\gamma)}{(\nu+2)(\gamma-1)}} r^{-\frac{\nu}{\gamma-1}}, \end{aligned}\right\} \qquad (7.19)$$

STAT

52

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

$$\left.\begin{aligned} k_1 &= -\frac{(\gamma+1)^{\frac{3\gamma(\gamma+1)}{(\gamma-1)(3\gamma-1)}}}{2^{\frac{6}{5(\gamma-1)}}\gamma^{\frac{7\gamma+1}{(\gamma-1)(3\gamma-1)}}(\gamma-1)}\left(\frac{2\gamma+1}{7-\gamma}\right)^{\frac{13\gamma^2-7\gamma+12}{5(\gamma-1)(2-\gamma)(3\gamma-1)}}, \\ k_2 &= \frac{0{,}32}{2^{6/5}}\frac{(\gamma+1)^{\frac{\gamma+1}{3\gamma-1}}}{\gamma^{\frac{4\gamma}{3\gamma-1}}}\left(\frac{2\gamma+1}{7-\gamma}\right)^{\frac{13\gamma^2-7\gamma+12}{5(2-\gamma)(3\gamma-1)}}, \\ k_3 &= \frac{k_2}{k_1(\gamma-1)} \end{aligned}\right\} \qquad (7.20)$$

for spherical symmetry:

$$k_1 = \frac{(\gamma+1)^{\frac{\gamma+1}{\gamma-1}}}{\gamma-1}\frac{\gamma^{\frac{3\gamma-4}{(\gamma-1)(2-\gamma)}}}{2^{\frac{2}{(\gamma-1)(2-\gamma)}}}, \quad k_2 = \frac{\gamma^{\frac{2(\gamma-1)}{2-\gamma}}}{2^{\frac{4-\gamma}{2-\gamma}}}, \quad k_3 = \frac{k_2}{k_1(\gamma-1)}$$

for cylindrical symmetry:

$$k_1 = \frac{(2\gamma-1)^{\frac{3\gamma-4}{(\gamma-1)(2-\gamma)}}}{2^{\frac{2}{3(\gamma-1)}}\gamma^{\frac{1}{\gamma-1}}}\frac{(\gamma+1)^{\frac{4+\gamma-3\gamma^2}{3(\gamma-1)(2-\gamma)}}}{(\gamma-1)}, \quad k_2 = \frac{2^{7/3}}{9}\frac{(2\gamma-1)^{\frac{3\gamma-4}{3(2-\gamma)}}}{(\gamma+1)^{2/3}\frac{\gamma+1}{2-\gamma}},$$

$$k_3 = \frac{k_2}{k_1(\gamma-1)}$$

for plane waves.

The relationship of the constants k_1, k_2 and k_3 to γ is shown in Fig.57.

Near the center of symmetry the velocity is close to zero and the pressure is constant. Under conditions of approach to the center of the explosion, the density very rapidly tends toward zero, and the temperature towards infinity. It is easily seen that entropy also tends towards infinity. Close to the center of the explosion large temperature gradients arise; owing to this fact the property of heat conductivity becomes very important. If heat conductivity is considered then the temperature obtained in the center of the explosion is terminal.

The mass of gas scatters from the center of the explosion (at r = 0 we have

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

$\rho \approx 0$), the pressure in the center is finite but tends toward zero with an increase in time.

It is clear from the foregoing that in the case of a strong explosion in a gas

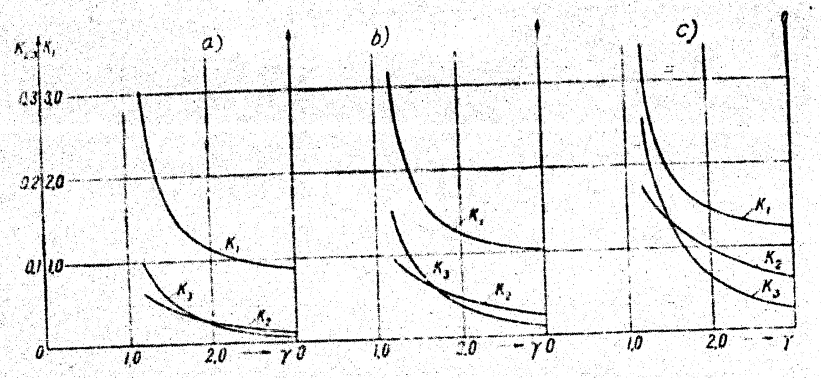


Fig.57 - The Quantities $k_1(\gamma)$, $k_2(\gamma)$, $k_3(\gamma)$ in the Formulas (7.19) which Give Asymptotic Values for Density, Pressure and Temperature Close to the Center of the Explosion

a) Spherical symmetry; b) Cylindrical symmetry; c) Plane waves

in which, prior to the explosion there is finite pressure, there must occur a motion of the gas back towards the center of the explosion. We observe well this effect in the case of underwater explosions where repeated pulsations of the gas bubble occur.

As a result of the scattering of the gas from the center and the drop in pressure, an Archimedean lift force sets in which causes a floating of the region of turbulent motion of the gas in the surrounding atmosphere. According to data in the photographs of an atomic bomb explosion in New Mexico the vertical rate of rise of the luminous nucleus was of the order of 35 m/sec.

The effect of the constant γ on the distribution of characteristics of the motion behind the shock wave front for spherical symmetry is presented in Fig.58, 59 and 60.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

$p \approx 0$), the pressure in the center is finite but tends toward zero with an increase in time.

It is clear from the foregoing that in the case of a strong explosion in a gas

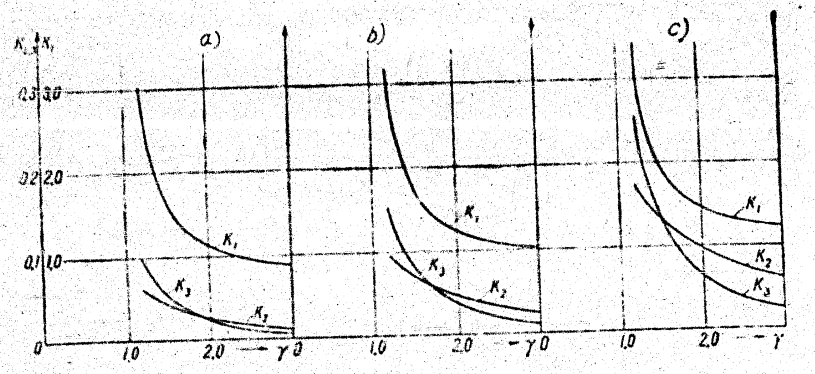


Fig.57 - The Quantities $k_1(\gamma)$, $k_2(\gamma)$, $k_3(\gamma)$ in the Formulas (7.19) which Give Asymptotic Values for Density, Pressure and Temperature Close to the Center of the Explosion

a) Spherical symmetry; b) Cylindrical symmetry; c) Plane waves

in which, prior to the explosion there is finite pressure, there must occur a motion of the gas back towards the center of the explosion. We observe well this effect in the case of underwater explosions where repeated pulsations of the gas bubble occur.

As a result of the scattering of the gas from the center and the drop in pressure, an Archimedean lift force sets in which causes a floating of the region of turbulent motion of the gas in the surrounding atmosphere. According to data in the photographs of an atomic bomb explosion in New Mexico the vertical rate of rise of the luminous nucleus was of the order of 35 m/sec.

The effect of the constant γ on the distribution of characteristics of the motion behind the shock wave front for spherical symmetry is presented in Fig.58, 59 and 60.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

The constant E, which must be expressed in terms of the charge energy E_0 (equal, in the accepted statement of the problem, to the total energy of the turbulent gas),

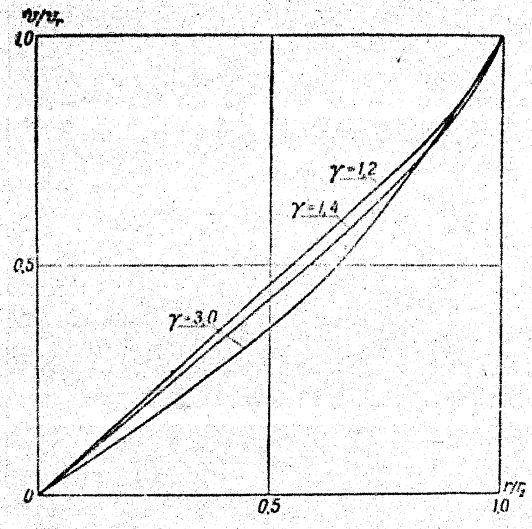


Fig.58 - Effect of the Constant γ on the Distribution of Velocity behind a Shock Wave Front for Spherical Symmetry

enters into the above-derived formulas from the dimensional characteristics of motion. The non-dimensional characteristics of motion are represented by universal curves independent of the explosion energy E_0 and of the quantity E which is proportional to it.

For the total energy we have the formulas

$$E_0 = \int_0^{r_2} \frac{\rho v^2}{2} 4\pi r^2 dr + \int_0^{r_2} \frac{p}{\gamma - 1} 4\pi r^2 dr \quad \text{in the spherical case,}$$

$$E_0 = \int_0^{r_2} \frac{\rho v^2}{2} 2\pi r dr + \int_0^{r_2} \frac{p}{\gamma - 1} 2\pi r dr \quad \text{in the cylindrical case,}$$

$$\frac{1}{2} E_0 = \int_0^{r_2} \frac{\rho v^2}{2} dr + \int_0^{r_2} \frac{p}{\gamma - 1} dr \quad \text{for plane waves.}$$

The first term is the kinetic energy of the gas; the second term is the heat energy of the gas. By introducing non-dimensional quantities we find

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

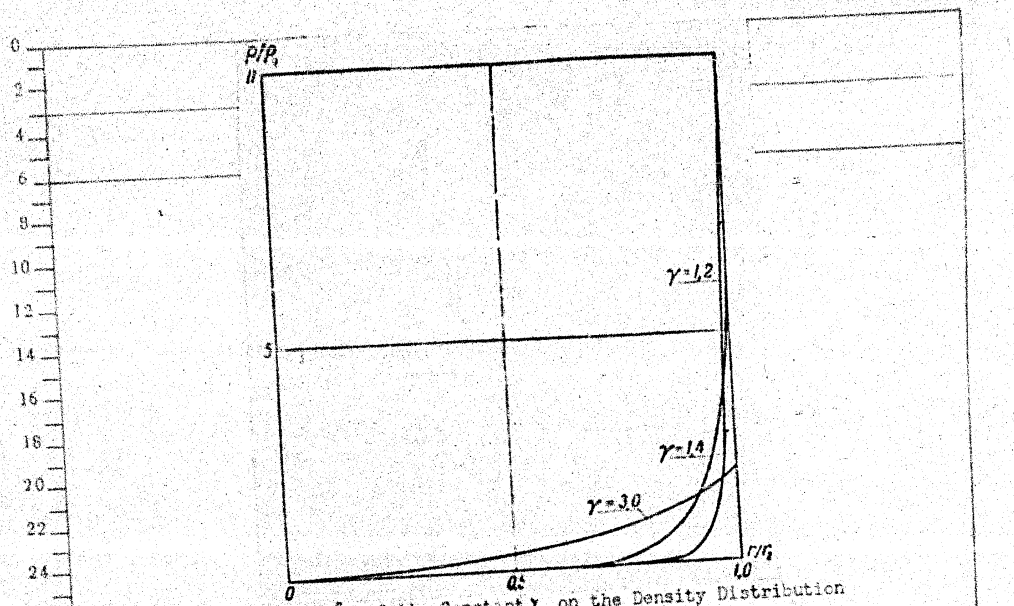


Fig.59 - Effect of the Constant γ on the Density Distribution behind a Shock Wave Front for Spherical Symmetry

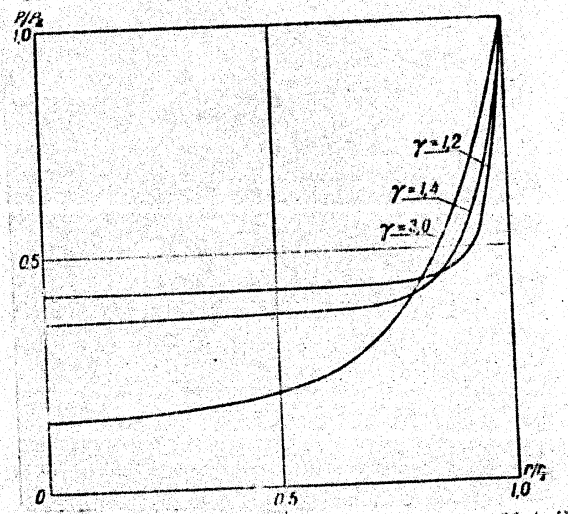


Fig.60 - Effect of the Constant γ on the Pressure Distribution behind a Shock Wave Front for Spherical Symmetry

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

$$E_0 = \alpha(\gamma)\, E \qquad (7.21)$$

where

$$\alpha = \frac{2\pi}{5}\int_1^\infty RV^2\frac{d\lambda}{\lambda^2} + \frac{4\pi}{5(\gamma-1)}\int_1^\infty P\frac{d\lambda}{\lambda^2}$$ in the spherical case,

$$\alpha = \frac{\pi}{4}\int_1^\infty RV^2\frac{d\lambda}{\lambda^2} + \frac{\pi}{2(\gamma-1)}\int_1^\infty P\frac{d\lambda}{\lambda^2}$$ in the cylindrical case,

$$\alpha = \frac{1}{3}\int_1^\infty RV^2\frac{d\lambda}{\lambda^2} + \frac{2}{3(\gamma-1)}\int_1^\infty P\frac{d\lambda}{\lambda^2}$$ for plane waves.

Figure 61 shows the relationship $\alpha(\gamma)$, calculated for spherical, cylindrical and plane symmetry.

In particular, in the cases of spherical symmetry at $\gamma = 1.4$ we have

$$E_0 = (0.186 + 0.665)\, E = 0.851\, E \text{ or } E = 1.175\, E_0$$

If we use the value of E according to formula (7.7) in accordance with the experimental data for the atomic bomb explosion in New Mexico, and assume $\gamma = 1.4$, then for the energy of the explosion* we obtain: $E_0 = 7.19 \times 10^{20}$ erg, which corresponds to the energy released in the explosion of 16,800 tons of trinitrotoluene.

We noted above that the properties of viscosity and heat conductivity can exert some effect on gas motion close to the center of an explosion. If we do not disregard viscosity and heat conductivity, then the coefficient of viscosity μ and the coefficient of heat conductivity χ enter into the equations of gas motion.

It should be noted that the coefficient of heat conductivity enters as a factor in the presence of temperature T.

---

* In evaluating the total energy released in an atomic bomb explosion it is necessary to keep in mind the fact that a considerable part of the energy is expended on radiation.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

POOR ORIGINAL

After eliminating the temperature T with the help of an equation of state (for simplicity we shall assume this to be Klapeyron's equation $T = \frac{P}{R\rho}$), there will enter

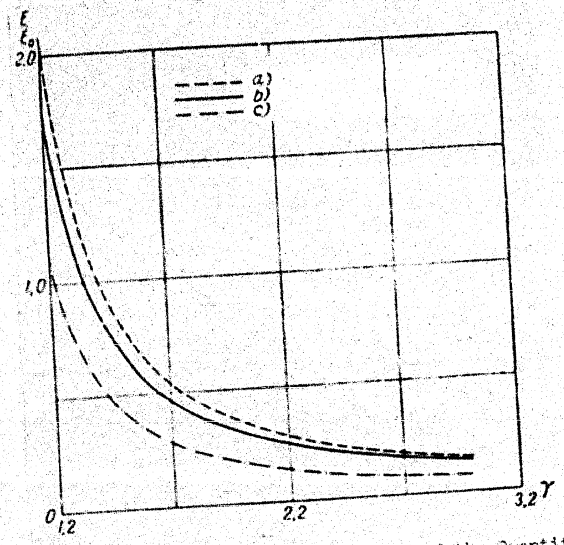


Fig.61 - The ratio $\frac{E}{E_0} = a$ as a Function of the Quantity γ

a) Cylindrical symmetry; b) Spherical symmetry; c) Plane symmetry

as a coefficient, not $\varkappa$ actually, but the ratio $\frac{\varkappa}{R}$ (R being the gas constant). As is well known, the coefficients μ and $\varkappa$ depend on the temperature. Ordinarily they are taken as proportional to the temperature T in some power (most often they are taken as proportional to $\sqrt{T}$ or they are constants). We shall assume that

$$\mu = \mu_1 T^\alpha \text{ and } \varkappa = \varkappa_1 T^\alpha$$

where μ_1 and $\varkappa_1$ are constants.

After elimination of T, the following new dimensional constants enter into the equations

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\frac{\mu_1}{R^\alpha} \text{ and } \frac{\varkappa_1}{R^{\alpha+1}}$$

Their dimensions are equal to

$$\left[\frac{\mu_1}{R^\alpha}\right] = \left[\frac{\varkappa_1}{R^{\alpha+1}}\right] = ML^{-1-2\alpha}T^{-1+2\alpha}.$$

In order that the motion be self modeling, it is sufficient that their dimensions be expressed in terms of the dimensions ρ_1 and E_0. It is easily seen that for this purpose it is necessary to assume in the spherical case*, $\alpha = \frac{1}{6}$ in the cylindrical case $\alpha = 0$ and in the plane case $\alpha = -\frac{1}{2}$.

In this manner, the problem of a strong explosion can be solved with the help of the integration of ordinary differential equations taking into consideration viscosity and heat conductivity, if we assume

$$\mu = \mu_1 T^{1/6}, \quad \varkappa = \varkappa_1 T^{1/6} \quad \text{for spherical waves} \tag{7.22}$$

$$\mu = \text{const.}, \quad \varkappa = \text{const.} \quad \text{for cylindrical waves} \tag{7.23}$$

$$\mu = \frac{\mu_1}{\sqrt{T}}, \quad \varkappa = \frac{\varkappa_1}{\sqrt{T}} \quad \text{for plane waves} \tag{7.24}$$

It is necessary, even so, to note the possibility of solving the problem of a strong explosion within the framework of the theory of an ideal liquid, using a more general form of the equation of state and with the relationship of the internal energy of the gas as a function of p and ρ**. The function internal energy $\varepsilon(p, \rho)$ enters immediately into the conditions at the shock wave and into the equation of

---

* This result was obtained in the work: Bam-Zelikovich, G.M., Distribution of Strong Explosion Waves, Symposium No.4, "Theoretical Hydromechanics", edited by L.I.Sedov, Oborongiz, 1949. When $\alpha = \frac{1}{6}$ we have $\left[\frac{\mu_1}{R^4}\right] = [(E\rho_1^2)^{1/3}]$

** Cf. G.M.Bam-Zelikovich, op.cit.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

heat influx. In the general case it is always possible to present it in the form

where $\bar{\varepsilon}$ is a dimensionless function of its own arguments and p^* is some constant with a pressure dimension.

Since the dimension p^* can not be expressed by means of the dimensions ρ_1 and E, it is then sufficient for self-modeling of the motion that ε not contain p^*; i.e., that the following would hold:

$$\varepsilon = \frac{p}{\rho_1}\,\varphi\left(\frac{\rho}{\rho_1}\right)$$

where φ is an arbitrary function of its argument.

This condition for the function ε imposes a corresponding limitation on the equation of state. Actually, since

$$\frac{d\varepsilon + pd\,\frac{1}{\rho}}{T} = dS$$

is a complete differential, then T and ε must satisfy the partial equation

$$T + \frac{\partial T}{\partial \rho}\left(\rho^2 \frac{\partial \varepsilon}{\partial \rho} - p\right) - \rho^2 \frac{\partial T}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \rho} = 0.$$

By substituting here $\varepsilon = p\varphi(\rho)$, we obtain:

$$T + \frac{\partial T}{\partial p}\,p\,[\varphi'(\rho)\rho^2 - 1] - \rho^2\varphi(\rho)\frac{\partial T}{\partial p} = 0.$$

It is possible to write the equivalent system of ordinary differential equations

$$\frac{dT}{T} = -\frac{dp}{p\,[\varphi'(\rho)\rho^2 - 1]} = \frac{d\rho}{\varphi(\rho)\rho^2}.$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

It has two primary integrals:

$$T e^{-\int \frac{d\rho}{\rho^2 \varphi(\rho)}} = C_1 \quad \text{AND} \quad p\varphi(\rho)\, e^{-\int \frac{d\rho}{\rho^2 \varphi(\rho)}} = C_2;$$

hence,

$$T = e^{\int \frac{d\rho}{\rho^2 \varphi(\rho)}} \Phi\left[p\varphi(\rho)\, e^{-\int \frac{d\rho}{\rho^2 \varphi(\rho)}} \right], \tag{7.25}$$

where φ and Φ are arbitrary functions of their arguments. It is obvious that the relationship (7.25) is satisfied, in particular, by all the equations of state in which the temperature is proportional to the pressure and depends on density by an arbitrary form. However, notwithstanding the presence of two arbitrary functions, many interesting equations of state (e.g., Van der Waals' equation) can not be written in the form of (7.25); i.e., with these equations of state, the problem of a strong explosion ceases to be self modeling.

Section 8. The Problem of a Strong Explosion in Lagrange Variables and Various Supplements

In the preceding paragraph there was given a complete solution of the problem of a strong explosion in Euler variables. This solution can also be presented easily in Lagrange variables. As a Lagrangian coordinate it is possible to take the initial coordinate of a particle r_0 or the entropy of a particle in the turbulent motion of a gas, equal to $c_v \ln \frac{p}{\rho^\gamma}$.

Let us examine first the connection between the quantities r_0 and $\frac{p}{\rho^\gamma}$ in the case of non-self-modeling motion during explosion in a gas at rest.

The precise conditions at the shock wave (7.1) can be written in the form

$$\left.\begin{aligned} v_2 &= \frac{2a_1}{(\gamma+1)\sqrt{q}}(1-q), \\ \rho_2 &= \frac{\gamma+1}{\gamma-1+2q}\rho_1, \\ p_2 &= \frac{p_1}{\gamma+1}\left[\frac{2\gamma-(\gamma-1)q}{q}\right]. \end{aligned}\right\} \tag{8.1}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

where $a_1 = \sqrt{\frac{\gamma p_1}{\rho_1}}$ is the speed of sound in a non-turbulent gas,

$$q = \frac{a_1^2}{c^2} \tag{8.2}$$

c is the velocity of the shock wave.

It is obvious that for an explosion which occurs in a point, on a straight line or on a plane, the dimensionless variable q can be regarded as a function of the parameter γ and the variable $\tau = \frac{p_1^{\frac{\nu+2}{2\nu}} \cdot t}{E_0^{1/\nu} \rho_1^{1/2}}$ or γ and the variable $s = \ln \frac{r_2 p_1^{1/\nu}}{E_0^{1/\nu}}$

where r_2 is the radius of the shock wave. The dimension $\frac{p_1}{E_0}$ equals:

$$\left[\frac{p_1}{E_0}\right] = \frac{1}{L^\nu}$$

hence, the argument to the logarithm represents an abstract combination.

For a self-modeling motion

$$q = 0, \quad \tau = 0 \text{ and } s = -\infty \tag{8.3}$$

whereby in cases of spherical, cylindrical and plane symmetry, at q = 0 we have:

$$\left.\begin{aligned} \frac{\eta}{\tau^{\frac{2\nu}{\nu+2}}} &= \frac{\gamma^{\frac{2}{\nu+2}} p_1^{\frac{2}{\nu+2}} E_0^{\frac{2}{\nu+2}}}{\rho_1 c^{\frac{2\nu}{\nu+2}}} = \left(\frac{2+\nu}{2}\right)^2 \cdot \gamma \alpha^{\frac{2}{\nu+2}}, \\ \frac{q}{e^{2s}} &= \frac{\gamma E_0}{\rho_1 c^2 r_2^\nu} = \left(\frac{2+\nu}{2}\right)^2 \cdot \gamma \alpha. \end{aligned}\right\} \tag{8.4}$$

Suitable values of the constant α are determined on the basis of the equalities (7.4) and (7.21). For non-self-modeling motions together with $\lambda = \frac{Et^2}{\rho r^{\nu+2}}$ it is possible to take as a second variable one of the dimensionless variables q, τ, s. On the basis of the conditions for jump (8.1) it is possible to write:

$$\frac{p_2}{p_2'^\gamma} = \frac{p_1}{\rho_1^\gamma} \cdot \frac{1}{(\gamma+1)^{\gamma+1}} \cdot \left[\frac{2\gamma-(\gamma-1)q}{q}\right] \cdot [\gamma-1+2q]^\gamma. \tag{8.5}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

From formula (8.5) it is obvious that distribution of entropy through the gas particles will be known - as long as the law of motion of the shock wave has been defined - that is equipollent to the mission of the functional relationship q(s), since at the moment the shock wave passes through any particle the equality $r_0 = r_2$ is valid. For a self-modeling motion we have $q = 0$, and on the basis of (7.4), (7.5), (7.6), from (8.5) and also from (8.1), at $p_1 = 0$ it follows that

$$\frac{p_1}{q} = \left(\frac{2}{2+\nu}\right)^2 \frac{E}{\gamma} \frac{1}{r_2^\nu}, \tag{8.6}$$

$$\frac{p}{\rho^\gamma} = \frac{8(\gamma-1)^\gamma}{(\nu+2)^2(\gamma+1)^{\gamma+1}} \frac{E}{\rho_1^\gamma} \frac{1}{r_0^\nu}. \tag{8.7}$$

Formula (8.7) defines for self-modeling motion the connection between entropy and the initial coordinate in the case of spherical, cylindrical and plane waves. On the basis of formulas (7.15), (7.16), (7.17) and (8.7) the complete solution for the problem of strong explosion under constant initial density with the help of the parameter V, varying in the range $\frac{2}{(2+\nu)\gamma} < V < \frac{4}{(2+\nu)(\gamma+1)}$, can be written in the following form:

$$\left.\begin{aligned}
\frac{r}{r_2} &= \left[\frac{(2+\nu)(\gamma+1)}{4} V\right]^{-\frac{2}{2+\nu}} \left[\frac{\gamma+1}{\gamma-1}\left(\frac{(2+\nu)\gamma}{2} V - 1\right)\right]^{-\alpha_2} \times \\
&\times \left[\frac{(2+\nu)(\gamma+1)}{(2+\nu)(\gamma+1) - 2[2+\nu(\gamma-1)]}\left(1 - \frac{2+\nu(\gamma-1)}{2} V\right)\right]^{-\alpha_1}, \\
\frac{r_0}{r_2} &= \left[\frac{(2+\nu)(\gamma+1)}{4} V\right]^{-\frac{2}{2+\nu}} \left[\frac{\gamma+1}{\gamma-1}\left(\frac{(2+\nu)\gamma}{2} V - 1\right)\right]^{\alpha_0} \times \\
&\times \left[\frac{(2+\nu)(\gamma+1)}{(2+\nu)(\gamma+1) - 2[2+\nu(\gamma-1)]}\left(1 - \frac{2+\nu(\gamma-1)}{2} V\right)\right]^{\alpha_2} \times \\
&\times \left[\frac{\gamma+1}{\gamma-1}\left(1 - \frac{2+\nu}{2} V\right)\right]^{-\left(\alpha_0+\alpha_2-\frac{2}{2+\nu}\right)},
\end{aligned}\right| \tag{8.8}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\left.\begin{aligned}
&\frac{v}{v_2} = f = \frac{(\nu+2)(\gamma+1)}{4} V \frac{r}{r_2},\\
&\frac{\rho}{\rho_2} = g = \left[\frac{\gamma+1}{\gamma-1}\left(\frac{(\nu+2)\gamma}{2}V-1\right)\right]^{\alpha_3}\left[\frac{\gamma+1}{\gamma-1}\left(1-\frac{\nu+2}{2}V\right)\right]^{\alpha_5}\times\\
&\times\left[\frac{(\nu+2)(\gamma+1)}{(2+\nu)(\gamma+1)-2[2+\nu(\gamma-1)]}\left(1-\frac{2+\nu(\gamma-1)}{2}V\right)\right]^{\alpha_6},\\
&\frac{p}{p_2} = h = \left[\frac{(2+\nu)(\gamma+1)}{4}V\right]^{\frac{2\nu}{2+\nu}}\left[\frac{\gamma+1}{\gamma-1}\left(1-\frac{\nu+2}{2}V\right)\right]^{\alpha_5+1}\times\\
&\times\left[\frac{(2+\nu)(\gamma+1)}{(2+\nu)(\gamma+1)-2[2+\nu(\gamma-1)]}\left(1-\frac{2+\nu(\gamma-1)}{2}V\right)\right]^{\alpha_6-2\alpha_7},\\
&\frac{T}{T_2} = \frac{p}{p_2}\cdot\frac{\rho_2}{\rho},
\end{aligned}\right\}$$

where

$$\left.\begin{aligned}
&\alpha_1 = \frac{(2+\nu)\gamma}{2\gamma\nu(\gamma-1)}\left[\frac{2\nu(2-\gamma)}{\gamma(2+\nu)^2}-\alpha_2\right], \quad \alpha_2 = \frac{1-\gamma}{2(\gamma-1)+\nu},\\
&\alpha_3 = \frac{\nu}{\nu-2(1-\gamma)}, \qquad \alpha_4 = \frac{\alpha_1(2+\nu)}{2-\gamma},\\
&\alpha_5 = \frac{2}{\gamma-2}, \qquad \alpha_6 = \frac{\gamma}{\nu-2(1-\gamma)},\\
&\alpha_7 = \frac{[2+\nu(\gamma-1)]\alpha_5}{\nu(2-\gamma)}.
\end{aligned}\right\} \tag{8.9}$$

In agreement with (7.18), the following equalities are valid:

$$\left.\begin{aligned}
&\frac{v_2}{v_{20}} = \left(\frac{r_2}{r_0}\right)^{-\frac{\nu}{2}},\\
&\frac{p_2}{p_{20}} = \left(\frac{r_2}{r_0}\right)^{-\nu},
\end{aligned}\right\} \tag{8.10}$$

where v_{20} and p_{20} are velocity and pressure behind the shock wave front at the moment the shock wave passes through a point with the coordinate r_0.

Figures 62, 63, 64, and 65 present the graphs for changes in the characteristics of motion of different gas particles in relation to the position of the shock

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

wave.

Below are also tables of numerical values for the characteristics of motion in the case of a strong explosion in a gas with constant density.

$\nu=1$, $\gamma=1,4$

$\lambda=\frac{r}{r_2}$	$\frac{r_0}{r_2}$	$\frac{v}{v_2}=f$	$\frac{\rho}{\rho_2}=g$	$\frac{p}{p_2}=h$	$f'(\lambda)$	$g'(\lambda)$	$h'(\lambda)$
1	1	1	1	1	1,5	7,5	4,5
0,9797	0,8873	0,9699	0,8625	0,9162	1,4688	6,1575	3,8131
0,9420	0,7151	0,9156	0,6659	0,7915	1,4067	4,3615	2,8352
0,9013	0,5722	0,8599	0,5160	0,6923	1,3367	3,1109	2,0959
0,8565	0,4501	0,8017	0,3982	0,6120	1,2597	2,2183	1,5250
0,8050	0,3427	0,7390	0,3019	0,5457	1,1758	1,5638	1,0723
0,7419	0,2448	0,6678	0,2200	0,4904	1,0862	1,0738	0,7067
0,7029	0,1980	0,6263	0,1823	0,4661	1,0396	0,8726	0,5483
0,6553	0,1514	0,5780	0,1453	0,4437	0,9921	0,6919	0,4022
0,5925	0,1040	0,5172	0,1074	0,4229	0,9438	0,5232	0,2648
0,5396	0,0741	0,4682	0,0826	0,4116	0,9144	0,4213	0,1838
0,4912	0,0529	0,4244	0,0641	0,4038	0,8947	0,3473	0,1289
0,4589	0,0415	0,3957	0,0536	0,4001	0,8849	0,3020	0,1003
0,4161	0,0293	0,3580	0,0415	0,3964	0,8750	0,2551	0,0453
0,3480	0,0156	0,2988	0,0263	0,3929	0,8651	0,1905	0,0237
0,2810	0,0074	0,2410	0,0153	0,3911	0,8602	0,1370	0,0111
0,2320	0,0038	0,1989	0,0095	0,3905	0,8584	0,1025	0,0089
0,1680	0,0012	0,1441	0,0042	0,3901	0,8574	0,0630	0,0029
0,1040	0,0002	0,0891	0,0013	0,3900	0,8572	0,0307	0,0005
0,0000	0,0000	0,0000	0,0000	0,3900	0,8571	0,0000	0,0000

To evaluate the effect of an explosion on a body falling into interaction with the field of turbulent motion of the gas, it is possible to examine the maximum pressure P_{max}, or the quantity of the total impulse I relative to a unit of area on the elementary stage of the body's surface being examined. Sometimes there is brought into existence also the vector J of the total overall impulse which is acting on the body.

If the geometrical form of the body is fixed and the body can be considered immobile and absolutely solid, then such a body is fully given by the linear dimen-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

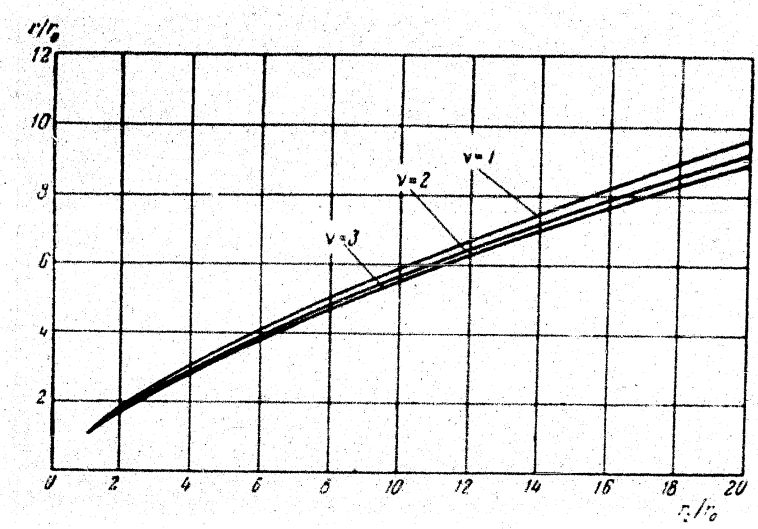


Fig.62 - Law of Motion of Gas Particles ($\gamma = 1.4$)

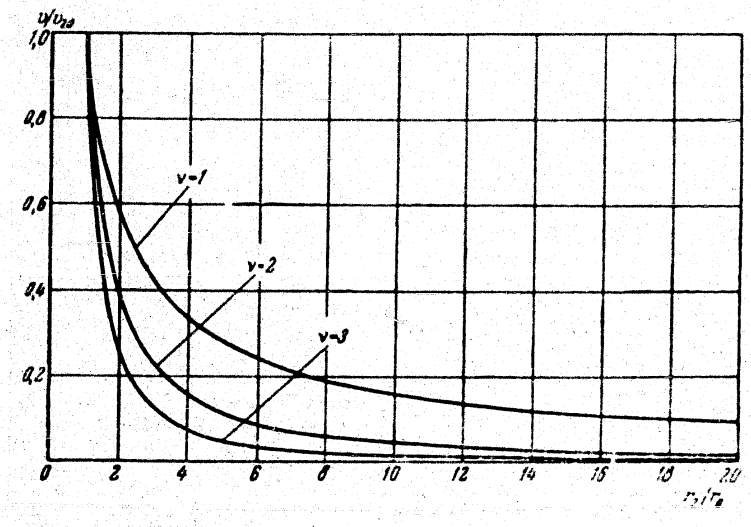


Fig.63 - Particle Density as a Function of Shock Wave Position ($\gamma = 1.4$)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

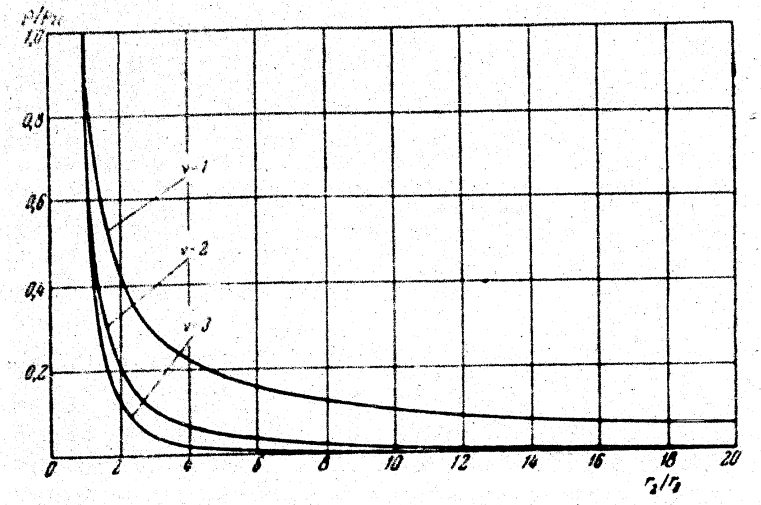


Fig.64 - Particle Velocity as a Function of Shock Wave Position (γ = 1.4)

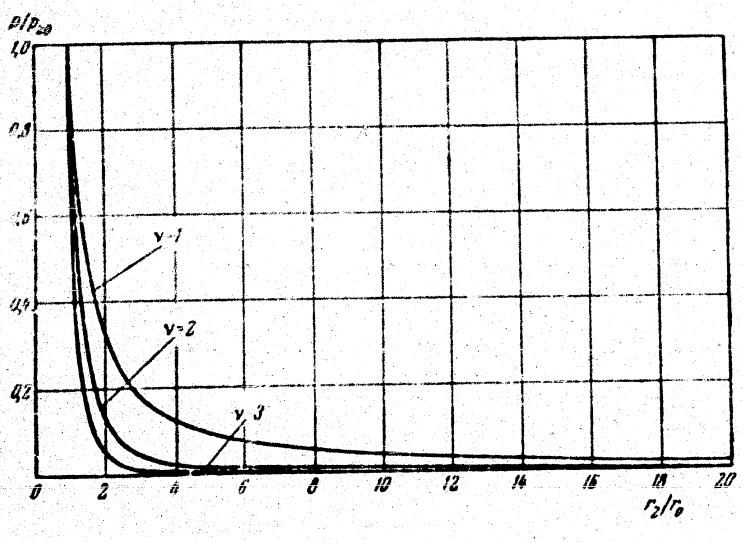


Fig.65 - Particle Pressure as a Function of Shock Wave Position

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$\nu=2,\ \gamma=1,4$

$\lambda=\frac{r}{r_s}$	$\frac{r_0}{r_s}$	$\frac{v}{v_s}=f$	$\frac{\rho}{\rho_s}=g$	$\frac{p}{p_s}=h$	$f'(\lambda)$	$g'(\lambda)$	$h'(\lambda)$
1	1	1	1	1	1,8333	14,1667	7,8333
0,9998	0,9954	0,9996	0,9973	0,9985	1,8325	14,1149	7,8102
0,9802	0,8911	0,9645	0,7653	0,8659	1,7516	9,8877	5,8478
0,9644	0,8164	0,9374	0,6285	0,7832	1,6848	7,5936	4,6993
0,9476	0,7460	0,9097	0,5164	0,7124	1,6132	5,8431	3,7665
0,9295	0,6789	0,8812	0,4234	0,6514	1,5371	4,4922	3,0009
0,9096	0,6140	0,8514	0,3451	0,5983	1,4569	3,4390	2,3668
0,8725	0,5115	0,7998	0,2427	0,5266	1,3206	2,1977	1,5608
0,8442	0,4461	0,7638	0,1892	0,4884	1,2313	1,6219	1,1574
0,8094	0,3695	0,7226	0,1414	0,4545	1,1394	1,1561	0,8136
0,7629	0,3009	0,6720	0,0975	0,4242	1,0455	0,7709	0,5170
0,7242	0,2480	0,6327	0,0718	0,4074	0,9885	0,5660	0,3567
0,6894	0,2071	0,5989	0,0545	0,3969	0,9503	0,4344	0,2551
0,6390	0,1577	0,5521	0,0362	0,3867	0,9120	0,2998	0,1556
0,5745	0,1081	0,4943	0,0208	0,3794	0,8832	0,1803	0,0798
0,5180	0,0748	0,4448	0,0123	0,3760	0,8698	0,1202	0,0423
0,4748	0,0552	0,4073	0,0079	0,3746	0,8640	0,0839	0,0249
0,4222	0,0370	0,3621	0,0044	0,3737	0,8602	0,0522	0,0123
0,3654	0,0220	0,3133	0,0021	0,3733	0,8583	0,0292	0,0052
0,3000	0,0111	0,2571	0,0008	0,3730	0,8574	0,0132	0,0016
0,2500	0,0058	0,2143	0,0003	0,3729	0,8572	0,0064	0,0005
0,2000	0,0027	0,1714	0,0001	0,3729	0,8572	0,0026	0,0001
0,1500	0,0010	0,1286	0,0000	0,3729	0,8571	0,0008	0,0000
0,1000	0,0002	0,0857	0,0000	0,3729	0,8571	0,0002	0,0000
0,0000	0,0000	0,0000	0,0000	0,3729	0,8571	0,0000	0,0000

$\nu=3,\ \gamma=1,4$

$\lambda=\frac{r}{r_s}$	$\frac{r_0}{r_s}$	$\frac{v}{v_s}=f$	$\frac{\rho}{\rho_s}=g$	$\frac{p}{p_s}=h$	$f'(\lambda)$	$g'(\lambda)$	$h'(\lambda)$
1	1	1	1	1	2,1666	20,8333	11,1666
0,9913	0,9498	0,9814	0,8379	0,9109	2,0981	16,4159	9,2305
0,9773	0,8785	0,9529	0,6457	0,7993	1,9864	11,5610	6,9475
0,9622	0,8103	0,9237	0,4978	0,7078	1,8659	8,1607	5,2109
0,9342	0,7038	0,8744	0,3241	0,5923	1,6548	4,6349	3,2119

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$\lambda = \frac{r}{r_2}$	$\frac{r_0}{r_2}$	$\frac{v}{v_2} = f$	$\frac{\rho}{\rho_2} = g$	$\frac{p}{p_2} = h$	$f'(\lambda)$	$g'(\lambda)$	$h'(\lambda)$
0,9086	0,6220	0,8335	0,2279	0,5241	1,4837	2,9536	2,1425
0,8747	0,5320	0,7872	0,1509	0,4674	1,3036	1,7841	1,3269
0,8559	0,4463	0,7397	0,0967	0,4272	1,1495	1,0716	0,7900
0,7956	0,3704	0,6952	0,0621	0,4021	1,0395	0,6690	0,4720
0,7493	0,2983	0,6496	0,0379	0,3856	0,9610	0,4090	0,2669
0,6788	0,2097	0,5844	0,0174	0,3732	0,8991	0,1981	0,1095
0,5794	0,1201	0,4971	0,0052	0,3672	0,8662	0,0679	0,0278
0,4560	0,0519	0,3909	0,0009	0,3656	0,8581	0,0142	0,0036
0,3000	0,0227	0,3086	0,0002	0,3655	0,8572	0,0030	0,0005
0,2960	0,0114	0,2538	0,0000	0,3655	0,8572	0,0008	0,0001
0,2000	0,0051	0,1714	0,0000	0,3655	0,8571	0,0001	0,0000
0,1040	0,0000	0,0892	0,0000	0,3655	0,8571	0,0000	0,0000
0,0000	0,0000	0,0000	0,0000	0,3655	0,8571	0,0000	0,0000

sion D. If the explosion is a point explosion and occurs in a gas which fills all of the space around the body, it is possible then to represent the system of defining parameters by the schedule:

$$E,\ \rho_1,\ \xi = \frac{D}{R},\ \varphi,\ \psi$$

where R, φ, ψ are polar coordinates of the center of the explosion in some system of coordinates connected with the body.

It is simple to verify that from dimension analysis these formulas follow:

$$\left.\begin{aligned} p_{\max} &= \frac{E}{R^3} \cdot f_1(\xi, \varphi, \psi); \\ I &= \sqrt{\frac{\rho_1 E}{R}} \cdot f_2(\xi, \varphi, \psi); \\ J &= \sqrt{\frac{\rho_1 E}{R}} \cdot D^2 f_3(\xi, \varphi, \psi). \end{aligned}\right\} \qquad (8.11)$$

Thus, in the case of a strong point explosion ($p_1 = 0$) the conditions of similarity for motions of the gas lead to constancy of the parameters $\xi = \frac{D}{R}, \varphi, \psi$.

Under the condition $\xi \to 0$, when the body contracts in a point, the quantities

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

P_{max} and I remain finite; hence at $\xi \to 0$, the functions of f_1, f_2, and f_3 maintain a finite value within the boundary.

The formulas (8.11) define fully the relationship of p_{max}, I and J to the energy of the explosion E and to the initial density of the gas ρ_1.

In the case of bodies of small size, the parameter ξ can be made equal to zero. In this case the formulas (8.11) give the relationship of p_{max}, I and J to the distance between the body and the center of the explosion.

If we consider the initial pressure p_1, then on the right side in formulas (8.11) the dimensionless argument $\omega = \frac{p_1 R^3}{E}$ will be added. With large values of E and small R, this parameter is close to zero; with any E and large R, this parameter is large. Hence, the above-indicated conditions of similarity and the relationship of the quantities being examined to the energy E may alter in the case of large values of R.

If the relationship of any of the quantities p_{max}, I or J to the explosion energy E is established from supplementary data (experimental data, for example), this will permit correspondingly, determination of the relationship of the functions f_1, f_2, f_3 to the parameter ω and, consequently, at $\xi \approx 0$ the relationship to the distance R will become known.

We examined above the general formulas for p_{max}, I and J in the case of interaction between non-steady state motion of a gas and a solid body. In the absence of a body it is possible to fix in the gas, imaginarily, some surface Σ, which does not interact with the motion of the gas, and examine analogous quantities p^0_{max}, I^0 and J^0 which represented characteristics of the field of the explosion; obviously they will not be equal to the corresponding quantities p_{max}, I and J, examined above.

In this case, for each element of the surface it is possible to write the formulas:

$$p^0_{max} = \frac{E}{r^3} f_1\left(\frac{p_1 r^3}{E}\right), \quad I^0 = \sqrt{\frac{p_1 E}{r}}\, f_2\left(\frac{p_1 r^3}{E}\right), \qquad (8.12)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

where r is the distance between the center of the explosion and the element being examined. The functions f_1 and f_2 depend only on the parameter $\omega = \frac{\rho_1 r^3}{E}$, since pressure in an ideal gas does not depend on the orientation of the pertinent area. In the case of a strong explosion $\omega \approx 0$ and, consequently, the functions f_1 and f_2 evolve to constants, which are easily computed with the help of the solution found above. It is obvious that p^0_{max} equals the value of the pressure behind the front of the jump at the moment of its passage through the point being examined.

For explosion along a straight line - the case of cylindrical waves - the dimension E varies; hence formulas (8.12) are replaced by the equations

$$p^0_{max} = \frac{E}{r^2} f_1\left(\frac{\rho_1 r^2}{E}\right), \quad I^0 = \sqrt{\rho_1 E}\cdot f_2\left(\frac{\rho_1 r^2}{E}\right), \tag{8.13}$$

and in the case of explosion along a plane, by the equations

$$p^0_{max} = \frac{E}{r} f_1\left(\frac{\rho_1 r}{E}\right), \quad I^0 = \sqrt{\rho_1 E}\cdot r\cdot f_2\left(\frac{\rho_1 r}{E}\right), \tag{8.14}$$

where r is respectively, the distance to the straight line or the plane of the explosion.

From eqs.(8.13) it follows that for a strong explosion, the impulse does not depend on the distance to the line of the explosion; and from eqs.(8.14) it follows that the impulse even increases during departure from the plane of the explosion. These effects are connected with an infinite extent of charge and with an infinite quantity for the total charge.

Section 9. <u>The Problem of Strong Explosion in a Medium with Variable Density</u>

The statement, method, and the results of solution for the problem of strong explosion is easily generalized on the case when the initial density ρ_1 depends on the initial coordinate of the particle r_0 according to the following law:

$$\rho_1 = \frac{A}{r_0^{\omega}} \tag{9.1}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

where ω is an abstract constant which can be positive or negative, and A is a positive constant with the dimension

$$[A] = ML^{\omega - 3}$$

If, in the case of strong explosion, we disregard the initial pressure p_1 and examine the adiabatic motion of an ideal gas, then the system of defining parameters is represented by the schedule

$$a, \gamma, A, E_0, r, t \tag{9.2}$$

From this it follows that the corresponding turbulent motion of the gas is self modeling. We shall examine further on only the case of spherical symmetry. Hence the dimension of the characteristic energy will be:

$$[E_0] = ML^2T^{-2}$$

All dimensionless quantities can be examined as functions of the following parameters:

$$\omega, \gamma, \lambda = \left(\frac{A}{aE_0}\right)^{\frac{1}{5-\omega}} \cdot \frac{r^{\frac{2}{5-\omega}}}{t}, \tag{9.3}$$

where a is a constant which can be eliminated.

From the statement of the problem it follows that for states with an identical phase and, in particular, for a shock wave, the following relationships are valid:

$$\left.\begin{aligned} &\lambda = \lambda_0 = \text{const.}, \quad r_0 = \lambda_0 \left(\frac{aE_0}{A}\right)^{\frac{1}{5-\omega}} \cdot t^{\frac{2}{5-\omega}}, \\ &c = \frac{dr_0}{dt} = \frac{2}{5-\omega}\frac{r_0}{t} = \frac{2\lambda_0^{\frac{5-\omega}{2}}}{5-\omega}\sqrt{\frac{aE_0}{A}}\, r_2^{\frac{\omega-3}{2}}. \end{aligned}\right\} \tag{9.4}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The constant a is determined from the condition that $\lambda_2 = 1$ at the shock wave. With a so defined, it is obvious that the following equation for λ is valid:

$$\lambda = \frac{r}{r_2}$$

The shock wave will move with deceleration if $\omega < 3$ and with acceleration, if $\omega > 3$. The case when $\omega < 3$ corresponds to a finite mass within any finite sphere containing a center of symmetry: with $\omega \geq 3$ this mass will be equal to infinity.

From eq.(9.4) it follows that the shock wave forming close to the center of symmetry at $t = 0$ propagates with a finite velocity at $t > 0$ toward the periphery only in the case when $\omega < 5$. In our further treatment we shall assume that $\omega < 5$.

In accordance with the general theory:

$$v = \frac{r}{t} V(\lambda), \quad \rho = \frac{A}{r^{\omega}} R(\lambda), \quad p = \frac{A}{r^{\omega-2} t^2} P(\lambda). \tag{9.5}$$

Substitution of these expressions in the system of equations (1.2) yields:

$$\left.\begin{aligned} \lambda\left[\left(V - \frac{2}{5-\omega}\right)V' + \frac{P'}{R}\right] &= V - V^2 + (\omega-2)\frac{P}{R}, \\ \lambda\left[V' + \left(V - \frac{2}{5-\omega}\right)\frac{R'}{R}\right] &= (\omega-3)V, \\ \lambda\left[\frac{P'}{P} - \gamma\frac{R'}{R}\right]\left(V - \frac{2}{5-\omega}\right) &= 2 + [\omega(1-\gamma) - 2]V. \end{aligned}\right\} \tag{9.6}$$

In the given case, at $z = \frac{\gamma P}{R}$, the system of equations (2.1), (2.2), (2.3) is written in the form

$$\frac{dz}{dV} = \frac{z\left\{\left[2(V-1) + 3(\gamma-1)V\right](V-\delta)^2 - (\gamma-1)V(V-1)(V-\delta) - \left[2(V-1) + \frac{3\delta}{\gamma}(\gamma-1)\right]z\right\}}{(V-\delta)\left[V(V-1)(V-\delta) + 3\left(\frac{\delta}{\gamma} - V\right)z\right]}, \tag{9.7}$$

$$\frac{d\ln\lambda}{dV} = \frac{z - (V-\delta)^2}{V(V-1)(V-\delta) + 3\left(\frac{\delta}{\gamma} - V\right)z}, \tag{9.8}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$(V-\delta)\frac{d\ln R}{dV} = -1 + (\omega-3)V\frac{[z-(V-\delta)^2]}{V(V-1)(V-\delta)+3\left(\frac{\delta}{\gamma}-V\right)}, \qquad (9.9)$$

where

$$\delta = \frac{2}{5-\omega}$$

As in the general case, the problem leads to the integration of an ordinary differential eq.(9.7).

Using the considerations of dimension theory and repeating the discussions of Section 7, we find that the system of equations (9.7), (9.8), and (9.9) permits the following integral:

$$V^2R\left(V-\frac{2}{5-\omega}\right)+\frac{2P}{\gamma-1}\left(V-\frac{2}{5-\omega}\right)+2PV=k\lambda^{-1}, \qquad (9.10)$$

where k is an arbitrary constant.

As in the case when $\rho = \rho_1$-const. ($\omega = 0$), it can be shown that during an explosion a shock wave is formed which propagates through gas at rest and bounds the region of turbulent conditions.

From the conditions at a strong shock wave (7.2) on the basis of eqs.(9.4) and (9.5), we find that behind the jump the quantities V, R, and $z = \frac{\gamma P}{R}$ have the following constant values:

$$V_2=\frac{4}{(5-\omega)(\gamma+1)}=\frac{2\delta}{\gamma+1}, \quad R_2=\frac{\gamma+1}{\gamma-1}, \quad z_2=\frac{8\gamma(\gamma-1)}{(5-\omega)^2(\gamma+1)^2}. \qquad (9.11)$$

After substituting these values in the integral (9.10) we find that

$$k = 0$$

It follows then that the equation of the integral curve which passes through the

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

point V_2, z_2 in the plane z, V has the form

$$z = \frac{(\gamma-1)V^2\left(V-\frac{2}{5-\omega}\right)}{2\left[\frac{2}{\gamma(5-\omega)}-V\right]} = \frac{(\gamma-1)V^2(V-\delta)}{2\left(\frac{\delta}{\gamma}-V\right)} \qquad (9.12)$$

From physical expressions it is clear that $z > 0$; hence the attainable values of V are situated on the section

$$\frac{\delta}{\gamma} = \frac{2}{\gamma(5-\omega)} \leqslant V \leqslant \frac{2}{5-\omega} = \delta; \qquad (9.13)$$

since $\gamma > 1$, the following inequalities are obvious

$$\frac{1}{\gamma} < \frac{2}{\gamma+1} < 1,$$

Hence, the value V_2 in accordance with eq.(9.11) will fall in the interval (9.13). The ends of this integral curve

$$\left(z=0,\ V=\frac{2}{5-\omega}\right) \quad \text{и} \quad \left(z=\infty,\ V=\frac{1}{\gamma}\frac{2}{5-\omega}\right)$$

are specific points of the differential equation (9.7).

Thus it is not difficult to prove that a point with the coordinates

$$V = \frac{2}{3\gamma-1} = V^*, \quad z = \frac{(\gamma-1)2\left(\frac{2}{3\gamma-1}-\delta\right)}{(3\gamma-1)^2\left(\frac{\delta}{\gamma}-\frac{2}{3\gamma-1}\right)} = z^*,$$

belonging to the integral curve (9.12) is a specific point of eq.(9.7); and with the values $\gamma = 1.4$ or $\gamma = \frac{5}{3}$ and values of ω close to zero it lies outside of the interval (9.13). At $\omega = 6 - 3\gamma$ this specific point coincides with the right end

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

of the interval (9.13); and with further increase of ω will rise along the integral curve (9.12). At a value of ω determined from the equation

$$\frac{4}{(5-\omega)(\gamma+1)} = \frac{2}{3\gamma-1} \quad \text{OR} \quad \omega = \frac{7-\gamma}{\gamma+1} \tag{9.14}$$

(at $\gamma = 1.4$, $\omega = 2\frac{1}{3}$; at $\gamma = \frac{5}{3}$, $\omega = 2$), the specific point being examined coincides with the values of V_2, z_2 behind the front of the jump.

It is easily seen that the following simple accurate solution of the system (9.6) corresponds to specific point V^*, z^* with any value of ω:

$$P = B\frac{z^*}{\gamma}\lambda^{\frac{(\omega-3)V^*}{V^*-1}}, \tag{9.15}$$

To this there responds a precise solution of the partial equations

$$v = V^*\cdot\frac{r}{t}, \quad \rho = \frac{AB}{r^\omega}\lambda^{\frac{(\omega-3)V^*}{(V^*-1)}}, \quad p = \frac{CBz^*}{\gamma r^{\omega-2}t^2}\lambda^{\frac{(\omega-3)V^*}{V^*-1}} \tag{9.16}$$

Here B is an arbitrary constant. The eqs.(9.16) yield a solution of the problem of a strong explosion at

$$\omega = \frac{7-\gamma}{\gamma+1}$$

Having determined, in this case, the constant B from the conditions at the shock wave, we obtain:

$$v = \frac{2}{3\gamma-1}\frac{r}{t}, \quad \rho = \frac{A(\gamma+1)}{r^\omega(\gamma-1)}\lambda^{\frac{8}{\gamma+1}}, \quad p = \frac{A}{r^{\omega-2}t^2}\frac{2(\gamma+1)}{(3\gamma-1)^2}\lambda^{\frac{8}{\gamma+1}},$$

or relating velocity, density, and pressure to the corresponding quantities behind the shock wave, we find:

$$\frac{v}{v_2} = \lambda, \quad \frac{\rho}{\rho_2} = \lambda, \quad \frac{p}{p_2} = \lambda^3. \tag{9.17}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

From these equations it is clear that in this case, the motion is continued to the center of symmetry, whereby, in the center of symmetry, density and pressure are equal to zero.

It is not difficult to see that the derivative $\frac{d \ln \lambda}{dV}$ in the point V_2, z_2 changes its sign with variation of ω, when the point V*, z* passes through the point V_2, z_2 along the integral curve (9.12). From this it follows that in the case of motion from the shock wave toward a center of symmetry - with λ diminishing - it is necessary to move up along the integral curve (9.12) if $V_2 < V^*$, and down, if $V_2 > V^*$. In the first case, during approach to the specific point $z = \infty$, $V = \frac{1}{\gamma} \cdot \frac{2}{(5-\omega)}$, the parameter λ tends toward zero; the motion is determined as far as the center of symmetry; in the second case it is necessary to move down along the integral curve (9.12) to specific point $V = \frac{2}{5-\omega}$, $z = 0$, in which the parameter λ assumes a finite value*.

It is a simple matter to define the combination $\frac{p}{\rho^\gamma}$, connected with entropy, as a function of the initial corrdinate r_0. On the basis of conditions at the shock wave (7.2) and eq.(9.4), we obtain

$$\frac{p}{\rho^\gamma} = \frac{2(\gamma-1)^\gamma}{(\gamma+1)^{\gamma+1}} \cdot \rho_1^{1-\gamma} \cdot c^2 = \frac{8(\gamma-1)^\gamma A^{-\gamma} a E_0}{(\gamma+1)^{\gamma+1}(5-\omega)^2} \cdot r_0^{\gamma\omega-3}. \qquad (9.18)$$

We shall use the relationship (9.18) to determine $\frac{r_0}{r_2}$ as a function of the parametric variable V.

It is curious to note that by measure of the propagation of a shock wave through the particles, the entropy of particles behind the jump front diminishes if $\omega < \frac{3}{\gamma}$, is maintained constant at $\omega = \frac{3}{\gamma}$, and increases if $\omega > \frac{3}{\gamma}$.

In the range

$$\frac{3}{\gamma} < \omega < 3$$

---

* From eqs.(9.20), (9.21), which give the solution, this is immediately apparent.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the shock wave retards whereas entropy behind the front of the wave increases. This is explained by the fact that, the weakening of the shock wave and the pressure drop behind the wave front notwithstanding, the density falls off so strongly that entropy increases as a result of it. In the case of such an effect, if

$$\frac{3}{\gamma} < \omega \leq \frac{7-\gamma}{\gamma+1}$$

then the turbulent motion of the gas occupies the entire inside of the shock wave sphere - all the way to the center of symmetry; but if

$$\frac{7-\gamma}{\gamma+1} < \omega \leq 3$$

then a vacuum is formed close to the center of the explosion.

We shall examine again the behavior of temperature T_2 behind the shock wave front. Taking into account the conditions at a jump (7.2), we obtain

$$RT_2 = \frac{p_2}{\rho_2} = \frac{2(\gamma-1)}{(\gamma+1)^2} c^2. \qquad (9.19)$$

Here R is the gas constant. The temperature T_2 is proportional to c^2; consequently, for the motions of the gas being examined the temperature behind the front of the jump can increase only with an increase in c^2, i.e., at $\omega > 3$.

By using integral (9.12) of eq.(9.7), it is possible, from eqs.(9.8) and (9.9), to determine $\lambda(V)$ and $R(V)$ with the help of simple quadratures, which permits the complete solution of the problem being examined to be obtained in closed form.

This solution is given by the equations

$$\left.\begin{aligned} \frac{r}{r_2} = &\left[\frac{(5-\omega)(\gamma+1)}{4} V\right]^{-\frac{2}{5-\omega}} \cdot \left[\frac{\gamma+1}{\gamma-1}\left(\frac{\gamma(5-\omega)}{2} V - 1\right)\right]^{-\alpha_2} \times \\ &\times \left[\frac{(5-\omega)(\gamma+1)}{7-\gamma-(\gamma+1)\omega}\left(1 - \frac{3\gamma-1}{2} V\right)\right]^{-\alpha_1}, \end{aligned}\right|$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\left.\begin{aligned}
&\frac{r_0}{r_2} = \left[\frac{(5-\omega)(\gamma+1)}{4}V\right]^{-\frac{2}{5-\omega}} \cdot \left[\frac{\gamma+1}{\gamma-1}\left(\frac{\gamma(5-\omega)}{2}V-1\right)\right]^{\alpha_6} \times \\
&\qquad \times \left[\frac{(5-\omega)(\gamma+1)}{7-\gamma-(\gamma+1)\omega}\left(1-\frac{3\gamma-1}{2}V\right)\right]^{\alpha_7} \times \\
&\qquad \times \left[\frac{\gamma+1}{\gamma-1}\left(1-\frac{5-\omega}{2}V\right)\right]^{-\left(\alpha_6+\alpha_7-\frac{2}{5-\omega}\right)}, \\
&\frac{v}{v_2} = f = \frac{(5-\omega)(\gamma+1)}{4}V\frac{r}{r_2}, \\
&\frac{\rho}{\rho_2} = g = \left[\frac{(5-\omega)(\gamma+1)}{4}V\right]^{\frac{2\omega}{5-\omega}} \left[\frac{\gamma+1}{\gamma-1}\left(1-\frac{5-\omega}{2}V\right)\right]^{\alpha_5} \times \\
&\qquad \times \left[\frac{\gamma+1}{\gamma-1}\left(\frac{\gamma(5-\omega)}{2}V-1\right)\right]^{\alpha_3+\omega\alpha_2} \times \\
&\qquad \times \left[\frac{(5-\omega)(\gamma+1)}{7-\gamma-(\gamma+1)\omega}\left(1-\frac{3\gamma-1}{2}V\right)\right]^{\alpha_4+\omega\alpha_1}, \\
&\frac{p}{p_2} = h = \left[\frac{(5-\omega)(\gamma+1)}{4}V\right]^{\frac{6}{5-\omega}} \left[\frac{\gamma+1}{\gamma-1}\left(1-\frac{5-\omega}{2}V\right)\right]^{\alpha_5+1} \times \\
&\qquad \times \left[\frac{(5-\omega)(\gamma+1)}{7-\gamma-(\gamma+1)\omega}\left(1-\frac{3\gamma-1}{2}V\right)\right]^{\alpha_4+\alpha_1(\omega-2)}, \\
&\frac{T}{T_2} = \frac{p}{p_2}\cdot\frac{\rho_2}{\rho}
\end{aligned}\right\} \qquad (9.20)$$

where

$$\left.\begin{aligned}
&\alpha_1 = \frac{(5-\omega)\gamma}{3\gamma-1}\left[\frac{2(6-3\gamma-\omega)}{\gamma(5-\omega)^2}-\alpha_2\right], \\
&\alpha_2 = \frac{1-\gamma}{(2\gamma+1-\gamma\omega)}, \quad \alpha_3 = \frac{-\omega+3}{(2\gamma+1-\gamma\omega)}, \\
&\alpha_4 = \frac{(5-\omega)(3-\omega)\alpha_1}{(6-3\gamma-\omega)}, \quad \alpha_5 = \frac{\omega(1+\gamma)-6}{(6-3\gamma-\omega)}, \\
&\alpha_6 = \frac{\gamma}{(2\gamma+1-\gamma\omega)}, \quad \alpha_7 = \frac{(3\gamma-1)\alpha_1}{(6-3\gamma-\omega)}.
\end{aligned}\right\} \qquad (9.21)$$

The eqs.(9.20) give a complete solution from the Euler and Lagrange viewpoints. These equations demonstrate that the distribution of dimensionless characteristics will be identical for different values of the energy E_0 of the explosion.

In Figs.66, 67, and 68 are presented the curves $\frac{v}{v_2} = f(\lambda,\omega)$, $\frac{\rho}{\rho_2} = g(\lambda,\omega)$, $\frac{p}{p_2} = h(\lambda,\omega)$ at $\gamma = 1.4$ and for $\omega = 2, 1, 0, -1, -2$; $\omega = 2\frac{1}{3}$; in all of these cases the motion of the gas extends all the way to the center of symmetry.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Figures 69, 70, and 71 show the curves for $\omega = 2\frac{1}{3}$, 2.5, 3, 3.5 at $\gamma = \frac{7}{5}$ and $\omega = 2$, 2.5, 3, 3.5 at $\gamma = \frac{5}{3}$: for each γ in the last three cases a vacuum is

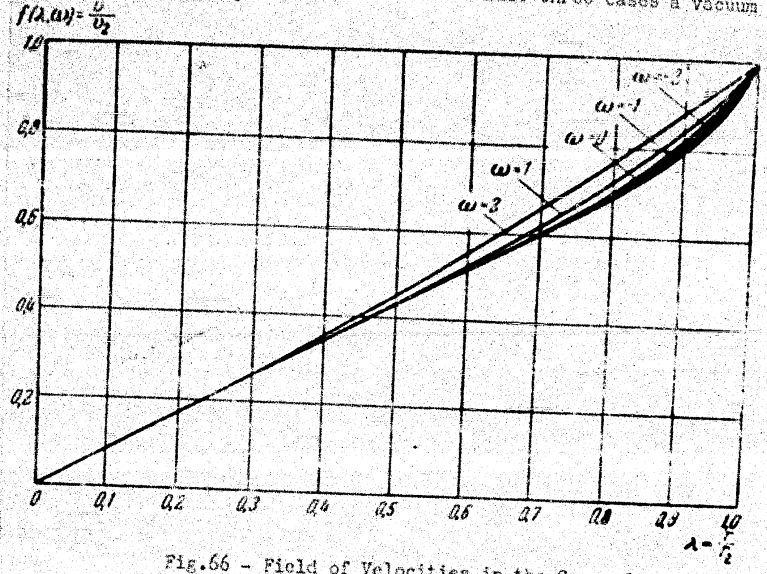


Fig.66 - Field of Velocities in the Case of a Point Explosion in a Gas with Variable Initial Density Distributed According to the Law $\rho_1 = \frac{A}{r^\omega}$, $\gamma = 1.4$. In cases where $\omega < 2\frac{1}{3}$, motion extends to the center of symmetry

formed close to the center of the explosion.

At $\omega < \frac{7-\gamma}{\gamma+1}$ the following asymptotic equations are valid close to the center of the explosion

$$\left.\begin{aligned} \frac{v}{v_2} &= \frac{\gamma+1}{2\gamma}\frac{r}{r_2}, \\ \frac{p}{p_2} &= \left(\frac{\gamma+1}{2\gamma}\right)^{\frac{6}{5-\omega}} \cdot \left(\frac{\gamma+1}{\gamma}\right)^{\alpha_2+1} \cdot \left(\frac{\omega-\frac{2\gamma-1}{\gamma}}{\omega-\frac{7-\gamma}{\gamma+1}}\right)^{\alpha_3+\alpha_1(\omega-2)} \\ \frac{\rho}{\rho_2} &= \left(\frac{r}{r_2}\right)^{\frac{\alpha_3}{\alpha_2}+\omega} \left(\frac{\gamma+1}{\gamma}\right)^{\alpha_2} \left(\frac{\gamma+1}{2\gamma}\right)^{-\frac{2}{5-\omega}\frac{\alpha_3}{\alpha_2}} \times \end{aligned}\right\} \quad (9.22)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\times \left[\frac{\omega - \frac{2\gamma+1}{2}}{\omega - \frac{7-\gamma}{\gamma+1}} \right]^{\alpha_4 - \frac{\alpha_1}{\alpha_2}\alpha_3} \Bigg\} \qquad (9.22)$$

From the eqs.(9.22) it follows that if $\omega > 0$, then the density at $r \to 0$ tends rapidly toward zero; the pressure is found to be other than zero. However,

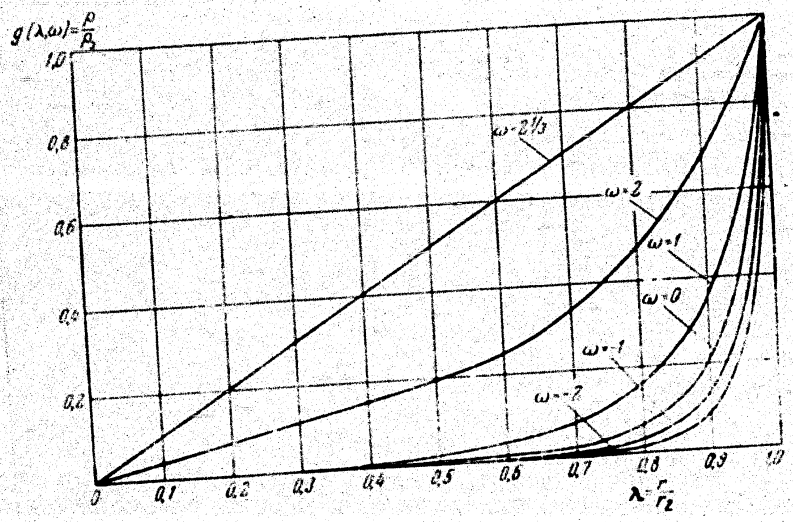


Fig.67 - Field of Densities in the Case of a Point Explosion in a Gas with Variable Initial Density Distributed According to the Law $\rho_1 = \frac{A}{r^\omega}$, $\gamma = 1.4$. In cases where $\omega < 2\frac{1}{3}$, motion extends to the center of symmetry

at $\omega = \frac{7-\gamma}{\gamma+1}$, pressure at the center tends toward zero.

At $\omega = \frac{7-\gamma}{\gamma+1}$ the asymptotic and precise equations coincide and are given by the equalities (9.17).

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

If $\omega > \frac{7-\gamma}{\gamma+1}$, then there is formed a vacuum expanding from the center of the explosion. At the boundary of the vacuum $V = \frac{2}{5-\omega}$, whereby the following

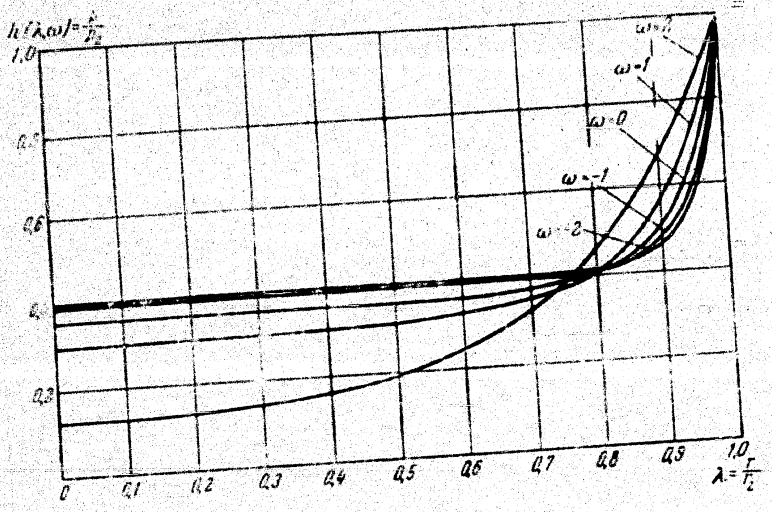


Fig.68 - Field of Pressures in the Case of a Point Explosion in a Gas with Variable Initial Density Distributed According to the Law $\rho_1 = \frac{A}{r^\omega}$, $\gamma = 1.4$. In cases where $\omega \leq 2\frac{1}{3}$ motion extends to the center of symmetry

asymptotic equations are valid:

$$\left.\begin{aligned} \frac{v}{v_2} &= \frac{\gamma+1}{\gamma}\frac{r^*}{r_2}, \\ \frac{\rho}{\rho_2} &= A\left(\frac{r}{r_2}-\frac{r^*}{r_2}\right)^{\alpha_5}, \\ \frac{p}{p_2} &= B\left(\frac{r}{r_2}-\frac{r^*}{r_2}\right)^{\alpha_5+1}, \end{aligned}\right\} \qquad (9.23)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\frac{r^*}{r_2} = \left(\frac{\gamma+1}{2}\right)^{-\frac{2}{5-\omega}} (\gamma - 1)^{-\alpha_2} \left(\frac{3\gamma+\omega-6}{\omega-\frac{7-\gamma}{\gamma+1}}\right)^{-\alpha_1},$$

$$A = \left(\frac{\gamma+1}{2}\right)^{\frac{2\omega}{5-\omega}} \cdot (\gamma+1)^{\alpha_3+\omega\alpha_2} \cdot \left(\frac{3\gamma+\omega-6}{\omega-\frac{7-\gamma}{\gamma+1}}\right)^{\alpha_1+\omega\alpha_2} \times$$

$$\times \left[\frac{\gamma(1-\gamma)}{(\gamma+1)(3\gamma+\omega-6)}\right]^{-\alpha_3} \cdot \left(\frac{r^*}{r_2}\right)^{-\alpha_5},$$

$$B = \left(\frac{\gamma+1}{2}\right)^{\frac{6}{5-\omega}} \cdot \left(\frac{3\gamma+\omega-6}{\omega-\frac{7-\gamma}{\gamma+1}}\right)^{\alpha_1+\alpha_3(\omega-2)} \times$$

$$\times \left[\frac{\gamma(1-\gamma)}{(\gamma+1)(3\gamma+\omega-6)}\right]^{-(\alpha_5+1)} \cdot \left(\frac{r^*}{r_2}\right)^{-(\alpha_5+1)},$$

in which $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are determined according to the eqs.(9.21): here r is the radius at $V = \frac{2}{5-\omega}$; within the corresponding sphere of radius r* a vacuum

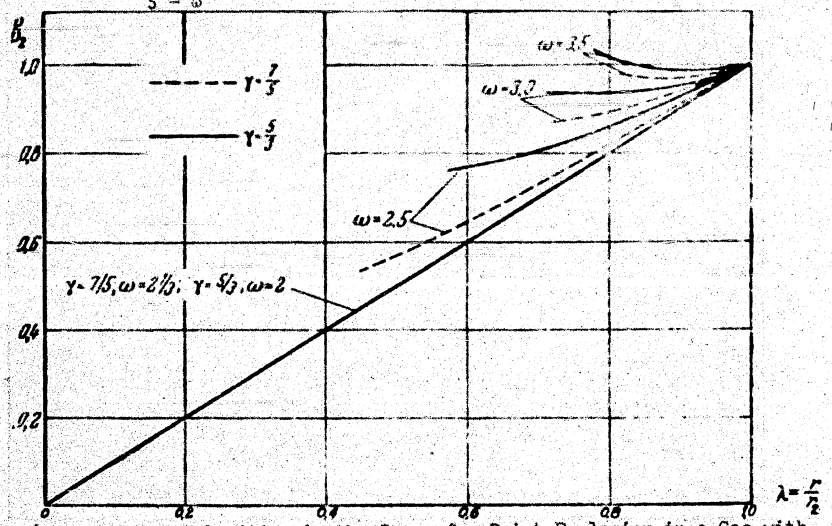


Fig.69 - Field of Velocities in the Case of a Point Explosion in a Gas with Variable Initial Density Distributed According to the Law $\rho_1 = \frac{A}{r^\omega}$; $\omega > 2\frac{1}{3}$ for $\gamma = 1.4$ and $\omega > 2$ for $\gamma = \frac{5}{3}$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

is obtained. The ratio $\frac{r^*}{r_2}$ as a function of ω is presented in Fig.72.

At $\omega < 3$, the pressure at $r = r^*$ is equal to zero; density is equal to zero

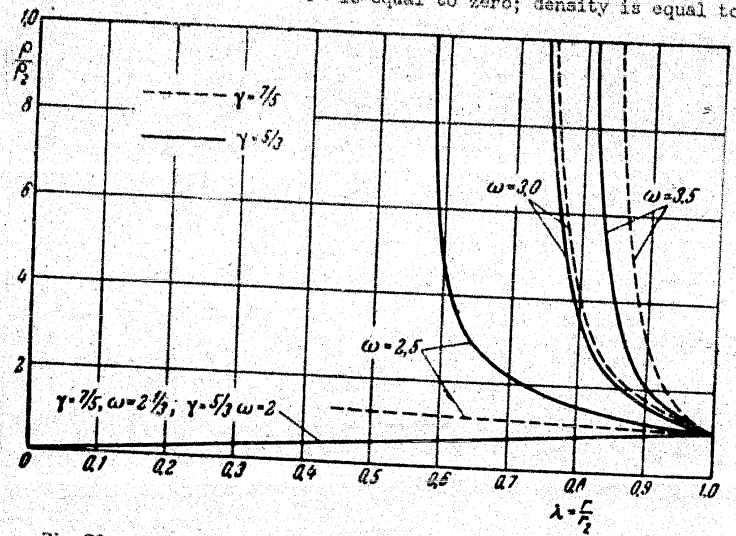


Fig.70 - Field of Densities in the Case of a Point Explosion in a Gas with Variable Initial Density Distributed According to the Law $\rho_1 = \frac{A}{r^\omega}$; $\omega > 2\frac{1}{3}$ for $\gamma = 1.4$ and $\omega > 2$ for $\gamma = \frac{5}{3}$

at $\omega < \frac{6}{1+\gamma}$ and to infinity at $\omega > \frac{6}{\gamma+1}$. At $\omega > 3$, pressure at the inner boundary is equal to infinity.

From the established asymptotic equations it is clear that the energy of turbulent motion at $\omega < 3$ is finite. It is likewise immediately obvious that at $\omega > 3$ the energy of turbulent motion is infinite since at $r = r^*$ the pressure is infinite at all times; and hence the work transferred to the gas at this boundary is equal to infinity.

Section 10. On the General Theory of One-Dimensional Motions of a Gas*

Solution of the boundary problems for the nonlinear system of eqs.(1.2) in

* See DAN SSSR; 1952, Vol.85, No.4; and DAN SSSR, 1952, Vol.87, No.1

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the absence of self modeling meets with considerable difficulties.

To obtain the required results it is usually necessary to use numerical methods, with the attraction of modern calculating machines. For the development of

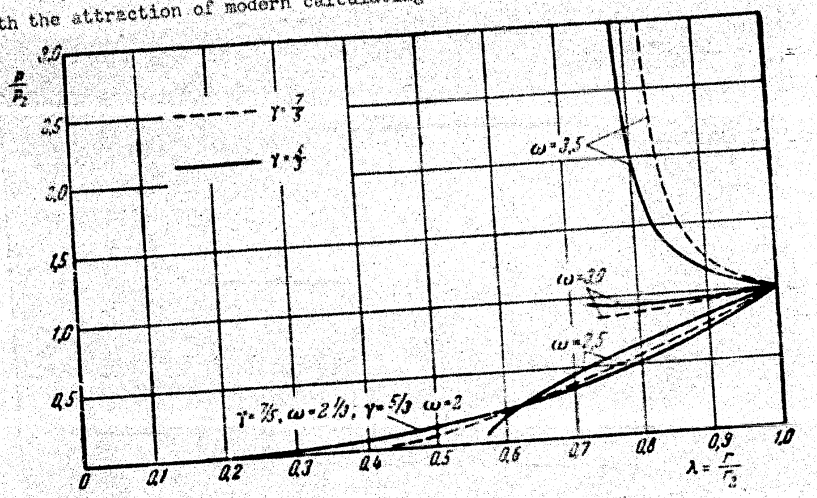


Fig.71 - Field of Pressures in the Case of a Point Explosion in a Gas with Variable Initial Density Distributed According to the Law $\rho_1 = \frac{A}{r^\omega}$; $\omega \geq 2\frac{1}{3}$ for $\gamma = 1.4$ and $\omega > 2$ for $\gamma = \frac{5}{3}$

such methods of computation and for construction of approximate analytic procedures of solution, the general relationships, of a different type, which occur for some general classes of motions of a gas may prove to be useful.

1. Behavior of the solutions of equations of one-dimensional nonsteady-state motions of a gas close to the center of symmetry.

We shall examine one-dimensional nonsteady-state motions of an ideal gas with the help of equations obtained by way of simple transformations from eqs.(1.2):

$$\frac{\partial \rho r^{\nu-1}}{\partial t} + \frac{\partial \rho u r^{\nu-1}}{\partial r} = 0, \tag{10.1}$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial r} + (\nu - 1)\frac{\rho u^2}{r} + \frac{\partial p}{\partial r} = 0 \tag{10.2}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

and

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} + \gamma p\left[\frac{\partial u}{\partial r} + (\nu-1)\frac{u}{r}\right] = 0. \qquad (10.3)$$

From eqs.(10.1) and (10.2) it is possible to express u (r,t) and p (r,t) through the density ρ (r,t).

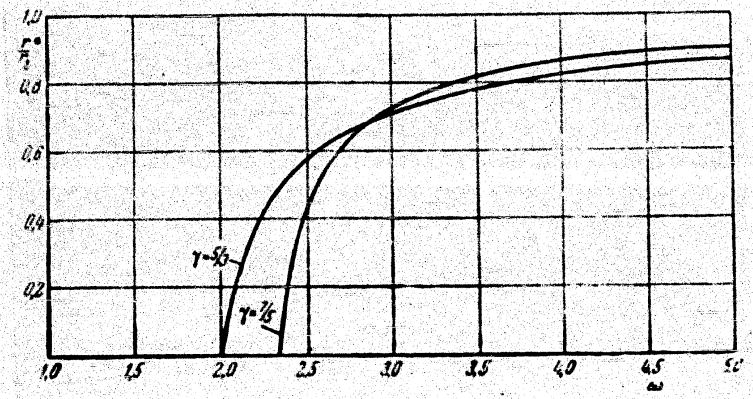


Fig.72 – r* is the Radius of the Inner Boundary Sphere; r_2 is the Radius of the Shock Wave Sphere. The ratio $\frac{r^*}{r_2}$ as a function of the index ω in the law

$$\rho_1 = \frac{A}{r^\omega} \text{ for } \gamma = 1.4 \text{ and } \gamma = \frac{5}{3}$$

By the same token, under the condition that u = 0 at r = 0 we have immediately:

$$u = -\frac{1}{r^{\nu-1}\rho}\int_0^r r^{\nu-1}\frac{\partial \rho}{\partial t}\,dr. \qquad (10.4)$$

From eq.(10.2), considering eq.(10.4), it follows that:

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$p(r, t) = p(0, t) - \rho u^2 - (\nu - 1) \int_0^r \frac{\rho u^2}{r} dr + \int_0^r \frac{1}{\xi^{\nu-1}} \int_0^\xi \eta^{\nu-1} \frac{\partial^2 \rho(\eta, t)}{\partial t^2} d\eta d\xi. \quad (10.5)$$

We shall examine the class of gas motions where, close to the center of symmetry, the density may be broken down into a power series of the form

$$\rho(r, t) = r^s [\omega_0(t) + \omega_1(t) r^m + \ldots + \omega_n(t) r^{nm} + \ldots] \quad (10.6)$$

where $\omega_0 \neq 0$, s and m are constant numbers which satisfy the inequalities

$$s + \nu + nm \neq 0, \; s + 2 + nm \neq 0 \; (n = 0, 1, 2, 3 \ldots) \quad (10.7)$$

Substitution of the series (10.6) in eq.(10.4) after integration yields:

$$u = -r \frac{\frac{\omega_0'}{s+\nu} + \frac{\omega_1'}{s+\nu+m} r^m + \ldots + \frac{\omega_n'}{s+\nu+nm} r^{nm} + \ldots}{\omega_0 + \omega_1 r^m + \ldots + \omega_n r^{nm} + \ldots}. \quad (10.8)$$

Owing to the condition (10.7), no logarithmic term appears in the course of integration.

It is obvious that the velocity u (r,t) can be represented close to r = 0 by a power series of the form

$$u = r [\omega_0(t) + \varphi_1(t) r^m + \ldots + \varphi_n(t) r^{nm} + \ldots] \quad (10.9)$$

Comparison of eq.(10.8) and eq.(10.9) leads to the following system of equations for the coefficients of φ_i:

$$\sum_{i=0}^{n} \varphi_i \omega_{n-i} + \frac{\omega_n'}{s+\nu+nm} = 0. \quad (10.10)$$

It is obvious that from the system of eqs.(10.10) the function $\varphi_n(t)$ can be expressed through the functions $\omega_0(t), \omega_1(t), \ldots, \omega_n(t)$, and their derivatives of the first order with the help of the determinant

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\varphi_n = \frac{1}{\omega_0^{n+1}} \begin{vmatrix} -\frac{\omega_n'}{s+\nu+nm}, & \omega_1 & \omega_2 & \dots & \omega_{n-1} & \omega_n \\ -\frac{\omega_{n-1}'}{s+\nu+(n-1)m}, & \omega_0 & \omega_1 & \dots & \omega_{n-2} & \omega_{n-1} \\ & 0 & \omega_0 & \dots & \omega_{n-3} & \omega_{n-2} \\ & \cdot & 0 & \dots & \cdot & \cdot \\ & \cdot & \cdot & & \cdot & \cdot \\ & \cdot & \cdot & & \omega_0 & \omega_1 \\ -\frac{\omega_0'}{s+\nu}, & 0 & 0 & \dots & 0 & \omega_0 \end{vmatrix} \qquad (10.11)$$

Substituting (10.6) and (10.9) in eq.(10.5) we find after integration (by virtue of the condition (10.7) logarithmic terms do not appear), that the pressure $p(r,t)$ is represented by a power series of the form

$$p(r,t) = \psi_{-1}(t) + r^{s+2}[\psi_0(t) + \psi_1(t)\, r^m + \dots + \psi_n(t)\, r^{nm} + \dots] \qquad (10.12)$$

wherein the following equations are valid for the functions $\psi_i(t)$:

$$\left.\begin{aligned} &\psi_{-1} = p(0,t), \\ &\psi_0 = \frac{\omega_0''}{(s+\nu)(s+2)} - \frac{(\nu+s+1)}{s+2}\varphi_0^2\omega_0, \\ &\psi_1 = \frac{\omega_1''}{(s+\nu+m)(s+2+m)} - \frac{\nu+s+1+m}{s+2+m}(2\varphi_0\varphi_1\omega_0 + \varphi_0^2\omega_1), \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &\psi_n = \frac{\omega_n''}{(s+\nu+nm)(s+2+nm)} - \frac{\nu+s+1+nm}{s+2+nm}\sum_{j=0}^{n}\omega_j\sum_{i=0}^{n-j}\varphi_i\varphi_{n-j-i}. \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{aligned}\right\} \qquad (10.13)$$

In the general equation for ψ_n the summation series can be changed according to the equality $\sum_{j=0}^{n}\omega_j\sum_{i=0}^{n-j}\varphi_i\varphi_{n-j-i} = \sum_{k=0}^{n}\varphi_{n-k}\sum_{i=0}^{i=k}\omega_i\varphi_{k-i}$.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Considering this and eq.(10.10) we obtain a general equation for ψ_n in the following simplified form:

$$\psi_n = \frac{\omega_n''}{(s+\nu+nm)(s+2+nm)} \quad \frac{\nu+s+1+nm}{s+2+nm} \sum_{k=0}^{n} \frac{\varphi_{n-k}\omega_k'}{s+\nu+km}. \qquad (10.14)$$

Thus, if φ_n is determined by the eqs.(10.11), and ψ_n by eq.(10.14), then the series (10.6), (10.9), and (10.12) satisfy eqs.(10.1) and (10.2).

Now, by substituting these series in eq.(10.3), we obtain

$$\left\{\psi_{-1}' - \gamma\psi_{-1} \sum_{n=0}^{\infty} \varphi_n (\nu - nm) r^{nm}\right\}$$

$$r^{s+2}\left\{\sum_{n=0}^{\infty}\left[\psi_n' - \sum_{i=0}^{n} \varphi_i \psi_{n-i}(\gamma\nu - s - 2 - im(\gamma-1) - nm)\right] r^{nm}\right\} = 0. \qquad (10.15)$$

This relationship, together with eqs.(10.11) and (10.14) can serve as a basis for obtaining equations for the functions $p(0,t)$, $\omega_0(t)$, $\omega_1(t)$, $\omega_2(t)$, ... For the formulation of these equations it is necessary to rely in the relationship between s and m.

Equation (10.15) is satisfied with s arbitrary, if both expressions inside the brackets are reduced to zero. Reduction of the first bracketed term to zero is equivalent to the system of equations:

$$\left.\begin{aligned} &\psi_{-1}' - \nu\gamma\psi_{-1}\varphi_0 = 0, \\ &(m - \nu)\psi_{-1}\varphi_1 = 0, \\ &(2m - \nu)\psi_{-1}\varphi_2 = 0, \\ &\cdots\cdots\cdots \\ &(nm - \nu)\psi_{-1}\varphi_n = 0. \\ &\cdots\cdots\cdots \end{aligned}\right\} \qquad (10.16)$$

From this, if $\psi_{-1} = p(0,t) \neq 0$, then the system (10.16), with calculation of eq.(10.11) at $km + \nu \neq 0$ ($k = 1,2, \ldots$) yields:

$$\varphi_0 = -\frac{\omega_0'}{(s+\nu)\omega_0}, \quad \varphi_1 = \varphi_2 = \ldots = \varphi_n = \ldots = 0. \qquad (10.17)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Consequently, in a given case the velocities are distributed relative to r according to the linear law

$$u = \varphi_0 \cdot r$$

On the basis of eqs.(10.17) and (10.10) it follows that:

$$\frac{\omega_n'}{s+\nu+nm} = \omega_n \frac{\omega_0'}{(s+\nu)\omega_0},$$

hence

$$\omega_0 = c_0\omega(l), \quad \omega_n = c_n\omega^{\frac{nm}{s+\nu}+1} \qquad (n = 1, 2, 3, \ldots) \tag{10.18}$$

and, consequently, on the basis of eq.(10.6) we have:

$$p = r^s\omega\left[c_0 + c_1\left(r\omega^{\frac{1}{s+\nu}}\right)^m + c_2\left(r\omega^{\frac{1}{s+\nu}}\right)^{2m} + \ldots\right] = r^s\omega \cdot P'\left[\left(r\omega^{\frac{1}{s+\nu}}\right)^{s+2}\right], \tag{10.19}$$

where

$$P(x) = c_0 x + (s+2)\left[c_1 \frac{x^{\frac{m+s+2}{s+2}}}{m+s+2} + c_2 \frac{x^{\frac{2m+s+2}{s+2}}}{2m+s+2} + \ldots\right]. \tag{10.20}$$

In view of the arbitrary quality of the coefficients c_0, c_1, ..., c_n,..., and the constant m, P(x) is some arbitrary function of its argument.

Further, from the first equation of (10.16), considering the equality $\omega_0 = c_0\omega$, it follows that:

$$\psi_{-1} = p(0, l) = c_{-1}\omega^{\frac{\gamma\nu}{s+\nu}}. \tag{10.21}$$

For w_n from eq.(10.14) we obtain:

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\psi_n = \frac{\omega_n''}{(s+\nu+nm)(s+2+nm)} - \frac{\nu+s+1+nm}{(s+2+nm)} \frac{\omega'}{(s+\nu)\omega} \frac{\omega_n'}{s+\nu+nm};$$

from which, on the basis of eq.(10.18) we find

$$\psi_n = \frac{c_n \omega^{\frac{nm}{s+\nu}}}{(s+\nu)(s+2+nm)} \left[\omega'' - \frac{\nu+s+1}{s+\nu} \frac{\omega'^2}{\omega} \right] \qquad (10.22)$$
$$(n = 0, 1, 2, \ldots).$$

Thus, equating to zero of the first bracketed term in eq.(10.15) leads to a solution which expresses, in accordance with eq.(10.17), (10.18), (10.21) and (10.22), all the functions $\omega_n(t)$, $\varphi_n(t)$ and $\psi_n(t)$ by way of one function $\omega(t)$, which can be arbitrary.

We shall demonstrate now that the obtained solution, in the case of proper selection of the function $\omega(t)$ reverts the second bracketed term in eq.(10.15) to zero.

By the same token, by virtue of eq.(10.17) the corresponding infinite system of equations has the form

$$\psi_n' - \frac{\omega'}{(s+\nu)\omega} \psi_n (\gamma\nu + s + 2 + nm) = 0 \qquad (n = 0, 1, 2, \ldots).$$

From eq.(10.22) it follows that all of these equations are satisfied if the function $\omega(t)$ is a solution of the ordinary differential equation

$$\omega'' - \left(\frac{\nu+s+1}{\nu+s} \right) \frac{\omega'^2}{\omega} = \frac{(\gamma-1)\nu(s+\nu)}{2} B \omega^{\frac{s+2+\nu\gamma}{s+\nu}}, \qquad (10.23)$$

where B is an arbitrary constant.

The general solution of eq.(10.23) can be represented in the form

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$t = \pm \frac{1}{s+\nu} \int_{\omega}^{} \frac{d\omega}{\omega^{1+\frac{1}{\nu+s}} \sqrt{A + B\omega^{\frac{(\gamma-1)\nu}{s+\nu}}}} = \pm \int \frac{d\mu}{\mu^{s} \sqrt{A + B\mu^{\nu(\gamma-1)}}}, \tag{10.24}$$

where A is an arbitrary constant and $\mu = \omega^{\frac{1}{\nu+s}}$ is a quantity which can be considered positive at $c_0 > 0$, since $\omega_0 > 0$ in its physical concept. The selection of the lower limit for the integral in eq.(10.24) is immaterial since it affects only the start of the time reading.

On the basis of eqs.(10.21), (10.22) and (10.23), the factoring of eq.(10.12) can be written in the following form:

$$p = \omega^{\frac{\gamma\nu}{s+2}} \left\{ c_{-1} + \frac{(\gamma-1)\nu}{2} B \left[\frac{c_0 \left(r\omega^{\frac{1}{s+\nu}} \right)^{s+2}}{s+2} + \frac{c_1 \left(r^{s+2}\omega^{\frac{s+2}{s+\nu}} \right)^{\frac{s+2+m}{s+2}}}{s+2+m} + \frac{c_2 \left(r^{s+2}\omega^{\frac{s+2}{s+\nu}} \right)^{\frac{s+2+2m}{s+2}}}{s+2+2m} + \ldots \right] \right\}, \tag{10.25}$$

which, in accordance with eq.(10.20) can be written:

$$p = \omega^{\frac{\gamma\nu}{s+\nu}} \left\{ c_{-1} + \frac{(\gamma-1)\nu}{2(s+2)} BP \left[\left(r\omega^{\frac{1}{s+\nu}} \right)^{s+2} \right] \right\}. \tag{10.25'}$$

Replacing ω with μ, the obtained solution can be written in the following form:

$$\left.\begin{aligned} dt &= \pm \frac{d\mu}{\mu^{s}(A + B\mu^{\nu(\gamma-1)})^{1/2}}, \\ \frac{dr}{dt} &= u = -\frac{1}{\mu}\frac{d\mu}{dt} \cdot r = \mp \mu \sqrt{A + B\mu^{\nu(\gamma-1)}} \cdot r, \\ \rho &= \mu^{\nu}(r\mu)^{s} P'[(r\mu)^{s+2}], \\ p &= \mu^{\gamma\nu} \left\{ c_{-1} + \frac{(\gamma-1)\nu}{2(s+2)} BP[(r\mu)^{s+2}] \right\}. \end{aligned}\right\} \tag{10.26}$$

The equations (10.26) define an exact solution of the system of equations (10.1)

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

(10.2), (10.3) dependent upon the arbitrary function $P(x)$*. It is easy to prove immediately that this solution occurs even in the case when the arbitrary function $P(x)$ is not broken down into a power series according to its argument.

Since the density ρ is positive, the function $P'(x)$ must satisfy the condition

$$P'(x) > 0$$

From the equations (10.26) it follows that

$$\frac{p}{\rho^{\gamma}} = \varphi(r_{\mu})$$

Since entropy is constant in a particle, the solution found (10.26) is characterized by the fact that

$$r = \frac{F(\xi)}{\mu(t)}$$

where ξ is a Lagrangian variable and the function $\mu(t) > 0$ is determined by eq.(10.24).

If ξ is the coordinate of a particle at some moment of time to which the value μ_0 corresponds, it is then obvious that the equality $F(\xi) = \mu_0 \xi$ is valid. Consequently, the law of motion of different particles of a gas has the form:

$$r = \frac{\mu_0}{\mu} \cdot \xi \qquad (10.27)$$

Equation (10.27) is obtained immediately from the second equation of (10.26).

From the last two equations of the solution (10.26) it follows that the pressure and density for any moment of time are connected by the relationship

---

*This solution was first published in our work, "On the Integration of Equations of the One-Dimensional Motion of a Gas", DAN, Vol.90, No.5, 1953, p.735.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\frac{dp}{dr} = B\frac{(\gamma-1)\nu}{2}\mu^{\nu(\gamma-1)+2}\cdot r\rho =$$
$$= B\frac{(\gamma-1)\nu}{2}\mu^{\nu\gamma+1}(\mu_0\xi)^{\nu+1}P'[(\mu_0\xi)^{\nu+2}]. \qquad (10.28)$$

The pressure gradient in a particle and in a given point in a space depends on time by way of μ. At $B > 0$ the pressure increases together with r; at $B < 0$ it drops with recession towards the periphery.

Three characteristic cases are possible for the relationship r(t):

I. $A > 0$, $B > 0$ } Pressure increases with recession from the

II. $A < 0$, $B > 0$ } center of symmetry.

III. $A > 0$, $B < 0$ Pressure drops with recession from the center of symmetry.

In case I it is possible to add the following form to eqs.(10.24) and (10.27):

$$\left.\begin{aligned} \tau = \sqrt{A}\cdot\mu_0\cdot t = \mp\int_0^y \sqrt{\frac{y^{\nu(\gamma-1)}}{1+y^{\nu(\gamma-1)}}}\,dy, \\ y = \frac{\mu_0}{\mu} = \frac{r}{\xi}, \text{ WHERE } \mu_0 = \left(\frac{A}{B}\right)^{\frac{1}{\nu(\gamma-1)}}. \end{aligned}\right\} \qquad (10.29)$$

From these equations it follows that the universal curves $y(\tau)$ for various ν, expressed in Fig.73 make it possible to obtain easily the law of motion for each particle at any $A > 0$ and $B > 0$.

In the case of $\tau = t = 0$, all the particles of the gas are concentrated in the center of symmetry. As t increases a scattering occurs, beginning with infinitely great speed. At $y = 1$ for different particles the coordinate r equals ξ. The corresponding moment of time is found from the relationship

$$\tau = \int_0^1 \sqrt{\frac{y^{\nu(\gamma-1)}}{1+y^{\nu(\gamma-1)}}}\,dy.$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

At $\tau \to \infty$ we have $y \to \infty$ and $\mu \to 0$; in this case the velocity of each particle within the boundary tends towards a finite value.

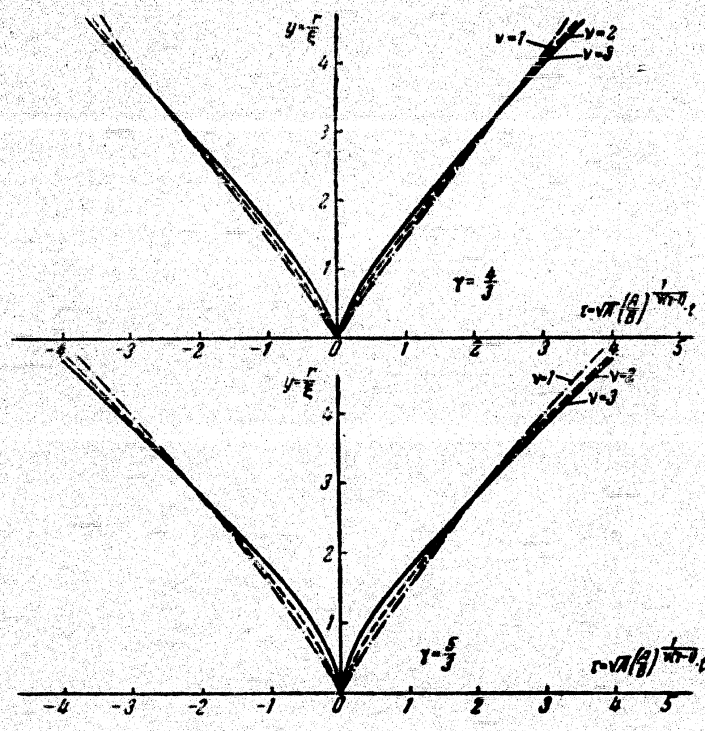


Fig.73 - Law of Motion of Gas Particles in Case I; $A > 0$, $B > 0$. At $t < 0$ there is focusing in the center; at $t > 0$ there is dispersion to infinity.

Since $\lim_{\tau \to \infty} \frac{dy}{d\tau} = 1$, then $\frac{dr}{dt} = \xi \sqrt{A}\, \mu_0$.

For each particle $r\mu$ = const. Hence, in accordance with eq.(10.26) it is obvious that density and pressure, at $t = 0$ and $\mu = \infty$, are equal to infinity; and at $t \to \infty$, when $\mu \to 0$, they tend toward zero.

In case II the following analogous form can be added to eqs.(10.24) and (10.27):

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\left.\begin{aligned} \tau = \sqrt{|A|}\,\mu_0 t = \mp \int_0^y \sqrt{\frac{y^{\nu(\gamma-1)}}{1-y^{\nu(\gamma-1)}}}\,dy, \\ y = \frac{\mu_0}{\mu} = \frac{r}{\xi}, \quad \text{где} \quad \mu_0 = \left(\frac{|A|}{B}\right)^{\frac{1}{\nu(\gamma-1)}}. \end{aligned}\right\} \qquad (10.30)$$

The corresponding universal curves for various ν are expressed in Fig.74. Close to the moment of time t = 0, cases I and II are completely analogous.

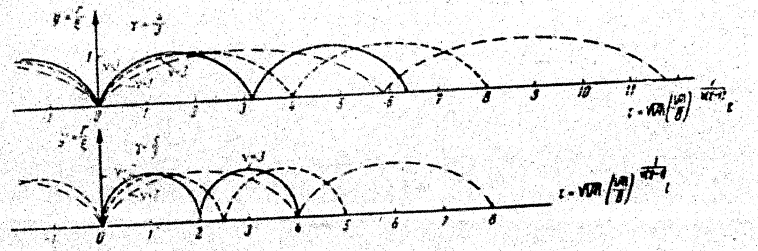


Fig.74 - Law of Motion of Gas Particles in Case II; A < 0, B > 0. Beginning from the moment t = 0, there is dispersion from the center, then a stop at a finite distance, after which there is focusing in the center.

However, later on, in the process of scattering, in contrast to case I, the coordinate r has a maximum value equal to $\frac{\xi}{\mu}$ (y = 1). This position is reached by all particles simultaneously at a moment of time τ* determined by the equation

$$\tau^* = \sqrt{|A|}\cdot\mu_0\cdot t^* = \int_0^1 \sqrt{\frac{y^{\nu(\gamma-1)}}{1-y^{\nu(\gamma-1)}}}\,dy.$$

At the moment τ * all the particles are arrested; and in the next moment of time, under the influence of the pressure gradient there arises a reverse motion towards the center as a result of which all the particles arrive at the center of symmetry

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

simultaneously at a moment of time $2\tau^*$.

With a further increase in time a repetition of this process can occur. An example of periodic pulsating motion of a gas is obtained.

Finally, in case III it is possible to add the following form to eqs.(10.24) and (10.27):

$$\left.\begin{aligned} \tau &= \sqrt{A}\left(\frac{A}{|B|}\right)^{\frac{1}{\nu(\gamma-1)}} t = \mp \int_1^y \sqrt{\frac{y^{\nu(\gamma-1)}}{y^{\nu(\gamma-1)}-1}}\, dy, \\ y &= \frac{\mu_0}{\mu} = \frac{r}{\xi}, \quad \text{где} \quad \mu_0 = \left(\frac{A}{|B|}\right)^{\frac{1}{\nu(\gamma-1)}}. \end{aligned}\right\} \qquad (10.31)$$

The corresponding universal curves $y(\tau)$ for various ν are expressed in Fig.75.

At $\tau = t = 0$ we have $y = 1$ and $\frac{d\mu}{dt} = 0$. The Lagrangian coordinate ξ represents the value of the coordinate r at that moment of time wherein all particles of the gas are at rest at the moment $t = 0$. At $t < 0$ the particles were moving toward the center of symmetry; at $t = 0$ a stop occurred; and then dispersion begins. In cases I and II there occurs a dispersion of the gas which at $t = 0$ is concentrated in the center of symmetry.

In the examined case III there is distributed in a space a mass of gas at rest, which, at the moment $t = 0$ begins to disperse since, in accordance with eq.(10.28), the pressure drops as r increases.

If on some sphere, the term $\left\{c_{-1} + \frac{(\gamma-1)\nu}{2(s+2)} B\rho\,[(r\mu)^{s+2}]\right\}$ is equated to zero, then on this movable sphere consisting of the identical particles of gas, the pressure will always be equal to zero. It is necessary to examine such a sphere as the external boundary, with a vacuum, of the dispersing mass of gas which is within the sphere.

Thus, the solution found permits finding in simple analytical form, the exact solution of the problem of dispersion of a gas in a vacuum in the case when the initial distribution of density and pressure is related by expression (10.28).

For an example, we shall give the functions P(x) the following partial form:

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$P(x) = \frac{\rho_0}{\mu_0^{\gamma}} \left(\frac{\gamma-1}{2}\right)^{\gamma} x^{\frac{2\gamma}{\gamma-1}} = \frac{\rho_0}{\mu_0^{\gamma}} \left(\frac{\gamma-1}{2}\right)^{\gamma} (r/t)^{\frac{2\gamma}{\gamma-1}}.$$

Then, for distribution of density and pressure the following equations are valid:

$$\rho = \rho_0 \left(\frac{r}{r_0}\right)^{\nu} = \frac{\rho_0}{\mu^{\nu}},$$

$$p = \left[\frac{1}{\gamma} - \frac{1}{2} \rho_0 \mu_0^{\gamma} (\gamma-1)^{\gamma} \left(\frac{\gamma-1}{2}\right)^{\gamma} (r/t)^2 \right] \mu^{\gamma\nu}.$$

The density has been found to be independent of geometrical coordinate and drops with passage of time. The pressure is distributed according to the parabolic law.

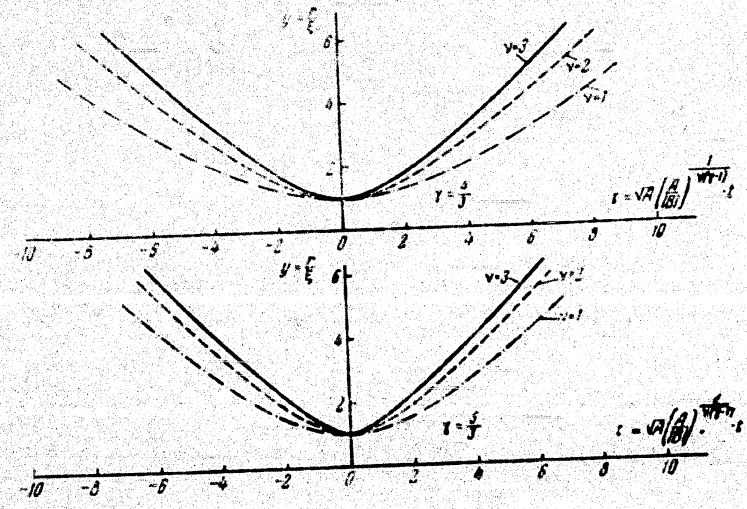


Fig.75 - Law of Motion of Gas Particles in Case III: A > 0 and B < 0. At t < 0, motion from infinity toward the center with stoppage of all particles at t = 0 at a finite distance from the center. At t > 0 there is dispersion to infinity.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The radius of the expanding sphere which is adjacent to a vacuum is defined by the equation

$$r^{\nu} = \sqrt{\frac{4c_{-1}}{\lambda\rho_0\nu(\gamma-1)\mu_0^{\gamma(\gamma-2)}}}\cdot\frac{1}{\mu}.$$

We shall again examine the case A = 0 or B = 0. If a = 0 and B > 0, then

$$\mu = \left[\pm\sqrt{B}\cdot t\left(1+\frac{\nu(\gamma-1)}{2}\right)\right]^{-\frac{1}{1+\frac{\nu(\gamma-1)}{2}}} = \mu_0\cdot\left(\frac{t}{t_0}\right)^{-\frac{2}{2+\nu(\gamma-1)}}. \tag{10.32}$$

Hence, $r\mu = \xi\mu_0$, $r = \xi\left(\frac{t}{t_0}\right)^{\frac{2}{2+\nu(\gamma-1)}}$, where μ_0 and t_0 are constants. The motion of the gas is self modeling and represents motion corresponding to case I. At $t_0 > 0$ we have dispersion; at $t_0 < 0$ we have motion toward the center of symmetry.

If B = 0 and A > 0, then

$$\mu = \mp\frac{1}{\sqrt{A}}\frac{1}{t} = \mu_0\frac{t_0}{t}, \quad r\mu = \xi\mu_0, \quad r = \xi\frac{t}{t_0}. \tag{10.33}$$

The corresponding motion is thus self modeling and belongs to type III. However, in this case at t = 0 all the particles of the gas concentrate in the center of symmetry. Each particle of the gas moves with a constant velocity equal to $\frac{\xi}{t_0}$.

In accordance with the equations (10.26) in this case we have:

$$p = c_{-1}\left(\frac{\mu_0 t_0}{t}\right)^{\gamma\nu}, \quad \rho = \left(\frac{\mu_0 t_0}{t}\right)^{\nu}\Phi(\mu_0\cdot\xi). \tag{10.34}$$

The pressure is constant throughout the space and drops with the passage of time; the density in each particle drops in inverse proportion to t^{ν}.

Exact solutions analogous to the solution (10.26) can be constructed in the problem of the motion of an ideal gas, taking into account the forces of Newtonian

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

attraction between particles of the gas*, if we assume $\gamma = \frac{4}{3}$. On the basis of this it is possible to draw the conclusion that the above-described types of laws for the motion of particles of a gas, which (types) are defined by pressure gradients, can maintain in the presence of forces of Newtonian attraction.

We shall revert now to eq.(10.15). The solution examined above occurs at any m and s.

If the conditions (10.7) are satisfied, then it is obvious that at $s + 2 < 0$ the solution (10.26) is a general solution of equation (10.15). This conclusion maintains its force also in the case when $s + 2 > 0$; but $s + 2 \neq km$ where k is some positive integer.

In cases when the conditions (10.7) are not satisfied and, in particular when $s = -2$, the appearance in eqs.(10.9) and (10.12) of terms containing ln r is possible.

If $s + 2 = km$, where $k \geq 1$ is an integer, then there will appear the corresponding terms of one order in the first and in the second bracketed terms, beginning with a term of the order of r^{km}. Equations (10.16), at $n = -1, 0 \ldots, (k-1)$ maintain their force and hence, in this case at $n < k$, eqs.(10.17), (10.18), (10.21) and (10.22) are valid.

At $n \geq k$ the following equations are obtained:

$$\left.\begin{aligned} \psi_0' + (\lambda m + \gamma\nu)\varphi_0\psi_0 + (\lambda m + \nu)\gamma\psi_{-1}\varphi_k = 0, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \psi_{n-k}' + \sum_{i=0}^{n-k}\varphi_i\psi_{n-k-i}(nm - im + i\gamma m + \gamma\nu) + (nm + \nu)\gamma\psi_{-1}\varphi_n = 0. \end{aligned}\right\} \quad (10.35)$$

These equations, together with the relationships (10.11) and (10.14) can be

---

* A corresponding solution has been studied by M.L.Lidov: DAN, 97, No.3, 1954, pp.409-411. Exact solutions of the system (10.1), (10.2), and (10.3) depending on two arbitrary functions were found by N.N.Kochina, DAN, 97, No.3, 1954, pp.407-408.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

studied as differential equations for the subsequent determination of $\omega_k, \omega_{k+1}, \ldots, \omega_n, \ldots$, if the function $\omega_0(t)$ is given.

In this manner, all the coefficients of $\omega_n(t)$, $\varphi_n(t)$ and $\psi_n(t)$ can be expressed through one function $\omega_0(t)$, the constants γ, k, m and the constants of integration. The indicated constants and the function $\omega_0(t)$ can be assigned arbitrarily.

If $\psi_{-1} = p(0,t) = 0$, then in eq.(10.15) the first bracketed term reverts to zero. Equating to zero of the second bracketed term is equivalent to the system of equations

$$\psi_n' + \sum_{i=0}^{n} \varphi_i \psi_{n-i} (s + 2 + \gamma\nu + nm + im(\gamma - 1)) = 0, \tag{10.36}$$

from which, taking into account eqs.(10.11) and (10.14), by subsequent integrations it is possible to find the functions $\omega_0(t)$, $\omega_1(t), \ldots, \omega_n(t), \ldots$; arbitrariness in the solution is obtained only owing to the constants of integration and the value of the index m. The series for density ρ eq.(10.6), for velocity u eq.(10.9) and for pressure p eq.(10.12) can be written in the form

$$\left.\begin{aligned} g &= \frac{\rho}{\rho_2} = \lambda^s (\alpha_0 + \alpha_1 \lambda^m + \alpha_2 \lambda^{2m} + \ldots), \\ f &= \frac{u}{u_2} = \lambda (\beta_0 + \beta_1 \lambda^m + \beta_2 \lambda^{2m} + \ldots), \\ h &= \frac{p}{\rho_2} = \gamma_{-1} + \lambda^{s+2} (\gamma_0 + \gamma_1 \lambda^m + \gamma_2 \lambda^{2m} + \ldots), \end{aligned}\right\} \tag{10.37}$$

where $\lambda = \frac{r}{r_2}$ and r_2, ρ_2, u_2 and p_2 are some characteristics and generally dependent upon the time of a quantity with a dimension corresponding to length, density, velocity and pressure. The dimensionless quantities α_i, β_i and γ_i can be examined in place of ω_i, φ_i and ψ_i as dimensionless functions of some parameter τ, which can be introduced in place of the time t.

In accordance with the foregoing, at $s + 2 = km$ ($k > 0$) all the coefficients of the series (10.37) can be expressed through the functions $r_2(\tau)$, $\rho_2(\tau)$, $p_2(\tau)$, $u_0(\tau)$, $\alpha_0(\tau)$. In particular, if in the nonsteady-state motion of the gas there is a shock

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

wave, then the values of the corresponding characteristics of motion at the shock wave can be taken in the capacity of r_2, ρ_2, u_2 and p_2.

If the motion of the gas is self modeling and all the characteristics of the motion as a whole, including r_2, ρ_2, u_2 and p_2 are defined as functions of the time t and of the constants, of which there are only two with independent dimensions

$$a = M^{\chi}T^{\zeta} \text{ and } b = L^{\eta}$$

(where χ, ζ and η are some real constants), then equations having the following form must be valid

$$\left.\begin{aligned} r_2 &= \delta_1 \frac{b}{t^{\eta}}, \qquad \rho_2 = \delta_2 a b^{-\chi-3} t^{\eta(\chi+3)-\zeta}, \\ u_2 &= \delta_3 \frac{b}{t^{\eta+1}}, \quad p_2 = \delta_4 a b^{-\chi-1} t^{\eta(\chi+1)-\zeta-2}. \end{aligned}\right\} \tag{10.38}$$

(Here δ_1, δ_2, δ_3 and δ_4 are abstract constants.) In this case all coefficients of α_i, β_i and γ_i do not depend on time and, hence, are constant abstract numbers.

At given δ_1, δ_2, δ_3 and δ_4, eq.(10.15) leads to algebraic equations which permit finding s and m, and permit expressing all the coefficients of α_i, β_i and γ_i through the constants α_0 and γ_{-1} (when $\gamma_{-1} \neq 0$). By the same token, while not limiting commonness, the abstract numbers δ_1, δ_2, δ_3 and δ_4 can be made equal to unity if we modify in like manner the determination of the constant coefficients of α_i, β_i and γ_i*. By making use of this, we find that eqs.(10.10) and (10.14) for self modeling of motion assume the form

$$\beta_n = -\frac{f_n}{s+\nu+nm}\frac{a_n}{a_0} - \frac{1}{a_0}\sum_{i=0}^{n-1} a_{n-i}\beta_i \tag{10.39}$$

---

* This is equivalent to altering the meaning of the dimensional parameters r_2, ρ_2, u_2 and p_2 while maintaining arbitrariness in selection of the constants a and b.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

and

$$\gamma_n = \frac{f_n(f_n-1)\,a_n}{(s+\nu+nm)(s+2+nm)} + \frac{(\nu+s+1+nm)}{s+2+nm}\sum_{k=0}^{n}\frac{f_k\beta_{n-k}a_k}{s+\nu+km}, \qquad (10.40)$$

where

$$f_n = \eta(3+s+nm+\chi)-\zeta \quad (n=0,\ 1,\ 2,\ \ldots). \qquad (10.41)$$

The basic equation (10.15) in this case takes the form

$$\gamma_{-1}\{\eta(\chi+1)-\zeta-2+\gamma\sum_{n=0}^{\infty}\beta_n(\nu+nm)\lambda^{nm}\}+\lambda^{s+2}\{\sum_{n=0}^{\infty}[(f_n-2)\gamma_n+ \qquad (10.42)$$
$$+\sum_{i=0}^{n}\beta_i\gamma_{n-i}(\gamma\nu+s+2+im(\gamma-1)+nm)]\lambda^{nm}\}=0.$$

Let $\gamma_{-1} \neq 0$ and $s+2 = km$ $(k \geq 1)$. Equation (10.42) is equivalent to the system of equations

$$\left.\begin{aligned}
&\eta(\chi+1)-\zeta-2+\gamma\nu\beta_0=0,\\
&\beta_1=0,\\
&\beta_2=0,\\
&\ldots\\
&\beta_{k-1}=0,\\
&\gamma\gamma_{-1}\beta_k(km+\nu)+(f_0-2)\gamma_0+\beta_0\gamma_0(km+\gamma\nu)=0,\\
&\ldots\\
&\gamma\gamma_{-1}(nm+\nu)+(f_{n-k}-2)\gamma_{n-k}+\\
&+\sum_{i=0}^{n-k}\beta_i\gamma_{n-k-i}(nm-im+i\gamma m+\gamma\nu)=0.\\
&\ldots
\end{aligned}\right\} \qquad (10.43)$$

From eq.(10.39) and the first of eqs.(10.43) we obtain

$$\beta_0 = -\frac{f_0}{s+\nu} = -\frac{[\eta(3+s+\chi)-\zeta]}{s+\nu}, \qquad (10.44)$$

$$\beta_0 = \frac{2+\zeta-\eta(\chi+1)}{\gamma\nu}. \qquad (10.45)$$

Since, in accordance with eq.(10.43), $\beta_1 = \beta_2 = \ldots = \beta_{k-1} = 0$ eqs.(10.39) for $n = 0, 1, 2, \ldots, k-1$ will yield:

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\beta_0 = -\left[\frac{\eta(3+s+nm+\chi)-\zeta}{s+\nu+nm}\right] \tag{10.46}$$

From this it follows that at $k > 1$ the following equality must be fulfilled:

$$\zeta = (3 - \nu + \chi)\,\eta \tag{10;47}$$

This is a supplementary condition imposed on the indexes in the dimension equations for the constants a and b. In this case, the index s remains arbitrary.

If condition (10.47) is not fulfilled, then $k = 1$ and, consequently

$$m = s+2 = \frac{\eta[\nu(\gamma-1)+2](\gamma+1)+(\nu-2)(\zeta+2)-\gamma\nu^2}{\eta(\chi+1-\gamma\nu)-\zeta-2}. \tag{10.48}$$

It is obvious that at $k = 1$ the constants a_0 and γ_{-1} remain arbitrary; the remaining constant coefficients in the analyses (10.37) are subsequently easily determined with the help of eqs.(10.39, (10.40) and (10.43).

If condition (10.47) is satisfied, then $m = \frac{s+2}{k}$, wherein k, s, a_0, a_1, ..., a_{k-1} and γ_{-1} remain arbitrary, $\beta_1 = \beta_2 = \ldots \beta_{k-1} = 0$; the remaining coefficients are determined from eqs.(10.39) (10.40) and (10.43).

If $\gamma_{-1} \neq 0$ and $s + 2 \neq km$ where k is an integer, then from eq.(10.42) we find:

$$\beta_0 = -\frac{f_0}{s+\nu} = \frac{\zeta-\eta(3+s+\chi)}{s+\nu}, \quad \beta_1 = \beta_2 = \ldots = \beta_n = \ldots = 0$$

and in addition, we obtain:

$$\gamma_n\left\{\frac{\zeta-\eta(3-\nu+\chi)}{s+\nu}nm - \frac{[-\zeta+\eta(3+s+\chi)][2+\nu(\gamma-1)]+2(s+\nu)}{s+\nu}\right\} = 0 \tag{10.49}$$

$$(n = 0, 1, 2, \ldots).$$

Equations (10.40) in this case assume the form

$$\gamma_n = \frac{a_n f_n}{(s+2+nm)(s+\nu+nm)}\left[f_n - 1 - \frac{(\nu+s+1+nm)f_0}{s+\nu}\right].$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In the general case eqs.(10.49) can be satisfied at arbitrary s, m and a_n ($n = 0, 1, 2, \ldots$), if the indexes ζ and η satisfy the relationships*:

$$\eta = -\frac{1}{1+\frac{\nu(\gamma-1)}{2}} \quad \text{и} \quad \zeta = \eta(3-\nu+\chi); \tag{10.49a}$$

the constant Y_{-1} may also be arbitrary. The corresponding solution is represented by the partial case of self-modeling motion in the exact solution (10.26).

If $Y_{-1} = 0$ and the pressure $p \neq 0$, then the index s is determined by the equation

$$s = \frac{-2\nu-[\eta(3+\gamma)-\zeta][2+\nu(\gamma-1)]}{\eta[2+(\gamma-1)\nu]+2}.$$

The coefficients a_0, a_1 and the index m remain indeterminate; all remaining coefficients are determined by eqs.(10.39), (10.40) and (10.42).

II. Concerning Coordinate Derivatives of u, ρ, p, T, S in Characteristic Movable Points of a One-Dimensional Nonsteady-State Motion of a Gas

We shall examine some movable point M, moving in a fixed space at a velocity c. With the help of eqs.(1.2) it is easy to express the primary derivatives on the coordinate r, of u, ρ, p, temperature T and entropy S through their values and the values of their derivatives in time t in the movable point M.

The following equations are valid for the arbitrary function F (r,t):

$$\frac{dF}{dt} = \frac{d}{dt}F(r_M(t), t) = \frac{\partial F}{\partial t} + c\frac{\partial F}{\partial r}, \text{ где } c = \frac{dr_M}{dt}.$$

On the basis of this relationship, eqs.(1.2) can be represented in the form

* It is obvious that in certain special cases the system (10.49) can be satisfied when the relationships (10.49a) are not satisfied.

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\left.\begin{aligned} \frac{\partial u}{\partial r}\rho + \frac{\partial \rho}{\partial r}(u-c) &= -\frac{(\nu-1)\rho u}{r} - \frac{d\rho}{dt}, \\ \frac{\partial u}{\partial r}(u-c) + \frac{1}{\rho}\frac{\partial p}{\partial r} &= -\frac{du}{dt}, \\ \frac{\partial S}{\partial r}(u-c) &= -\frac{dS}{dt}. \end{aligned}\right\} \qquad (10.50)$$

In addition to these relationships, from the equation of state in the form $p = p(\rho,S)$ and in the form $T = T(\rho,S)$ we have:

$$\left.\begin{aligned} \frac{\partial p}{\partial r} &= \left(\frac{\partial p}{\partial \rho}\right)_S \frac{\partial \rho}{\partial r} + \left(\frac{\partial p}{\partial S}\right)_\rho \frac{\partial S}{\partial r}, \\ \frac{\partial T}{\partial r} &= \left(\frac{\partial T}{\partial \rho}\right)_S \frac{\partial \rho}{\partial r} + \left(\frac{\partial T}{\partial S}\right)_\rho \frac{\partial S}{\partial r}. \end{aligned}\right\} \qquad (10.51)$$

After solution of the linear system of eqs.(10.50) and (10.51) relative to the partial derivatives in r, we arrive at equations valid in the movable point M for adiabatic motions of any gas:

$$\left.\begin{aligned} \frac{\partial u}{\partial r} &= -\frac{(\nu-1)\cdot\rho u\cdot\left(\frac{\partial p}{\partial \rho}\right)_S}{r\cdot\Delta} - \frac{1}{\Delta}\left[(c-u)\rho\frac{du}{dt} + \frac{dp}{dt}\right], \\ \frac{\partial \rho}{\partial r} &= -\frac{(\nu-1)\rho^2 u(c-u)}{r\cdot\Delta} - \\ &\quad -\frac{1}{\Delta}\left[(c-u)\rho\frac{d\rho}{dt} + \rho^2\frac{du}{dt} + \left(\frac{\partial p}{\partial S}\right)_\rho \frac{\rho}{c-u}\frac{dS}{dt}\right], \\ \frac{\partial p}{\partial r} &= -\frac{(\nu-1)\rho^2 u(c-u)\left(\frac{\partial p}{\partial \rho}\right)_S}{r\cdot\Delta} - \\ &\quad -\frac{1}{\Delta}\left[(c-u)\rho\frac{d\rho}{dt} + \rho^2\frac{du}{dt} - (c-u)\rho\left(\frac{\partial \rho}{\partial S}\right)_p\frac{dS}{dt}\right]\left(\frac{\partial p}{\partial \rho}\right)_S, \\ \frac{\partial T}{\partial r} &= -\frac{(\nu-1)\rho^2 u(c-u)\left(\frac{\partial T}{\partial \rho}\right)_S}{r\cdot\Delta} - \\ &\quad -\frac{1}{\Delta}\left[\rho^2\left(\frac{\partial T}{\partial \rho}\right)_S\frac{dU}{dt} + (c-u)\rho\frac{dT}{dt} - \right. \end{aligned}\right\} \qquad (10.52)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$-\frac{\rho}{c-u}\left(\frac{\partial p}{\partial \rho}\right)_T\left(\frac{\partial T}{\partial S}\right)_\rho \frac{dS}{dt}\Big],$$

$$\frac{\partial S}{\partial r}=\frac{1}{(c-u)}\frac{dS}{dt}, \quad \text{or} \quad \Delta=\rho\left[\left(\frac{\partial p}{\partial \rho}\right)_S-(c-u)^2\right].$$

These equations can be applied to the case when the point M coincides with the front of a strong discontinuity. On the basis of the Ciugonio conditions all the quantities behind the jump can be expressed in the parameters of the states through which the jump propagates, and in the velocity $c(r_2)=\frac{dr_2}{dt}$, which is determined by the law of motion of the jump (r_2 is the coordinate of the jump).

In particular, we shall examine the case when a shock wave propagates from a center ($c>0$) through an ideal gas at rest with density ρ_1 and pressure p_1. As before, we shall designate

$$\frac{a_1^2}{c^2}=\frac{\gamma p_1}{\rho_1 c^2}=q, \quad \frac{v}{v_2}=f, \quad \frac{\rho}{\rho_2}=g, \quad \frac{p}{p_2}=h.$$

Having made use of the conditions at the shock wave (8.1), eqs.(10.52) can be converted to the following form:

$$\left.\begin{aligned}
r_2\frac{\partial f}{\partial r}&=-\frac{(\nu-1)\left[1+\frac{\gamma-1}{\gamma+1}(1-q)\right]}{1-q}+\frac{3+q}{2q(1-q)^2}\cdot r_2\frac{dq}{dr_2},\\
r_2\frac{\partial g}{\partial r}&=-\frac{2(\nu-1)}{\gamma+1}-\\
&-\frac{1}{\gamma+1-2(1-q)}\left[\frac{2\left[1+\frac{\gamma-1}{\gamma+1}(1-q)\right]}{1-\frac{2}{\gamma+1}(1-q)}-\frac{3(1+q)}{q}\right]\frac{r_2}{1-q}\frac{dq}{dr_2},\\
r_2\frac{\partial h}{\partial r}&=-\frac{2\gamma(\nu-1)}{\gamma+1}+\\
&+\frac{\gamma}{q}\left[\frac{q+1}{\gamma+1-2(1-q)}+\frac{2}{\gamma+1+(\gamma-1)(1-q)}\right]\frac{r_2}{1-q}\frac{dq}{dr_2},\\
r_2\frac{\partial S}{\partial r}&=-\frac{2c_p(1-q)^2(\gamma^2-1)}{[\gamma+1-2(1-q)]^2[\gamma+1+(\gamma-1)(1-q)]\cdot q}\cdot r_2\frac{dq}{dr_2}.
\end{aligned}\right\} \qquad (10.53)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In the same manner, it is possible to obtain equations for second derivatives analogous to eqs.(10.52) and (10.53).

From eq.(10.53) it follows that if the jump moves with a constant velocity c in a nonturbulent medium having a constant temperature T_1, then q = const., and hence

$$\left.\begin{aligned} \frac{\partial f}{\partial \lambda} &= -(\nu-1)\frac{\left[1+\frac{\gamma-1}{\gamma+1}(1-q)\right]}{1-q} < 0, \\ \frac{\partial g}{\partial \lambda} &= -\frac{2(\nu-1)}{\gamma+1} < 0, \\ \frac{\partial h}{\partial \lambda} &= -\frac{2\gamma(\nu-1)}{\gamma+1} < 0, \\ \frac{\partial S}{\partial \lambda} &= 0, \\ \lambda &= \frac{r}{r_2}. \end{aligned}\right\} \qquad (10.54)$$

Equating to zero is achieved only in the case of plane waves ν = 1. In the case of motion of a plane shock wave with constant velocity, the derivatives of the characteristics of turbulent motion of the gas behind the front are equal to zero.

In the case of spherical and cylindrical waves at q = const., only $\frac{\partial S}{\partial r} = 0$; the derivatives $\frac{\partial v}{\partial r}$, $\frac{\partial \rho}{\partial r}$, $\frac{\partial \rho}{\partial r}$ are other than zero but tend toward zero as r_2 increases, as $\frac{A}{r_2}$ [A is a suitable constant; see eq.(10.54)].

If the velocity of the shock wave c drops, then the quantity $q = \frac{a_1^2}{c^2}$ increases and, hence, $\frac{dq}{dr_2} > 0$. In this case, the direction of tangents to the corresponding curves in Fig.76, which (direction) is shown for constant velocity c, turns counterclockwise.

It is obvious that at a given q, the intensity of attenuation of a compression jump increases as the values of the derivatives $\frac{\partial f}{\partial \lambda}$, $\frac{\partial g}{\partial \lambda}$, $\frac{\partial h}{\partial \lambda}$ increase. As the shock wave recedes from the center of symmetry, near which, over a finite time interval, a turbulence with finite energy has formed, attenuation of the jump occurs whereby the jump propagation velocity c tends towards the speed of sound, and hence q → 1.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Asymptotic laws of attenuation of a shock wave can be determined with the help of eq.(10.53) if the limit behavior of the left terms is known at $q \to 1$.

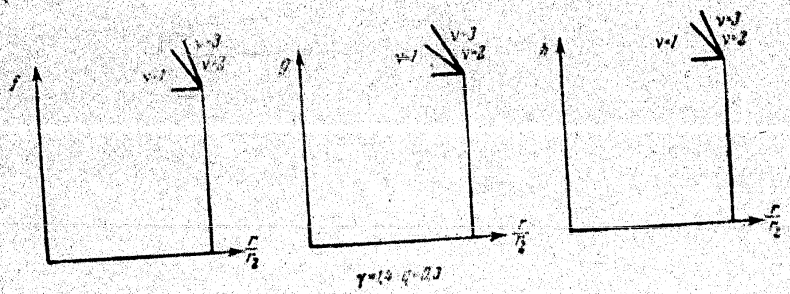


Fig.76 - Elements of the Velocity Distribution Curves for Gas Behind a Shock Wave Front under the Condition of Constant Velocity of the Front, in the Plane, Cylindrical and Spherical Cases

Approximate solutions* for problems of the nonsteady-state motion of gas within a shock wave ($0 < r < r_2$) in certain cases can be constructed with the help of interpolation equations for f, g, h or for any other functions expressed in terms of f, g, h.

It is possible to introduce into the interpolation equations as parametric the unknown functions $w_0(t)$ and $q(t)$, which determine the behavior of the solution close to the center of symmetry and close to the shock wave.

To determine these functions it is possible to use a different type of integral relationships in the same manner as in the methods of Hitts and Galerkin and in the approximation theories of the boundary layer.

Conditions at the shock wave [eqs.(10.53)] and their generalization for derivatives on the radius r of an order higher than the first, permit calculation of gas motion behind the jump front if the law of motion of the jump is known. (Motion of

---

* See Sedov, L.I., DAN SSSR, 1952 Vol.85, No.4.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the jump - shock wave - can be determined experimentally or on the basis of additional assumptions.)

Section 11. Asymptotic Laws of Attenuation of Shock Waves

In the general case of turbulent motion of a gas bounded by a shock wave being propagated through a gas at rest, the asymptotic laws of behavior of the velocity of the shock wave as a function of the coordinate of the shock wave r_2, and consequently also the change in intensity of the shock wave can be very diverse, and depend, essentially, upon the conditions determining the motion of the gas inside the shock wave.

The laws of degeneration of plane shock waves were studied and were established as early as 1913 by Crussard* in the proposition that turbulent motion of a gas behind the front of a jump represents a Riman traveling wave containing a point in which the velocity of the gas equals zero.

It was demonstrated by Riman** (for one-dimensional motions of a gas with plane waves in which the gas pervades the entire space) that if initial turbulences were continuous and distributed over a finite section along the x axis, that during continuous motion, at some finite time, the initial turbulences are transformed into two traveling waves which propagate in different directions. If in the traveling wave propagating in the positive direction of the x axis, at some moment of time the motion of the gas is continuous and has intervals at which the pressure drops as the x coordinate increases, thus in the traveling wave, at the expense of upsetting the wave there arise shock waves - compression jumps.

The attenuation of spherical and cylindrical shock waves was established for

* See Crussard, Compt.Rend. 156, 447, 611, 1913. In the book by Ya.B.Zel'dovich, "Introduction to the Theory of Shock Waves and Gas Dynamics", 1948, there is a bibliography of works devoted to the problem of attenuation of plane shock waves.

** See Riman, B., Works, Costekhizdat, Moscow, 1948, p.376.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the first time by L.D.Landau* in 1945 in the proposition that the turbulent motion of a gas behind the front of a shock wave is weakened and that this motion tends towards a traveling wave in which turbulences at a fixed moment of time are situated at an interval of finite length, and which differs from an acoustical wave only by the more specific value of the velocity of sound.

Subsequently, a great deal of work appeared on this problem in which there was studied the attenuation of spherical and cylindrical waves in the same or analogous propositions to that of Landau. In the work of Landau it was also shown that the corresponding methods, reasoning and results are carried over directly to the case of attenuation of curvilinear shock waves which form during flow past bodies by a supersonic advancing flow of gas in the plane-parallel and axisymmetrical cases.

Below is given the theory of attenuation of shock waves on the basis of eqs.(10.53) for derivatives behind the shock wave front.

For the system (1.2) on characteristics along which turbulences are propagated in the direction of growth of the coordinate r, the following differential relationship is valid:

$$dr - (a + u)\,dt = 0 \qquad (11.1)$$

The equation for these characteristics in the plane r,t can be written in the form

$$\xi(r,t) = \text{const.}$$

in which the function $\xi(r,t)$ is determined on the basis of the equality

$$\mu\,d\xi = dr - (a + u)\,dt$$

Hence

$$dt = \frac{dr - \mu\,d\xi}{a + u} \qquad (11.2)$$

* Landau, L.D., Shock Waves at a Great Distance from Their Point of Origin, Prikl. Mat.i Mekh., Vol.9, No.4, 1945.

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

where μ is some integrating factor.

Instead of the independent variables r,t we shall introduce the independent variables r, ξ. The equations for substitution of the partial derivatives have the form

$$\frac{\partial f(r,t)}{\partial t} = -\frac{\partial f(\xi,r)}{\partial \xi}\frac{(a+u)}{\mu}, \quad \frac{\partial f(r,t)}{\partial r} = \frac{\partial f(\xi,r)}{\partial r} + \frac{\partial f(\xi,r)}{\partial \xi}\cdot\frac{1}{\mu}.$$

Using this, the equations of motion (1.2) can be written in the following form:

$$\left.\begin{aligned} a\frac{\partial u}{\partial \xi} - \frac{1}{\rho}\frac{\partial p}{\partial \xi} &= \mu\left(u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r}\right), \\ a\frac{\partial \rho}{\partial \xi} - \rho\frac{\partial u}{\partial \xi} &= \frac{\mu}{r^{\nu-1}}\frac{\partial r^{\nu-1}\rho u}{\partial r}, \\ a\frac{\partial}{\partial \xi}\left(\frac{p}{\rho^{\gamma}}\right) &= \mu u\frac{\partial}{\partial r}\frac{p}{\rho^{\gamma}}. \end{aligned}\right\} \qquad (11.3)$$

The condition of integration (11.2) gives yet another equation

$$\frac{\partial (a+u)}{\partial \xi} = (a+u)\frac{\partial \mu}{\partial r} - \mu\frac{\partial (a+u)}{\partial r}. \qquad (11.4)$$

It is easy to see that while $\nu = 1$ in the case of plane waves, the system (11.3) and eq.(11.4) admit a solution in which u, ρ, p depend only on ξ. This solution is the traveling wave of Riemann, it can be presented in the following form:

$$\left.\begin{aligned} p &= \frac{p_1}{\rho_1^{\gamma}}\rho^{\gamma}, \quad a^2 = \frac{\gamma p_1}{\rho_1^{\gamma}}\rho^{\gamma-1}, \quad u = \frac{2}{\gamma-1}(a-a_1), \\ \mu &= \frac{d\ln(a+u)}{d\xi}\cdot r + \left[\frac{F(u)}{a+u}\right]_i'(a+u) = \\ &= \frac{d\ln(a+u)}{d\xi}\cdot r + \left[\frac{\Omega(\xi)}{a+u}\right]_i'(a+u) \end{aligned}\right\} \qquad (11.5)$$

and $\quad r-(a+u)t = F(u) = \Omega(\xi)$ *if* $u = F[r-(a+u)t] = \Phi(\xi)$,

where $F(u) = \Omega(\xi)$, and consequently, also $F[r-(a+u)t] = \Phi(\xi)$ are arbitrary functions of their own arguments (p_1 and ρ_1 are constants).

In particular, in this case, while not disturbing commonness, it can be

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

assumed

$$\Omega(\xi) = \xi = r - (a + u)\,t \tag{11.6}$$

while preserving the function $F(u)$ and correspondingly the inverse function $\Phi(\xi)$ as arbitrary functions. For the derivative $\left(\frac{\partial u}{\partial r}\right)_{t\,=\,\text{const.}}$ on the basis of eqs.(11.5) and (11.6) we have

$$\frac{\partial u}{\partial r} = \Phi'(\xi)\left[1 - \frac{\partial (a+u)}{\partial r}\Big|_{t=\text{const.}} \cdot t\right] = \Phi'(\xi)\left[1 - \frac{\gamma+1}{2}\frac{\partial u}{\partial r}\cdot t\right].$$

Hence

$$r\frac{\partial u}{\partial r} = \frac{\Phi'(\xi)\cdot r}{1 + \frac{\gamma+1}{2}\,t\Phi'(\xi)}. \tag{11.7}$$

If at $r_2 \sim \infty$ the shock wave degenerates into a sound wave, and the motion of the gas behind the jump front degenerates into Riman flow, then $c \to 1$ and $\frac{a_1 t}{r_2} \to 1$.

We shall designate by ξ_0 the boundary value of ξ at the shock wave in the case where $r_2 \to \infty$. Since according to the proposition, the shock wave degenerates into a sound wave and it is obvious that while $\xi \to \xi_0$ it must follow that $u \to 0$ and consequently

$$\Phi(\xi_0) = 0$$

Further we shall find the asymptotic law of attenuation of a shock wave under the assumption that

$$\lim_{r_2 \to \infty} r_2\Phi'(\xi) \to \infty, \tag{11.8}$$

If $\Phi'(\xi_0) \neq 0$, then the limiting relationship (11.8) is not an assumption.

If

$$u = \Phi(\xi) = c\cdot(\xi - \xi_0)^n + \ldots,$$

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

where c and $n > 1$ are constants, then $\Phi'\left(\frac{x}{t}\right) = c\left(\frac{x}{c}\right)^{\frac{n-1}{n}} + \ldots$; hence, the assumption (11.8) is equivalent to the condition

$$r_2 u^{\frac{n-1}{n}} \longrightarrow \infty,$$

which will be fulfilled if the velocity u behind the front of the jump decreases more slowly than

$$\frac{k}{r_2^{1+\frac{1}{n-1}}}.$$

Further, we shall show that the assumption (11.8) leads to a law of decreasing velocity which is slower than $\frac{k}{r^{1+\frac{1}{n-1}}}$; therefore, in this case the assumption (11.8) is likewise immaterial.

On the basis of the equations (10.53) and (11.7) at $\nu = 1$ we will obtain the asymptotic equation

$$\frac{2a_1}{\gamma+1} \approx \frac{4a_1}{\gamma+1}\frac{r_2}{1-q}\frac{dq}{dr_2}.$$

Hence, after integration we find

$$1-q \approx \sqrt{\frac{r_0}{r_2}} \text{ when } a_1^2\left(\frac{dt}{dr_2}\right)^2 \approx 1-\sqrt{\frac{r_0}{r_2}}, \qquad (11.9)$$

where r_o is a constant.

On the basis of eq.(11.9) and the conditions at the jump (8.1) we obtain asymptotic equations for the law of motion of the jump and for the characteristic motion of the gas behind the jump.

$$\left.\begin{aligned} a_1(t-t_0) &= r_2\left(1-\sqrt{\frac{r_0}{r_2}}-\frac{1}{8}\frac{r_0\ln r_2}{r_2}+\ldots\right), \\ v_2 &= \frac{2a_1}{\gamma+1}\sqrt{\frac{r_0}{r_2}}+\ldots, \\ \rho_2 &= \rho_1+\frac{2\rho_1}{\gamma+1}\sqrt{\frac{r_0}{r_2}}+\ldots, \end{aligned}\right\} \qquad (11.10)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$p_2 = p_1 + \frac{2}{\gamma+1}\rho_1 \sqrt{\frac{r_0}{r_2}} + \dots \Bigg]$$

The equations (11.10) occur in accordance with the solution (11.5) and give the sought asymptotic laws for plane waves. From eq.(11.10) it is obvious that the condition (11.8) is satisfied by any n.

In the case of spherical or cylindrical waves, the equations (11.3) at $\mu = 1$ after linearization in the vicinity of a nonturbulent state of rest with pressure p_1 and density ρ_1 acquire the form

$$\left.\begin{aligned} a_1 \frac{\partial u}{\partial \xi} - \frac{1}{\rho_1}\frac{\partial p}{\partial \xi} &= \frac{1}{\rho_1}\frac{\partial p}{\partial r}, \\ a_1 \frac{\partial \rho}{\partial \xi} - \rho_1 \frac{\partial u}{\partial \xi} &= \frac{\rho_1}{r^{\nu-1}}\frac{\partial r^{\nu-1} u}{\partial r}, \\ \frac{p}{p_1} = \left(\frac{\rho}{\rho_1}\right)^{\gamma} \quad &if \quad \frac{p-p_1}{p_1} = \gamma\frac{\rho-\rho_1}{\rho_1}. \end{aligned}\right\} \tag{11.11}$$

In contrast to the case of plane waves at $\nu = 1$, the equations (11.11) have no solutions depending solely on the variable ξ.

We shall examine the solutions of eqs.(11.11) for traveling waves in the case where the velocity u of turbulent motion is represented by the series

$$u = a_1\left[\frac{\Phi_1(\xi)}{r^m} + \frac{\Phi_2(\xi)}{r^{2m}} + \dots\right], \tag{11.12}$$

where $\Phi_1(\xi)$ is an arbitrary function and $m > 0$ is a fixed number subject to determination.

We shall assume

$$\rho = \rho_1\left[1 + \frac{\Psi_1(\xi)}{r^m} + \frac{\Psi_2(\xi)}{r^{2m}} + \dots\right]. \tag{11.13}$$

The last of eqs.(11.11) gives

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$p = p_1 \left[1 + \frac{\gamma \Psi_1(\xi)}{r^m} + \frac{\gamma \Psi_2(\xi)}{r^{2m}} + \dots \right]. \qquad (11.14)$$

Substitution of these series in the first two equations of eq.(10.11) leads to the relationships

$$\left.\begin{aligned} &\frac{\Phi_1' - \Psi_1'}{r^m} + \frac{\Phi_2' - \Psi_2'}{r^{2m}} + \dots - \frac{m\Psi_1}{r^{m+1}} + \frac{2m\Psi_2}{r^{2m+1}} + \dots = 0, \\ &\frac{\Phi_1' - \Psi_1'}{r^m} + \frac{\Phi_2' - \Psi_2'}{r^{2m}} + \dots \\ &\dots - \frac{(m+1-\nu)\Phi_1}{r^{m+1}} - \frac{(2m+1-\nu)\Phi_2}{r^{2m+1}} - \dots = 0. \end{aligned}\right\} \qquad (11.15)$$

Hence, it follows that

$$\Phi_1 = \Psi_1 + C \qquad (11.16)$$

where C is the constant of integration.

Inasmuch as the function Φ_1 is arbitrary, it is necessary that in eqs.(11.15) the terms of the series m + 1, 2m + 1, ... be factored with one of the terms of the series 2m, 3m, ... Consequently, it must follow that

$$m + 1 = km$$

where k is an integer.

Equating to zero in eq.(11.15) of the coefficients at r^{km} gives

$$\left.\begin{aligned} &\Phi_k' - \Psi_k' + m\Psi_1 = 0, \\ &\Phi_k' - \Psi_k' - (m+1-\nu)\Phi_1 = 0. \end{aligned}\right\}$$

From this and from eq.(11.16) it follows that

$$2m + 1 - \nu = 0 \text{ and } C = 0$$

Thus, we find that in the case of cylindrical waves at $\nu = 2$, when $m = 0.5$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

and k = 3, the following equations are valid

$$\Psi_n = \frac{2-n}{n}\Phi_n, \quad \Phi'_{n+2} = \frac{n^2-4}{4(n+1)}\Phi_n, \quad \Phi'_2 = 0, \tag{11.17}$$

wherein the function $\Phi_1(\xi)$ remains arbitrary, the functions $\Phi_{2n+1}(\xi)$ with odd values of the index (n = 1, 2, 3, ...) are expressed in terms of the function $\Phi_1(\xi)$ by means of the n-multiple indeterminate integral

$$\int^{\xi}\int^{\xi_1}\int^{\xi_2}\cdots\int^{\xi_{n-1}}\Phi_1 d\xi_{n-1}\cdots d\xi_1\, d\xi,$$

the functions $\Phi_{2n}(\xi)$ are polynomials in ξ to the power n - 2; each subsequent polynomial is obtained by integration of the preceding according to the recurrent eq.(11.17).

In the case of spherical waves at $\nu = 3$ the following equations are valid:

$$\left.\begin{array}{ll} m=1, & k=2, \\ \Psi_n = \frac{2-n}{n}\Phi_n & (n=1, 2, 3, \ldots), \\ \Phi'_n = \frac{n(n-3)}{2(n-1)}\Phi_{n-1} & (n=2, 3, 4, \ldots). \end{array}\right\} \tag{11.18}$$

The function $\Phi_1(\xi)$ remains arbitrary; from the equations (11.18) it follows that

$$\Phi_2(\xi) = -\int \Phi_1(\xi)\, d\xi$$

and that the consequent functions $\Phi_n(\xi)$ at $n > 2$ are polynomials in ξ to the power n - 3; each consequent polynomial results from the preceding by integration according to the recurrent eq.(11.18).

Thus, for the linearized equations there is determined a class of solutions, represented by the series (11.12), (11.13) and (11.14) which contain an arbitrary function $\Phi_1(\xi)$ and arbitrary constants, emerging during the determination of the coefficients at higher powers $\frac{1}{r}$.

From the equations (11.12), (11.13) and (11.14) it follows that for the quan-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

tity a + u the following solution is valid:

$$a+u=\sqrt{\frac{\gamma p}{\rho}}+u=a_1\left[1+\frac{\gamma+1}{2}\cdot\frac{\Phi_1(\xi)}{\frac{\nu-1}{2}}+\ldots\right]. \tag{11.19}$$

In the acoustic approximation we can examine eqs.(11.11) independent of eq.(11.4) if we assume that $\mu = 1$ and $\xi = r - a_1 t$, which is equivalent to replacing, in eq.(11.4), the quantity a + u by a_1. On the basis of eq.(11.19) from eq.(11.4) and (11.2), we can find more exact expressions, corresponding to the first term of the solutions for the functions u, p, ρ in eqs.(11.12), (11.13) and (11.14).

In the course of solution of the nonlinear equations (11.3), the analyses of eqs.(11.12), (11.13) and (11.14) vary; however, the first terms retain their form with the more precise $\mu(\xi, r)$ and retain the connection between t, r and ξ. These relationships in the case of cylindrical waves at $\nu = 2$ have the form:

$$\left.\begin{aligned}\mu&=1+\sqrt{r}\left[(\gamma+1)\Phi_1'(\xi)+O\left(\frac{\ln r}{\sqrt{r}}\right)\right],\\ at+\text{const.}&=r-\sqrt{r}\left[(\gamma+1)\Phi_1(\xi)-\frac{(\gamma+1)^2}{4}\Phi_1^2(\xi)\frac{\ln r}{\sqrt{r}}+\ldots\right].\end{aligned}\right\} \tag{11.20}$$

In this case and later on O(f (r)) designates quantities which tend toward zero as r increases; the order of tendency to zero equals f (r).

From eqs.(11.2) and (11.20) for all $\Phi'_1(\xi) \neq 0$ it follows that

$$\left.\frac{\partial\xi}{\partial r}\right|_{\mu=\text{const.}}=\frac{1}{\mu}=\frac{1}{(\gamma+1)\Phi_1'(\xi)\sqrt{r}}\left[1+O\left(\frac{\ln r}{\sqrt{r}}\right)\right]. \tag{11.21}$$

Analogous equations in the case of spherical waves for $\nu = 3$ have the form

$$\left.\begin{aligned}\mu&=1+\frac{\gamma+1}{2}\Phi_1'(\xi)\ln r+O\left(\frac{\ln r}{r}\right),\\ at+\text{const.}&=r-\frac{\gamma+1}{2}\Phi_1(\xi)\ln r+O\left(\frac{\ln r}{r}\right)\end{aligned}\right\} \tag{11.22}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

and at $\Phi'(\xi) \neq 0$

$$\left.\frac{\partial \xi}{\partial r}\right|_{t=\text{const.}} = \frac{1}{\mu} = \frac{2}{(\gamma+1)\Phi_1'(\xi)\ln r}\left[1+O\left(\frac{1}{\ln r}\right)\right]. \qquad (11.23)$$

From eq.(11.12) it follows that

$$\left.\begin{aligned} r\frac{\partial u}{\partial r}\Big|_{t=\text{const.}} &= a_1\frac{\Phi_1'(\xi)}{\sqrt{r}}\left(\frac{\partial \xi}{\partial r}\right)_{t=\text{const.}}\left[1+O\left(\frac{1}{\sqrt{r}}\right)\right] \quad \text{where } \nu=2,\\ r\frac{\partial u}{\partial r}\Big|_{t=\text{const.}} &= a_1\frac{\Phi_1'(\xi)}{r}\left(\frac{\partial \xi}{\partial r}\right)_{t=\text{const.}}\left[1+O\left(\frac{\ln r}{r}\right)\right] \quad \text{where } \nu=3. \end{aligned}\right\} \qquad (11.24)$$

For asymptotic laws in the case of cylindrical waves ($\nu = 2$), from eqs.(10.53), (11.21) and (11.24) we obtain the asymptotic equation

$$\frac{a_1}{\gamma+1} = -\frac{2a_1}{\gamma+1} + \frac{4a_1}{\gamma+1}\frac{r_2}{1-q}\frac{dq}{dr_2}.$$

Hence, after integration we have

$$1-q=\left(\frac{r_0}{r_2}\right)^{3/4}+\dots \quad \text{или} \quad a_1^2\left(\frac{dt}{dr_2}\right)^2 = 1-\left(\frac{r_0}{r_2}\right)^{3/4}+\dots, \qquad (11.25)$$

where r_0 is a constant.

From eq.(11.25) and conditions at the jump (8.1), we find for cylindrical waves the following asymptotic laws of motion of the jump and values of the characteristics of motion of the gas behind the jump

$$\left.\begin{aligned} a_1(t-t_0) &= r_2(1-2r_0^{3/4}r_2^{1/4}+\dots),\\ v_2 &= \frac{2a_1}{\gamma+1}\left(\frac{r_0}{r_2}\right)^{3/4}+\dots,\\ \rho_2 &= \rho_1+\frac{2\rho_1}{\gamma+1}\left(\frac{r_0}{r_2}\right)^{3/4}+\dots,\\ p_2 &= p_1+\frac{2\gamma p_1}{\gamma+1}\left(\frac{r_0}{r_2}\right)^{3/4}+\dots \end{aligned}\right\} \qquad (11.26)$$

The equations (11.26) agree with eqs.(11.12), (11.13) and (11.14) representing motion of the gas behind a jump front. In the case of spherical waves ($\nu = 3$) from eqs.(10.53), (11.23) and (11.24) we obtain the asymptotic equation

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\frac{2a_1}{\gamma+1}\frac{1}{\ln r_2} = -\frac{4a_1}{\gamma+1} + \frac{4a_1}{\gamma+1}\frac{r_2}{1-q}\frac{dq}{dr_2}.$$

Hence, after integration, we find

$$1-q = \frac{k}{r_2\sqrt{\ln r_2}} + \ldots; \quad a_1^2\left(\frac{dt}{dr_2}\right)^2 = 1 - \frac{k}{r_2\sqrt{\ln r_2}} \ldots, \qquad (11.27)$$

where k is a constant.

By means of eqs.(11.27) and conditions at the jump (8.1) corresponding asymptotic laws in the case of spherical symmetry assume the form

$$\left.\begin{aligned} a_1(t-t_0) &= r_2 - k\sqrt{\ln r_2} + \ldots, \\ r_2 &= \frac{2a_1}{\gamma+1}\frac{k}{r_2\sqrt{\ln r_2}} + \ldots, \\ \rho_2 &= \rho_1 + \frac{2\rho_1}{\gamma+1}\frac{k}{r_2\sqrt{\ln r_2}} + \ldots, \\ p_2 &= p_1 + \frac{2\gamma}{\gamma+1}p_1\frac{k}{r_2\sqrt{\ln r_2}} + \ldots \end{aligned}\right\} \qquad (11.28)$$

The equations (11.28) agree with eqs.(11.12), (11.13) and (11.14) determining motion of the gas behind a jump.

In both of the cases investigated ($\nu = 2$ and $\nu = 3$) the velocity v_2 decreases more rapidly than according to the acoustic laws; hence, at $r_2 \to \infty$ we have $\xi \to \xi_0$, whereby $\Phi_1(\xi_0) = 0$. From eqs.(11.20) and (11.22) it follows that corresponding limiting characteristics in the plane r,t tend toward straight lines.

The shock wave and lines of the characteristics, in contrast to the limiting characteristics, at $r \to +\infty$ diverge from any fixed straight line at an infinite distance.

The method stated above can be used to obtain asymptotic laws or to make more exact the laws found above, also in the case where the motion of the gas behind the jump front is determined by certain well-known laws which are different from those laws examined above or which make these laws more precise.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Section 12. Problem of the Point Explosion Taking Counterpressure into Account

We shall examine the problem of a point explosion in a medium with a constant initial density ρ_1 and a constant initial pressure p_1 which is other than zero. In the case where pressure p_1 which enters into the conditions at the shock wave, is taken into acccunt, self modeling is lost.

As two independent variables we shall take the following values

$$\lambda = \frac{r}{r_2} \quad \text{and} \quad q = \frac{a_1^2}{c^2}, \tag{12.1}$$

where r_2 is the coordinate of the shock wave, $c = \frac{dr_2}{dt}$ and $a_1^2 = \frac{\gamma p_1}{\rho_1}$.

The function $q(r_2)$ is not yet known and determines the law of motion of the shock wave.

As the sought-for functions we shall take the dimensionless relative values of velocity, density and pressure

$$f(\lambda, q) = \frac{v}{v_2}, \quad g(\lambda, q) = \frac{\rho}{\rho_2} \quad \text{and} \quad h(\lambda, q) = \frac{p}{p_2}, \tag{12.2}$$

where

$$r_2 = \frac{2a_1}{\gamma+1}\frac{1-q}{\sqrt{q}}, \quad \rho_2 = \frac{(\gamma+1)\rho_1}{\gamma-1+2q}, \quad p_2 = \frac{p_1}{\gamma+1}\frac{2\gamma-(\gamma-1)q}{q}, \ \cdots \tag{12.3}$$

In conformance with eq.(8.1) these quantities equal the velocity, density and pressure behind the shock wave front.

In these variables, the basic system of equations of turbulent motion (1.2) is transformed to the form

$$\left.\begin{aligned} &\left[\frac{2(1-q)}{\gamma+1} f - \lambda\right](1-q)\frac{\partial f}{\partial \lambda} + \\ &+ \frac{|\gamma+1+(\gamma-1)(1-q)|\,[\gamma+1-2(1-q)]}{2\gamma(\gamma+1)} \cdot \frac{1}{g}\frac{\partial h}{\partial \lambda} + \\ &+ \left[(1-q)\frac{\partial f}{\partial q} - \frac{2-(1-q)}{2q} f\right] r_2 \frac{dq}{dr_2} = 0, \end{aligned}\right\}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\left.\begin{aligned}
&\frac{2(1-q)}{\gamma+1}\frac{\partial f}{\partial\lambda}+\left[\frac{2(1-q)}{\gamma+1}f-\lambda\right]\frac{1}{g}\frac{\partial g}{\partial\lambda}+\frac{2(1-q)}{\gamma+1}\cdot\frac{(\nu-1)}{\lambda}f+\\
&\qquad+\left[\frac{1}{g}\frac{\partial g}{\partial q}-\frac{2}{\gamma+1-2(1-q)}\right]r_2\frac{dq}{dr_2}=0,\\
&\left[\frac{2(1-q)}{\gamma+1}f-\lambda\right]\frac{1}{h}\frac{\partial h}{\partial\lambda}-\left[\frac{2(1-q)}{\gamma+1}f-\lambda\right]\frac{\gamma}{g}\frac{\partial g}{\partial\lambda}+\left[\frac{1}{h}\frac{\partial h}{\partial q}-\right.\\
&\left.-\frac{\gamma}{g}\frac{\partial g}{\partial q}-\frac{2\gamma(\gamma-1)(1-q)^2}{[\gamma+1-2(1-q)][\gamma+1+(\gamma-1)(1-q)]\cdot q}\right]r_2\frac{dq}{dr_2}=0.
\end{aligned}\right\}\qquad(12.4)$$

It is necessary to find the solution of the system (12.4) in the plane q, λ inside the square $0 \leqslant q \leqslant 1$ and $0 \leqslant \lambda \leqslant 1$, which satisfies the following boundary condition:

at the jump when $\lambda = 1$

$$f(1, q) = g(1, q) = h(1, q) = 1; \qquad (12.5)$$

in the center of symmetry when $\lambda = 0$

$$f(0, q) = 0 \qquad (12.6)$$

and the initial conditions when $q = 0$

$$\left.\begin{aligned}
f(\lambda, 0) &= f_0(\lambda),\\
g(\lambda, 0) &= g_0(\lambda),\\
h(\lambda, 0) &= h_0(\lambda),
\end{aligned}\right\}\qquad(12.7)$$

where $f_0(\lambda)$, $g_0(\lambda)$, $h_0(\lambda)$ are corresponding functions for a self-modeling motion, found and studied when solving the problem of a strong explosion in Section 7 and 8.

Equation (12.4) and conditions (12.5), (12.6) and (12.7) can be satisfied by making

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\left.\begin{aligned} f(\lambda, q) &= f_0(\lambda) + f_1(\lambda)\, q + \ldots, \\ g(\lambda, q) &= g_0(\lambda) + g_1(\lambda)\, q + \ldots, \\ h(\lambda, q) &= h_0(\lambda) + h_1(\lambda)\, q + \ldots, \\ \frac{r_1^{\nu} p_1}{E_0} &= \left(\frac{2}{2+\nu}\right)^2 \frac{q}{\gamma\, a(\gamma)}\, e^{A_1 \tau + \ldots}, \end{aligned}\right\} \qquad (12.8)$$

where E_0 is the energy of the explosion, and $a(\gamma)$ is a constant determined by eq.(7.21).

If we take into account eq.(8.4) then when $\varepsilon = 0$ the equations (12.8) will give a corresponding self-modeling solution. Below, by means of eq.(12.4) and boundary conditions (12.5) and (12.6) we shall determine the functions $f_1(\lambda)$, $g_1(\lambda)$, $h_1(\lambda)$ and the constant A*.

The boundary conditions give

$$f_1(0) = 0, \quad f_1(1) = 0, \quad g_1(1) = 0, \quad h_1(1) = 0 \qquad (12.9)$$

Equations (12.4) lead to the following system of ordinary linear differential equations containing the unknown constant A:

$$\left.\begin{aligned} &\left[\frac{2}{\gamma+1} f_0 - \lambda\right] g_0 \frac{df_1}{d\lambda} + \frac{\gamma-1}{\gamma+1} \frac{dh_1}{d\lambda} + \left[\frac{2}{\gamma+1} f_0' + \frac{\nu}{2}\right] g_0 f_1 + \\ &\quad + \left[\frac{2}{\gamma+1} f_0 f_0' - \lambda f_0' - \frac{\nu}{2} f_0\right] g_1 + \\ &+ \left[\frac{\gamma^2+4\gamma-1}{2\gamma(\gamma+1)} h_0' - \nu f_0 g_0 - \frac{2}{\gamma+1} f_0 f_0' g_0 + \frac{\nu}{2} f_0 g_0 A\right] = 0, \\ &\frac{2g_0}{\gamma+1} \frac{df_1}{d\lambda} + \left[\frac{2}{\gamma+1} f_0 - \lambda\right] \frac{dg_1}{d\lambda} + \frac{2}{\gamma+1} \left[\frac{\nu-1}{\lambda} g_0 + g_0'\right] f_1 + \\ &\quad + \left[\frac{2}{\gamma+1} \cdot \frac{\nu-1}{\lambda} \cdot f_0 + \frac{2}{\gamma+1} f_0' + \nu\right] g_1 - \lambda g_0' - 5\nu g_0 = 0, \\ &\left[\frac{2f_0}{\gamma+1} - \lambda\right] g_0 \frac{dh_1}{d\lambda} - \left[\frac{2f_0}{\gamma+1} - \lambda\right] \gamma h_0 \frac{dg_1}{d\lambda} + \\ &\quad + \frac{2}{\gamma+1} [g_0 h_0' - \gamma h_0 g_0'] f_1 + \\ &+ \left[\frac{2f_0}{\gamma+1} - \lambda\right] h_0' g_1 - \nu(\gamma+1) h_0 g_1 - \gamma \left[\frac{2f_0}{\gamma+1} - \lambda\right] g_0' h_1 - \\ &\quad - \frac{2}{\gamma+1} (g_0 h_0' - \gamma h_0 g_0') f_0 + \nu h_0 g_0 \left[A + \frac{4\gamma^2 - (\gamma-1)^2}{2\gamma(\gamma-1)}\right] = 0. \end{aligned}\right\} \qquad (12.10)$$

* A solution of this problem was given in the dissertation of N.S.Burnova, defended at Moscow State Univ. in 1953: Ref.Zhur.Mat. No.3, 1954, Abstract No.2535.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The coefficients of these equations are the known functions of λ expressed through the functions f_0, g_0, h_0, corresponding to a self-modeling solution. The constant A enters only into the free terms linearly; therefore the sought solution of system (12.10) can be presented in the form

$$\left.\begin{aligned} f_1 &= f_{11} + Af_{12}, \\ g_1 &= g_{11} + Ag_{12}, \\ h_1 &= h_{11} + Ah_{12}. \end{aligned}\right\} \qquad (12.11)$$

We shall determine the functions $f_{11}(\lambda)$, $g_{11}(\lambda)$, $h_{11}(\lambda)$ as a solution of Cauchy's problem for system (12.10) at $A = 0$ and at initial data

$$f_{11}(1) = 0,\ g_{11}(1) = 0,\ h_{11}(1) = 0$$

The functions $f_{12}(\lambda)$, $g_{12}(\lambda)$, and $h_{12}(\lambda)$ we shall determine as a solution of Cauchy's problem for an analogous system of equations with correspondingly varied free terms and with the initial data

$$f_{12}(1) = 0,\ g_{12}(1) = 0,\ h_{12}(1) = 0$$

The conditions (12.9) will be satisfied if the constant A is determined from the condition

$$\lim_{\lambda \to 0} (f_{11} + Af_{12}) = 0 \qquad (12.12)$$

The solution of the indicated Cauchy problems can be produced by a numerical method. However, continuation of the calculations to the center of symmetry is difficult; therefore, to obtain a solution near the center and to determine the constant A from condition (12.12) it is necessary to obtain asymptotic equations near the center of symmetry for the general solution of the systems of ordinary equations under consideration, which (systems) are obtained from system (12.10). From the equations (8.8) it follows that near the center of symmetry, analyses having the follow-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

ing form are valid*.

$$\left.\begin{aligned} f_0 &= \lambda(\beta_0 + \beta_1\lambda^m + \beta_2\lambda^{2m} + \ldots) \\ g_0 &= \lambda^s(\alpha_0 + \alpha_1\lambda^m + \alpha_2\lambda^{2m} + \ldots), \\ h_0 &= \gamma_{-1} + \lambda^m(\gamma_0 + \gamma_1\lambda^m + \gamma_2\lambda^{2m} + \ldots), \end{aligned}\right\} \qquad (12.13)$$

Here $s = \frac{\nu}{\gamma - 1}$, $m = s + 2$, and β_k, α_k, γ_k are abstract constants depending only on γ and ν.

In the spherical case at $\nu = 3$ and for $\gamma = 1.4$ we have

$$\left.\begin{aligned} s &= 7{,}5, & m &= 9{,}5, \\ \alpha_0 &= 0{,}3107, & \alpha_1 &= 0{,}2918, & \alpha_2 &= 0{,}2082, \\ \beta_0 &= 0{,}8571, & \beta_1 &= 0{,}1530, & \beta_2 &= 0{,}0043, \\ \gamma_{-1} &= 0{,}3655, & \gamma_0 &= 0{,}3004, & \gamma_1 &= 0{,}1979. \end{aligned}\right\} \qquad (12.14)$$

By using the analysis (12.13) we find that the equations for f_{11}, g_{11}, h_{11}, near the center of symmetry assume the form

$$\left.\begin{aligned} f'_{11} &= [-2 - 9{,}0562\lambda^{9,5} + \ldots]\frac{f_{11}}{\lambda} + \\ &\quad + [6{,}1537 + d_1\lambda^{9,5} + \ldots]\lambda^2 g_{11} + \\ &\quad + [-7{,}0358 - 1{,}9604\lambda^{9,5} + \ldots]h_{11} + \\ &\quad + [2{,}9388 + 4{,}1298\lambda^{9,5} + \ldots], \\ g'_{11} &= [6{,}7966 + d_2\lambda^{9,5} + \ldots]\lambda^{9,5} f_{11} + \\ &\quad + [18 + d_3\lambda^{9,5} + \ldots]\frac{g_{11}}{\lambda} + \\ &\quad + [-6{,}3759 + d_4\lambda^{9,5} + \ldots]\lambda^{9,5} h_{11} + \\ &\quad + [-21{,}8045 + d_5\lambda^{9,5} + \ldots]\lambda^{9,5}, \\ h'_{11} &= [-5{,}1932\lambda^{9,5} + \ldots]\frac{f_{11}}{\lambda^3} + [9{,}1837 + d_6\lambda^{9,5} + \ldots]\lambda g_{11} + \\ &\quad + [-3{,}7474\lambda^{9,5} + \ldots]\frac{h_{11}}{\lambda} + [-3{,}8234\lambda^{9,5} + \ldots]\frac{1}{\lambda}, \end{aligned}\right\} \qquad (12.15)$$

and the equations for the functions f_{12}, g_{12}, h_{12} are written thus:

---

*The coefficients of these analyses are easily determined with the help of the equations of Section 10. See eqs.(10.39, (10.40), (10.43).

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$
\left.\begin{aligned}
f'_{12} &= [-2-9{,}0562\lambda^{0,5}+\ldots]\frac{f_{12}}{\lambda}+\\
&\quad+[6{,}1537+d_1\lambda^{0,5}+\ldots]\lambda^2 g_{12}+\\
&\quad+[-7{,}0358-1{,}9604\lambda^{0,5}+\ldots]h_{12}+\\
&\quad+[-2{,}5714-1{,}7208\lambda^{0,5}+\ldots],\\
g'_{12} &= [6{,}7966+d_2\lambda^{0,5}+\ldots]\lambda^{5,5}f_{12}+\\
&\quad+[18+d_3\lambda^{0,5}+\ldots]\frac{g_{12}}{\lambda}+\\
&\quad+[-6{,}3759+d_4\lambda^{0,5}+\ldots]\lambda^{0,5}h_{12}+\\
&\quad+[-2{,}3303+d_7\lambda^{0,5}+\ldots]\lambda^{0,5},\\
h'_{12} &= [-5{,}1932\lambda^{0,5}+\ldots]\frac{f_{12}}{\lambda^2}+[-3{,}7474\lambda^{0,5}+\ldots]\frac{h_{12}}{\lambda}+\\
&\quad+[9{,}1837+d_6\lambda^{0,5}+\ldots]\lambda g_{12}+[-2{,}5680\lambda^{0,5}+\ldots]\frac{1}{\lambda},
\end{aligned}\right\} \qquad (12.16)
$$

where d_1, d_2, d_3, d_4, d_5, d_6, d_7 are constant numerical values which are not further used. The sequent terms of the analysis in brackets have the order λ_{19}. Near the center of symmetry, the general solution of these equations can be presented in the form

$$
\left.\begin{aligned}
f_{11} &= \frac{C_1}{\lambda^2}+(0{,}9796-2{,}3453C_2)\lambda+\ldots,\\
g_{11} &= -0{,}5034C_1\lambda^{0,5}+(1{,}4425+2{,}1253C_2)\lambda^{1,5}+C_2\lambda^{19}+\ldots,\\
h_{11} &= C_2-1{,}5103\lambda^{0,5}+[0{,}4566+2{,}9421C_2]\lambda^{0,5}+\ldots;
\end{aligned}\right\} \qquad (12.17)
$$

$$
\left.\begin{aligned}
f_{12} &= \frac{C_5}{\lambda^2}-(0{,}875+2{,}3453C_4)\lambda+\ldots,\\
g_{12} &= -0{,}5034C_5\lambda^{0,5}+(0{,}7768+2{,}1253C_4)\lambda^{1,5}+C_6\lambda^{18}+\ldots.\\
h_{12} &= C_4-1{,}5103C_5\lambda^{0,5}+(0{,}9491+2{,}9421C_4)\lambda^{0,5}+\ldots.
\end{aligned}\right\} \qquad (12.18)
$$

where C_1, C_2, C_3, C_4, C_5, C_6 are arbitrary constants.

The solution can be constructed in the following manner. We solve by a numerical method the problem of Cauchy in the interval from $\lambda = 1$ to $\lambda = \tilde{\lambda} \ll 1$. At $\lambda = \tilde{\lambda}$ we join the solution obtained by numerical calculation to the solution obtained according to the asymptotic eqs.(12.17) and (12.18); by this, the constants C_1, C_2, C_3, C_4, C_5, C_6 are determined. The constant A is determined according

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

to the formula

$$A = -\frac{C_1}{C_5}$$

The results of numerical calculation give the following value for the quantity A:

$$A = 1.909$$

The results of the numerical calculations for the functions $f_1(\lambda)$, $g_1(\lambda)$, $h_1(\lambda)$ are presented in Fig.77.

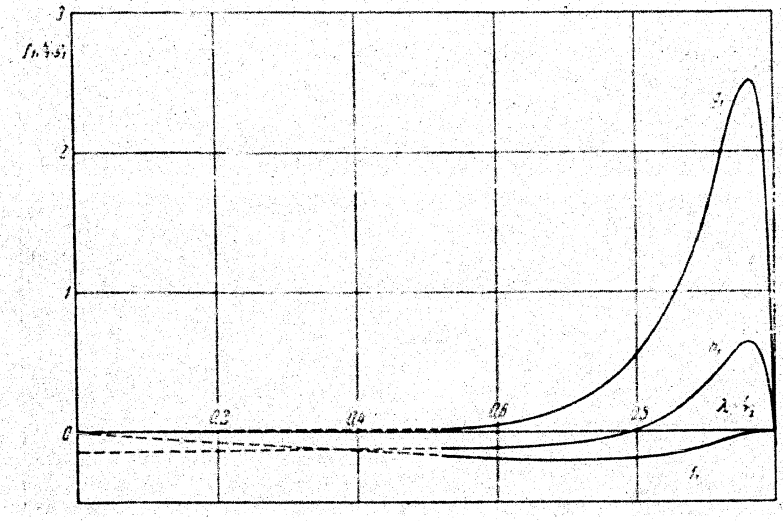


Fig.77 - Curves for the Derivatives $\left(\frac{\partial \frac{p}{\rho_2}}{\partial q}\right)_{q=0} = g_1(\lambda)$, $\left(\frac{\partial \frac{p}{\rho_2}}{\partial q}\right)_{q=0} = h_1(\lambda)$ and $\left(\frac{\partial \frac{v}{v_2}}{\partial q}\right)_{q=0} = f_1(\lambda)$ which Determine the Effect of Counterpressure in the Initial Stage of a Point Explosion

With the help of this graph, in accordance with the equations (12.8) for small q it is possible to estimate the change with passage of time, of the characteristics of gas motion.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

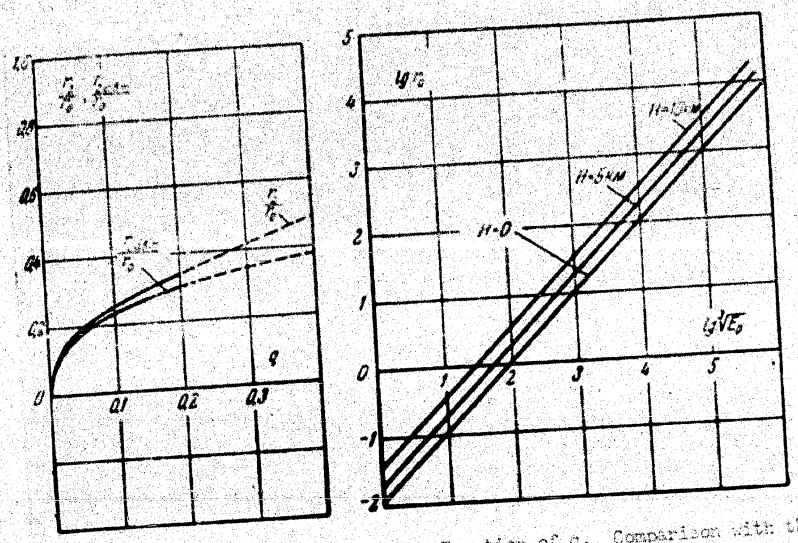


Fig.78 - Radius of a Shock Wave r_2 as a Function of q. Comparison with the self-modeling case, $r_o = \sqrt[3]{\frac{E_o}{p_1}}$ is the dynamic linear dimension in meters, H is the height of the standard atmosphere, E_o is the energy of the explosion in ergs.

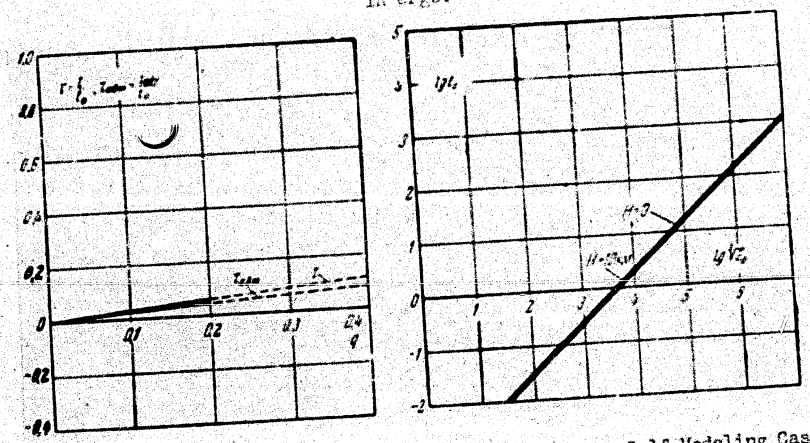


Fig.79 - Time t as a Function of q; comparison with the Self-Modeling Case. $t_o = E_o^{1/3} \rho_1^{1/2} p_1^{-5/6}$ is the dynamic time in seconds, H is the height of the standard atsmosphere, E_o is the energy of the explosion in ergs.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

CHAPTER V

APPLICATIONS TO PROBLEMS OF ASTROPHYSICS

Section 1. Certain Data of Observations

In modern astrophysics analysis and understanding of the internal pressures in stars, of the evolution of stars and of the evolution of various nebulae are impossible within the framework of the dynamics of systems of discrete material points or within the framework of the hydrostatics of fluid masses - theories which up to recent times have served as the main source of a different kind of models and representations in classical astronomy.

At the present time study of the motions of celestial objects as gaseous bodies must give the clue for the solution of the main problems of cosmogony, and only in this way is it possible to clarify and to interpret the series of observed effects. Now it has become obvious that at the basis of the concepts for the investigation of celestial phenomena must lie the posing and solution of a number of dynamic problems on the movements of gas which are able to be considered as theoretical models encompassing the essential properties of the motion and evolution of stars and nebulae. For the construction and investigation of such models it is necessary to utilize the methods, apparatus and representations of modern theoretical gas dynamics - aerodynamics - and to pose and solve the corresponding mechanical problems in application to the problems of astrophysics.

In connection with this already clearly perceived requirement an international

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

conference was held in 1949 in Paris at which occured the meeting of astrophysicists with the most prominent notions of aerodynamics, occupied before this mainly with the problems of aviation and with the theories of the propagation of explosive waves.

As a result of this conference there was published a symposium of reports* with discussions in which it was noted that in investigations of the most important celestial phenomena it is necessary to set up problems on the motion in a gravitational field of gaseous masses with large relative velocities and with the presence of shock waves, problems on the turbulent motions of gas that take into account electromagnetic fields, and other problems.

Preliminary analysis has shown that for the clarification of celestial phenomena it is insufficient to utilize the already well known and studied phenomena of the movement of gas - it is necessary to construct new models and to set up and to solve new problems of gas dynamics.

On the basis of the methods of the theory of dimensions which have been developed in the preceding chapter in applications to the theory of the one-dimensional motions of a gas with spherical symmetry, we have studied certain motions of a gas which to all appearances are able to reflect the essential outlines discovered in astronomical observations.

Below we consider certain applications to the theory of the brightness and internal structure of stars, to the theory of the variation of brightness of cepheid stars, and to the theory of the flare-ups of novae and supernovae. We now expound certain principal data on these phenomena well known from astrophysics.

The principal characteristic of stars is their brightness and intensity, which are connected directly with the perception of the eye during observations. The brightness can be measured by means of special devices called photometers. Since ancient times the intensity of brightness of the stars has rested on the principle

---

* This symposium has been translated into Russian, Problemy Kosmicheskoy Aerodinamiki, Publishers of Foreign Literature, Moscow, 1953.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

of the concept of so called stellar magnitude.

At first the classification of the stars was carried out simply by eye, so that the stars of the first magnitude appeared to be brighter than the stars of the second magnitude by so many times as they seemed to be brighter than the stars of the third magnitude, and so on. In this manner all the stars were able to be arranged in a series in correspondence with their apparent brightness. Already in ancient times the stars were divided into six classes during observations by the naked eye. The brightest stars were referred to the first class and called stars of the first magnitude, after which followed the stars of the second magnitude and so on. It is obvious that many stars fell into intervals between the classes and in correspondence with these they were ascribed apparent magnitudes expressed by non-whole numbers. From the indicated determination it is obvious that with the increase in the stellar magnitude the apparent brightness decreases. At the present time in the case of photographing with prolonged exposure of the images of stars obtained with the aid of powerful modern telescopes it is possible to study weak stars which can be referred to stellar magnitudes of the 22nd or even 23rd apparent magnitude.

The above described distribution of stars depends upon the methods of determination of brightness. In the determination of apparent magnitude by eye or by means of preliminary photographing it is possible to arrive at various conclusions, since the human eye is more sensitive to the red and yellow rays of light, whereas the photographic plate possesses enhanced sensitivity to the blue and violet rays. Therefore we distinguish the visual apparent magnitude of stars as photographic, photoelectric apparent magnitude, etc.

The stellar magnitudes determined with the aid of apparatus identically sensitive to all lengths of light waves, with the introduction of corrections for the absorption in the terrestrial atmosphere and in the optical system of instruments, are called bolometric. Bolometric magnitudes characterize the total radiation of stars that reaches as far as the upper layers of the Earth's atmosphere. In what

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

follows we shall utilize the concept of bolometric stellar magnitude.

It is obvious that the stellar magnitude m is determined by the quantity of radiant energy E_m which is incident per unit time upon the Earth from the star under consideration. This energy is proportional to the total energy which is radiated by the star per unit time, that is, to the luminosity L^* of a star, and is inversely proportional to the square of the distance l from the Earth to the star.

On the basis of the properties of the human eye, which is determined by the Weber-Fechner law, it has been found that during variation of the stellar magnitudes m which are determined by the intensity of the perceptions in accordance with the numbers of an arithmetic progression, the corresponding intensities of the irritant or stimulant vary in accordance with the numbers of a geometric progression.

In agreement with this law there has been established the following relation

$$m = -2{,}5 \lg E_m + \text{const.}, \tag{1.1}$$

which can be represented in the following form

$$\frac{E_{m_2}}{E_{m_1}} = 10^{\frac{m_1 - m_2}{2{,}5}} = (2{,}512)^{m_1 - m_2}. \tag{1.2}$$

The apparent magnitude of the Sun is $m_2 = -26.7$. The apparent magnitude of the star Sirius - the brightest visible star - is equal to -1.6; hence it follows that $m_1 - m_2 = 25.1$ and hence approximately $E_{Sun}/E_{Sirius} = 10^{10}$: in other words, the apparent brightness of the Sun is ten billion times greater than the apparent brightness of Sirius.

The apparent magnitude of the stars depend essentially upon the distance of the star to the Earth; therefore in order to obtain the comparative characteristics of stars we introduce the concept of absolute visual magnitude M, which is equal to the above defined apparent magnitude of the star if considered to be a

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

star placed at a distance of 10 parsecs from the Earth*.

On the basis of the formulas

$$m = -2{,}5 \lg \frac{\varrho^*}{l^2} + \text{const.} \quad \text{и} \quad M = -2{,}5 \lg \frac{\varrho^*}{10^2} + \text{const.}$$

we obtain

$$M = m + 5 - 5 \log l \; ; \tag{1.3}$$

In formula (1.3) the distance of the star from the Earth l is expressed in parsecs.

The absolute magnitude of the Sun equals 4.9; this means that at a distance of 10 parsecs the Sun would be represented to us by a weak little starlet of the fifth magnitude, hardly accessible to observation by the unaided eye. The absolute magnitude of Sirius is 1.3. Hence it follows that

$$\frac{\varrho^*_{\text{Sirius}}}{\varrho^*_{\text{Sun}}} = 10^{\frac{4.9-1.3}{2.5}} = 28.$$

Consequently, the luminosity of the star Sirius is 28 times greater than the luminosity of the Sun. The magnitude of the apparent brightness of a star m can be measured directly; if from any supplementary data at all we are able to find the absolute magnitude M, then with the aid of formula (1.3) it is easy to compute the distance l to the star.

Along with the luminosity of a star L^* the most important characteristics are the mass of the star M and its radius R. With the aid of a different kind of observations and suitable analysis astronomers at the present time have at their disposal for a considerable number of stars data on the quantities L^*, M and R. Consideration of these data shows that, for various groups of stars, relations of the following form exist**.

---

* One parsec is equated to 3.26 light years which is equal to $3.08.10^{13}$ km and corresponds to the distance to the Earth from that star for which the parallax is $1''$.

** See P.P.Parenago and A.G.Masevich, Issledovaniye Zavisimosti Massa-Radius-Svetimost' (Study of the Relation Mass-Radius-Luminosity), Works of the State Astronomical Institute in Shternberg, Vol.XX, Moscow University Press, 1951.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\mathfrak{L}^* = f_1(\mathfrak{M}) \text{ AND } \mathfrak{R} = f_2(\mathfrak{M}). \tag{1.4}$$

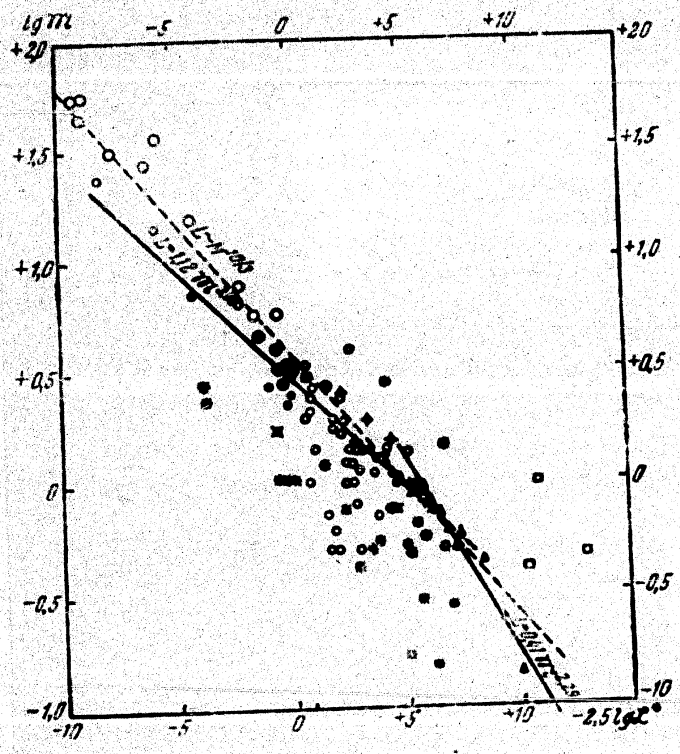


Fig.81. The Mass-Luminosity Diagram

● Main sequence O-G4; ▲ Main sequence G7-M; ◆ Above the main sequence; ■ Below the main sequence; O supergiants; ◍ giants; ◦ subgiants; ● subdwarfs; □ white dwarfs

Figures 81 and 82 give diagrams taken from the works of P.P.Parenago and A.G.Masevich, in which relations of this kind are given. In Section 3 here we shall present certain theoretical considerations concerning the relations in (1.4).

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

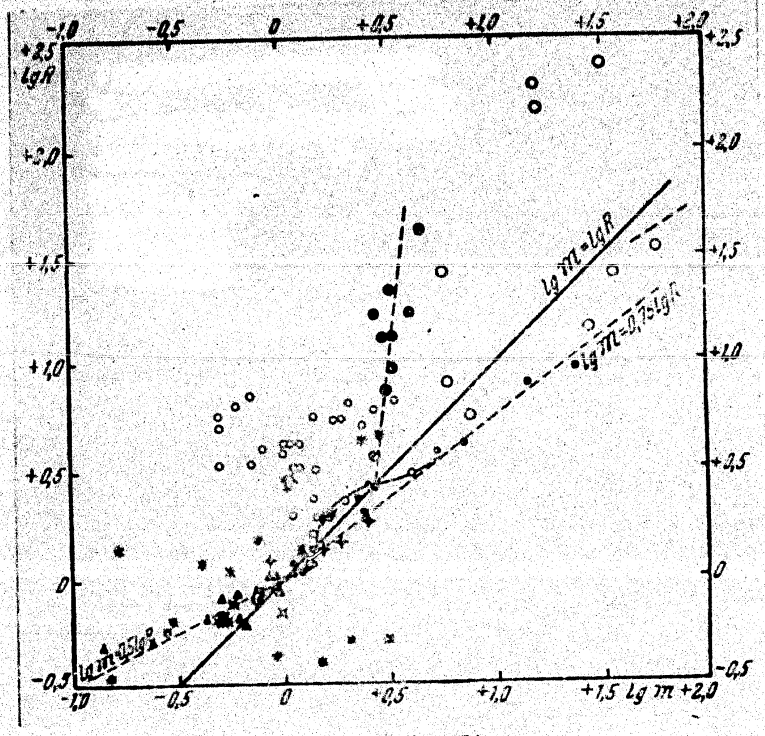


Fig.82 - The Mass-Radius Diagram

● Main Sequence O–G4; ▲ Main Sequence G7–M; ◆ Above the main sequence; ■ Below the main sequence; ⊙ Supergiants; ● Giants; ○ Subgiants; ● Subdwarfs;

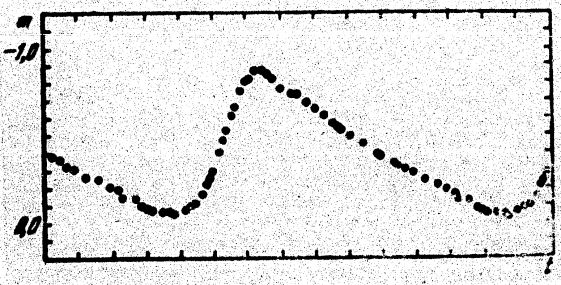


Fig.83 - The Photovisual Curve of the Brightness of the Star δ Cephei

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Let us now consider certain data on the so called variable stars. In 1784 the astronomer Gudrayk published for the first time a communication* on observations in which he disclosed that a star designated by the Greek letter δ situated near the head of Cepheus varies its brightness periodically. At the present time are known very many similar variable stars called cepheids; they are found in various regions of our Galaxy and in other galaxies.

Empirical laws established among the periods and absolute apparent magnitudes of the cepheids have permitted astronomers with the aid of formulas (1.3) to find the distances to other galaxies** and to obtain very valuable data for the investigation of the problem of the evolution of stars and for the solution of many other problems.

In Fig.83 are shown the results of photometric measurements of the brightness m of the star δ Cephei in the course of time. The corresponding curves for other cepheids in general possess the same character. By contemporary observations it has been discovered that the period of δ Cephei is equal to 5 days, 8 hours, 53 minutes and 27.46 seconds; this quantity here possesses almost a constant value. In investigations of astronomers it has been demonstrated that although fluctuations in the period are real they are however small; for example, δ for Cephei over the past 20 years: the diminution in the period is of the order of 1 second. The periods of the characteristic cepheids are found to range from one half a day to 80 days.

Spectroscopic measurements on the basis of the Doppler effect show that the variation in the brightness of cepheids is accompanied by a variation with the same period in the radial velocities of the radiating particles of the gas, which testifies to the presence of radial motion of the gas in the photosphere of the cepheids.

---

* Phil. Trans., 76 (1786), 48-61.

** At the present time this is the only method for the determination of such great distances, which are of the order of hundreds of millions of light years.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The amplitude of the fluctuations in the radial velocity is of the order of several tens of kilometers per second; for δ Cephei this quantity is equal to 39 km per second. The curves of brightness and the curves of radial velocities are very similar to each other.

Table 1

Data for Characteristic Cepheids*

a)	b)	c)	d)	e)	f)	g)	h)	i)
l Car	**F8—K0**	**35,52**	**−5,52**	**1,2**	**8,59**	**145**	**50**	**1,7**
Y Oph	**F8—G7**	**17,12**	**−4,00**	**0,61**	**1,79**	**59**	**23**	**1,4**
X Cyg	**F8—K0**	**16,39**	**−3,92**	**0,69**	**6,12**	**41**	**19**	**1,7**
ζ Gem	**cG 1v**	**10,15**	**−3,16**	**0,42**	**1,82**	**43**	**15**	**1,0**
S Sge	**F8—G7**	**8,38**	**−2,87**	**0,50**	**1,47**	**33**	**13**	**2,0**
W Sgr	**F2—G5**	**7,59**	**−2,72**	**0,85**	**1,93**	**24**	**11**	**2,5**
η Aql	**F2—G9**	**7,18**	**−2,62**	**0,51**	**1,77**	**25**	**11**	**2,0**
X Sgr	F5—G9	7,01	−2,60	0,67	1,33	28	11	2,0
Y Sgr	F5—G8	5,77	2,30	0,74	1,35	27	10	1,4
δ Cep	F4—G6	5,37	−2,19	0,61	1,27	23	9	2,5
T Vul	F3—G5	4,44	−1,95	0,71	0,97	17	8	1,7
α UMi	cFV	3,97	−1,81	0,08	0,17	20	8	1,7
SU Cyg	F0—G1	3,85	−1,78	0,74	0,71	12	7	2,5
RT Aur	F1—G5	3,73	−1,74	0,80	0,86	14	7	3,3
SZ Tau	F4—G2	3,15	−1,58	—	0,46	14	7	1,0
SU Cas	F2—F9	1,95	−1,15	0,33	0,30	9,2	5	1,0
RR Lyr	B9—F2	0,567	−0,35	0,85	0,17	4,3	4	2,0

a) Star b) Spectral Class c) Period, in Days d) M, Stellar Magnitude e) Amplitude of M f) 10^6km δR g) 10^6km R h) $\frac{M}{m}$ i) Asymmetry of Brightness Curve

* All the numerical data and this table are taken from the book of S.Rosseland, Theory of Pulsations of Variables, translated into Russian in 1952 by the Moscow Publishers of Foreign Literature, under the title Teoriya Pul'satsiy Peremennykh Zvezd.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Thus the data of observations without doubt points to the presence of considerable movements of the gas in the photosphere of cepheids. In addition observations have established that the fluctuations in the brightness are accompanied by a variation of their spectra and of their effective temperatures. Variation in temperature and radius of the photosphere conditions the variation in the brightness. The ratio of luminosities (the total light energy radiated by a star) of a cepheid in the period of an oscillation changes by several times; for delta(δ) Cephei this variation is equal to 2.

In table 1 are given certain quantities for typical cepheids.

In this table the letter M represents the mass of the star, $M_\odot$ the mass of the Sun, R the radius of the star (the radius of the Sun is $R_\odot = 6.26 \cdot 10^5$ km), δR half of the difference between the greatest value and the least value of the star's radius, M the mean absolute visual magnitude. In the graph of the asymmetry of the brightness curve is given the ratio of the duration of drop in brightness from the maximum to the minimum to the duration of its rise.

The graph of the so called spectral class is connected with the temperature characteristic of the star. Cepheids, just as all variable stars, are stars of very high luminosity and, as a rule, are supergiants. According to their mass and size the cepheids are considerably larger than our own Sun.

The magnitudes of the radii of stars were calculated in this table under the assumption that the photosphere (which expands or contracts together with the star) consists of gas particles of one and the same kind. Here it is evident that the variations in the radius δR possess the order of millions of kilometers. If the boundary of the expanding photosphere coincides with the front of the shock wave, then the variation of the radius of the photosphere will be larger, since the velocity of the jump or discontinuity is greater than the measured velocity of the gas particles behind the discontinuity. It is obvious that the variation in the luminosity of a star because of the variation in the area of the photosphere for cep-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

heids can be quite considerable. This effect is aggravated when in front of the shock wave - namely, the boundary of the photosphere - there is a layer of comparatively cold gas, which absorbs especially greatly the radiated energy at the moment of minimum radius of the photosphere.

The name of novae is given to stars which abruptly increase their brightness by 10-12 stellar magnitudes, and in certain cases even by 15 magnitudes; this means that the visual brightness of such stars increases by 10,000 to 100,000 times. The characteristic property of the flare-ups of novae is their extreme abruptness; the time of increase of the brightness possesses the order of one or two days, but the liberation of the energy which causes the flare-up occurs apparently all in only several hours. After the attainment of maximum brightness occurs the slow drop in brightness to the original amount.

In Fig.84 is shown the variation in brightness of the star Nova Aquilae, which flared up in 1918.

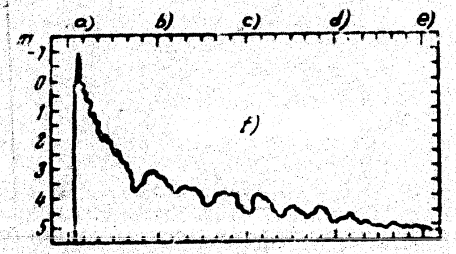


Fig.84. The Curve of Brightness Variation of Nova Aquilae

a) June 5 b) July 5 c) August 4 d) September 3 e) October 3 f) Nova Aquilae 1918

The variation in brightness is accompanied by abrupt changes in the spectra. Characteristic is the displacement of all lines toward the violet side, which shift is due to the occurrence of large radial velocities of the radiating gas. These velocities are of the order of several hundreds to three-four thousands of kilometers

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

per second. For a certain time after the outburst there appear in the spectrum of a nova bright forbidden lines which are characteristic for the radiation of an extremely rarefied gas and for the spectra of gaseous nebulae. The spectra of novae before outburst and after outburst for many years belong to the class O of very hot stars. In the case of Nova Aquilae it was found by means of the Sandster method according to lines of He-II that for three months after the outburst the temperature of the star was equal to 65,000° (the temperature of the Sun is equal to 6000°).

The diameter of novae at the moment of their maximum is comparable with the diameter of the Earth's orbit. According to various calculations astronomers evaluate the energy released during the outbursts of novae to be 10^{45} to 10^{46} ergs. This energy is equal to the energy radiated by the Sun in 10,000 to 100,000 years. In our Galaxy about 100 novae flare up each year.

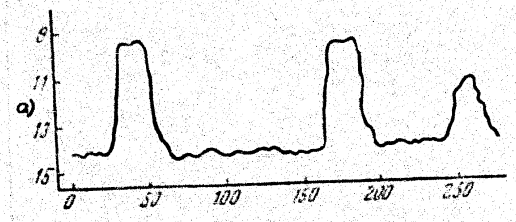


Fig.85. Curves of the Variation in Brightness of Novae of the Type U Geminorum

a) apparent magnitude

Among the novae and cepheids exists an intermediate type of variable star - the so called repetitive stars which flare up several times, sometimes every several years. An example of such a variable star is the star Mira Ceti ("The Wonderful"), in which the time between outbursts is 320-370 days; at the time of an outburst the brightness varies by 1500 times.

It is possible to indicate still another, namely the so called dwarf novae of

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the type U Geminorum, which repeat their weak outbursts - stellar sneezing - every one-two weeks. In Fig.85 is shown the brightness curve for such stars.

Finally we point still to another, namely to the star R Coronae Borealis, for which the curve of the brightness variation is shown in Fig.86. The abrupt decreases in brightness are due to the absorption of light by the masses of carbon ejected by the star.

The outbursts of novae represent amazing colossal cosmic explosions; however, a still more gigantic monstrous catastrophy difficult to represent is the outburst of so called super stars. During the outbursts of super stars the luminosity grows by billions of times and is comparable with the total luminosity of all the stars of the Galaxy.

The very brightest observed super star flared up in NGC 5253 (it attained an absolute magnitude of -18.4). This star at its maximum brightness was brighter than our own Sun by 15 billion times.

In Fig.87 is given a typical curve of the variation of brightness of a super star. The curves of the drop in brightness of super stars are monotonic; for novae during fall in brightness one observes fluctuations in it. The outbursts of super stars is an abrupt phenomenon. In our Galaxy over the past 900 years there have been observed it seems only two supernovae: the first was the star which flared up according to Chinese annals in 1054 in the constellation Taurus. Now at the same place is observed a nebulosity called the Crab Nebula. In our day this nebula has continued to expand with great velocity. In the center of the Crab Nebula there is a weak but very hot star with temperature higher than 100,000°. As the second supernova, observed by Tycho Brahe, is the star that flared up in 1572 in Cassiopeia and shone in the sky for about a year (it was visible even in the daytime during the full brightness of the Sun).

In other galaxies astronomers have already observed about 40 super stars. With contemporary instruments because of the great distance it is possible to make obser-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

vations of supernovae in other galaxies only at the moments of their maximum brightness.

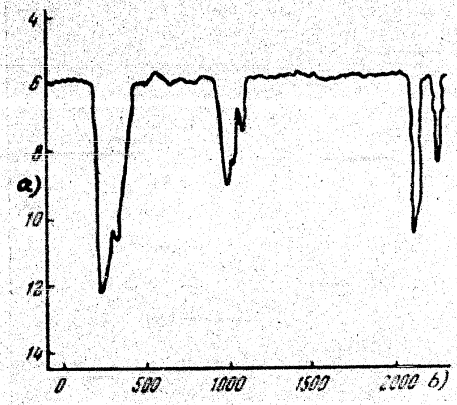


Fig.86. Curve of the Variation in Brightness of R Coronae Borealis

a) apparent magnitude b) days

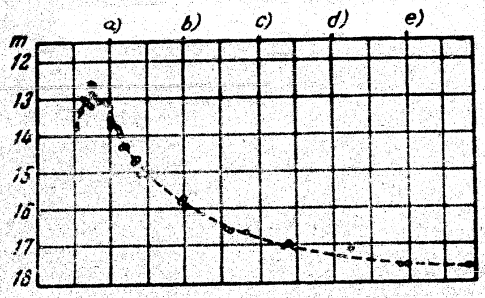


Fig.87. Curve of the Brightness of the Super Star in the Spiral Nebula NGC 1003

a) September 22 b) November 1 c) December 11 d) January 20 e) March 2

According to the evaluations of astronomers the diameters of the supernovae at

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the moment of maximum can exceed by 250 times the corresponding diameters of ordinary novae and can exceed by 5-6 times the diameter of our solar system - the diameter of the orbit of the planet Pluto. In a short interval of time of the order of several days during outbursts of supernovae there is released energy evaluated to be a magnitude of 10^{50} ergs; such energy is radiated by the Sun in a billion years. With the aid of spectroscopic data it has been discovered that during the outbursts of supernovae the gas particles in the radiating photosphere are moving with the tremendous velocity of the order of 6000 km/sec.

The above described observations of the outbursts of supernovae, novae and of the fluctuations in the luminosity of cepheids indicate that the essence of these phenomena is very directly connected with the motion of colossal masses of gas from which are formed corresponding variable stars.

Below we consider the stating of the problem and in several cases their solution in the case of the non-steady motions of gas, which can be considered as approximate models of the fluctuations of cepheids and outbursts of novae and supernovae.

Section 2. Concerning the Equations of Equilibrium and Motion of a Mass of Gas That Models a Star

In order to construct a quantitative description of the equilibrium and motion of gaseous masses that form stars it is necessary to establish the equations of equilibrium and motion. We shall derive below the equations of equilibrium and the equations of motion of masses of gas that gravitate in accordance with Newton's. For the moving masses we consider only the case of radial motion in the presence of spherical symmetry.

For motion with spherical symmetry the equation of continuity which expresses the law of conservation of mass possesses the following form,

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial r} + \frac{2\rho u}{r} = 0, \qquad (2.1)$$

STAT

15

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

where ρ denotes the density, u is the radial velocity of the particles of the gas, r is the distance to the center of symmetry, and t is the time. During equilibrium, that is, during rest, this equation is satisfied identically.

In the case of tremendous linear scales and velocity scales of the movements of gaseous masses it is completely valid to disregard the viscosity of the gas; therefore the equation of impulses (momenta) taking into consideration the gravitational forces we can write under the assumption that the gas is not viscous, that is, ideal, as follows:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{fm}{r^2} = 0, \qquad (2.2)$$

where $f = 6.670 \cdot 10^{-8}$ cm^3/gram$\cdot$sec^2 is the gravitational constant, p is the total pressure equal to the sum of the molecular pressure and light pressure, and m(r,t) is the mass of the gas within a sphere of considered radius r. As is well known, in the case of spherical symmetry the action of the entire masses upon a point mass at a distance of r from the center of symmetry is equal to the attraction of a material point located at the center of symmetry and having a mass which is equal to m(r,t). In order to determine m(r,t) we have the following equation

$$\frac{\partial m}{\partial r} = 4\pi r^2 \rho \quad \text{or} \quad m = 4\pi \int_0^r r^2 \rho \, dr. \qquad (2.3)$$

The temperature of the substance within a star is very high - it is of the order of millions of degrees; but close to the outer surface of the star the temperature is of the order of several thousand degrees. At such temperatures the substance of a star can be considered as a perfect gas even in the case where the pressure and the density are extremely great. Therefore, assuming further that local thermodynamic equilibrium holds true, we can take the equation of state in the following form

$$p = p_{\text{molec}} + p_{\text{light}} = \frac{R\rho T}{\mu} + \frac{a}{3}T^4, \qquad (2.4)$$

STAT

16

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

where T is the absolute temperature, $R = 8.3144 \cdot 10^7$ erg/grad·mol is the absolute gas constant, and μ is the molecular weight (that is, the mass of one mole of the gas μ = gram/mole) which defines the chemical composition of the gas*, and $a = 7.569 \cdot 10^{-15}$ erg/cm^3·grad^4 is the constant of density of radiation (Stefan's constant).

The light pressure is comparable with the molecualr pressure only in the case of very great temperatures or only in the case of extremely small densities. Therefore in studying the structure and internal motions of a gas in stars we can ordinarily disregard the light pressure in comparison with the molecular pressure, which is equivalent to replacing equation (2.4) by the Clapeyron equation

$$p = \frac{R_\mu T}{\mu}. \qquad (2.4')$$

Instead of the equation of conservation of energy we make use of a consequence of this equation and of a theorem of vires vivae (kinetic energies), namely we make use of the equation of heat flow which in the general case possesses the following form

$$\frac{dE}{dt} + \frac{dA^{(i)}}{dt} = \frac{dQ^{(e)}}{dt}, \qquad (2.5)$$

where E is the internal energy of a particle without internal potential gravitational energy, $dA^{(i)}/dt$ is the work per unit time of the forces of internal stresses (here the work of the internal gravitational forces** do not enter), and $dQ^{(e)}/dt$ is the external flow of heat per unit time. By virtue of the assumptions made above the equation of heat flow can be written in the following form

---

* Thanks to the high temperatures within a star complete ionization of the light elements sets in. In the case of complete ionization for all the elements , μ is close to two; for only hydrogen it is $\mu = 0.5$ and for helium it is $\mu = 1.33$.

** The work of the internal gravitational forces and the increment of the internal gravitational energy are abbreviated in the left part of equation (2.5).

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\frac{d}{dt}(c_v T) + p \cdot \frac{d}{dt}\frac{1}{\rho} = \varepsilon, \qquad (2.5')$$

where c_v is the heat capacity per unit mass of the gas at constant volume, which depends upon the chemical composition of the substance; ε is the total energy that is released per unit mass of the gas in unit time. The quantity ε can be different from zero because of radiation and absorption, because of nuclear or chemical reactions, and because of heat conduction. If the star gas is in equilibrium, then the total energy balance reduces to the following equation: $\varepsilon = 0$. Since the star radiates energy into the surrounding space, sources of energy must exist during equilibrium within the star. The nature of these sources and their distribution within the star at the present time are still not completely clear. However, investigation of the equilibrium of stars for various laws governing the distribution of the sources of energy indicates that the distribution of pressures and of the density within a star and, in particular, their values at the center of the star, depend weakly upon the law governing the distribution of the sources of energy. Calculations show that if one assumes the distribution of sources to be uniform over the entire mass of the star or assumes that the same quantity of energy is released at one point, namely at the center of the star, then the characteristics of the state are obtained in first approximation. To this it is possible to add that the quantity of energy released because of the physical and chemical processes depends very greatly upon the temperature. At the center of a star the temperature is greatest; therefore, the principal part of the energy is released close to the center of the star*. As calculations show, this position must be well

* See V.A.Ambartsumyan, E.R.Mustel', A.B.Severnyy, and V.V.Sobolev, Teoreticheskaya Astrofizika (Theoretical Astronomy), Moscow, State Technical Press, 1952; see also S.Chandrasekar, Introduction to the Study of the Structure of Stars, translated into Russian by the Foreign Literature Press, Moscow, 1950.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

justified in actuality.

Therefore, in what follows we shall sometimes employ schemes in which it is assumed that the energy can be released only at the center of the star.

The energy can be transferred by radiation from the center to the periphery of the star; in this process the distribution of energy over the spectrum of frequencies can change thanks to absorption and to its own radiation, but during equilibrium the radiated, absorbed and transferred (by heat conduction*) energy gives the total balance, which is equal to zero. Further we shall assume as an approximate condition that also during non-stationary processes such a situation is preserved; in other words, we shall study adiabatic motions of gas ($\varepsilon = 0$).

It is not difficult to show that the total energy which is radiated by cepheids over the periods of the variation in brightness is small in comparison with the total reserve of gravitational and internal thermal energy of the entire star. By this we can explain also the weak influence of the laws governing the distribution of sources of stellar energy upon the distribution of density and pressure in the stellar depths for ordinary stars and for cepheids. Therefore we can assume that in non-steady movements of a star on the whole the energy released at the center and radiated to outer space during the period of time of one fluctuation or oscillation does not play an essential role. In the consideration of non-steady movements we assume as the last assumption that the molecular weight, μ and the coefficient of heat conduction c_v are constant over the entire mass of the star.

From the above made assumptions it follows that in order to describe the non-steady movements of gaseous models of the stars we can make use of the following system of equations:

---

* The energy transferred by heat conduction is small in comparison with the energy that is transferred by radiation.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\left.\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial r} + \frac{2\rho u}{r} &= 0, \\ \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{fm}{r^2} &= 0, \\ \frac{\partial m}{\partial r} - 4\pi r^2 \rho &= 0, \\ \frac{\partial \frac{p}{\rho^\gamma}}{\partial t} + u\frac{\partial \frac{p}{\rho^\gamma}}{\partial r} &= 0, \end{aligned}\right\} \qquad \text{(I)}$$

where $\gamma = c_p/c_v$ is the Poisson constant. The last one of equations (I) follows immediately from the equations (2.4') and (2.5'), when $\varepsilon = 0$. The system (I) consists of four equations with four unknown functions: ρ, u, p, m.

In the case of equilibrium the equations (I) are simplified and reduced to one equation:

$$\frac{1}{\rho}\frac{dp}{dr} + \frac{4\pi f}{r^2}\int_0^r r^2 \rho\, dr = 0,$$

which contains two unknown functions ρ and p. In order to find the distribution of temperatures we must rely upon data on the distribution within a star of the sources of energy and upon the equations of the theory of radiant energy transfer.

In the presence of spherical symmetry and of thermodynamic irreversibility of phenomenon, the system of equations that describe the distribution of character-

$$\left.\begin{aligned} \frac{1}{\rho}\frac{dp}{dr} + \frac{fm}{r^2} &= 0, \\ m &= 4\pi \int_0^r r^2 \rho\, dr, \\ p &= \frac{R\rho T}{\mu} + \frac{a}{3}T^4, \\ \frac{d}{dr}\frac{R^1 T^4}{3} &= -\frac{\varkappa_1 Q \rho}{4\pi r^2}, \quad \varkappa_1 = \frac{\varkappa R^1}{ac}, \\ \frac{dQ}{dr} &= 4\pi r^2 \rho \varepsilon^*, \end{aligned}\right\} \qquad \text{(II)}$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

istics of state within a star during equilibrium can be taken in the preceding form*:

where κ is the coefficient of absorption, c is the velocity of light, ε^* is the intensity of the sources of energy which is released by unit mass of stellar substance in unit time, $L(r)$ is the flux of energy through a sphere of radius r.

In order to obtain the solutions to the equations of equilibrium (II) we can take as the independent variable the mass m and make use of the following boundary conditions:

$$\text{at the center of the star} \quad m = 0 \; \mathfrak{L} = \mathfrak{L}_0, \tag{2.6}$$

$$\text{at the surface of the star} \quad m = \mathfrak{M} \; \rho = 0 \text{ and } T = 0. \tag{2.6'}$$

If the sources of energy are distributed within the star continuously, then the quantity L_0 must be set equal to zero. In the presence at the center of the star of a concentrated source of energy we have $L_0 \neq 0$.

Since the temperature within the star is many times greater than the temperature at the surface, many astrophysicists ordinarily assume that at the surface of the star one can take $T = 0$. For given μ, κ, and ε^*, the solution to the system(II) depends upon three arbitrary constants for which there exist one boundary condition (2.6) at the center and two boundary conditions (2.6') at the surface of the star.

If the molecular weight μ, the coefficient of absorption κ_1, and the intensity of the sources of energy ε^* are given as functions of ρ and T:

$$\mu = \mu(\rho, T), \quad \kappa_1 = \kappa_1(\rho, T) \text{ and } \varepsilon^* = \varepsilon^*(\rho, T), \tag{2.7}$$

then the system of equation (II) and the boundary conditions (2.6) and (2.6')

---

* See V.A.Ambartsumyan, E.R.Mustel', A.B.Severnyy, V.V.Sobolev, Teoreticheskaya Astrofizika (Theoretical Astrophysics), State Technical Press, 1952, page 5.

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

completely determine the solution, which possesses the following form:

$$\left.\begin{aligned} \rho &= \rho(m, \mathfrak{M}, \mathfrak{L}_0, a_1, a_2, \ldots), \\ T &= T(m, \mathfrak{M}, \mathfrak{L}_0, a_1, a_2, \ldots), \\ \mathfrak{L} &= \mathfrak{L}(m, \mathfrak{M}, \mathfrak{L}_0, a_1, a_2, \ldots), \\ r &= r(m, \mathfrak{M}, \mathfrak{L}_0, a_1, a_2, \ldots), \end{aligned}\right\} \qquad (2.8)$$

where a_1, a_2, ... are physical constants entering into the system (II) and into the functional relations (2.7).

If $L_0 = 0$, then formulas of the following form for the luminosity and radius of the star follow from equations (2.8)

$$\left.\begin{aligned} \mathfrak{L}^* &= \mathfrak{L}(\mathfrak{M}, \mathfrak{M}, a_1, a_2, \ldots), \\ \mathfrak{R} &= r(\mathfrak{M}, \mathfrak{M}, a_1 a_2, \ldots). \end{aligned}\right\} \qquad (2.9)$$

Thus as a result of the solution of the posed problem it is possible to determine the relation between the luminosity of a star and its mass, radius of the star and its mass, and consequently we can obtain the possibility of theoretically comprehending the empirical results which were described in section 1 of this chapter.

The relations (2.9) depend parametrically upon the constants a_1, a_2, ..., the values of which can be different for different stars. For those groups of stars in which these constants are identical the relations (2.9) determine a unique dependence of luminosity upon mass of a star.

If $L_0 \neq 0$ and is given independently of M, then the functions (2.9) depend upon L_0, as upon an additional parameter. If all the sources of energy are concentrated at the center, namely a model with point source of energy, then in place of (2.9) we obtain formulas of the form

$$\left.\begin{aligned} \mathfrak{L}^* &= \mathfrak{L}_0, \\ \mathfrak{R} &= f(\mathfrak{M}, \mathfrak{L}_0, a_1, a_2, \ldots). \end{aligned}\right\} \qquad (2.10)$$

Thus, for a model with point source of energy it is impossible to establish as

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

a result of the solution of the system of equations (II) two relations of the form $L^*(M)$ and $R(M)$. These relations can be established in this case also if one is given additionally the dependence $L_0(\rho_0, T_0)$ where ρ_0 and T_0 are the density and temperature at the center of the star.

Section 3. The Theoretical Formulas for the Laws of Luminosity-Mass and Radius-Mass

For the purpose of setting up the task formulated in the preceding paragraph, the actual determination of the laws (2.9) is of great interest. This problem was treated by Strömgren* for the case of the following assumptions:

1. In the equation of state one rejects the term $aT^4/3$ which corresponds to the pressure of radiation.

2. The molecular weight, μ is constant everywhere within the star.

3. For the coefficient of absorption and for the sources of energy the following formulas hold true

$$\varkappa_1 = B\rho\,(RT)^{-3-s} \quad \text{AND} \quad \varepsilon^* = \varepsilon_0\,\rho^{\alpha}\,(RT)^{\beta},$$

where B, s, ε_0, α, β are constants.

Relying on these assumptions Strömgren established the formula

$$\mathfrak{L}^* = \mathfrak{D}\,\frac{1}{B}\,\frac{\mathfrak{M}^{5+s}}{\mathfrak{R}^{4}}\,\mu^{7+s}, \tag{3.1}$$

where the constant D depends only upon f, s, α, β.

It turned out, however**, that it is possible to show instead of the single

---

* B.Strömgren, Handbuch der Astrophysik, Vol. 7, 1936, page 159; Erg. Exact. Naturwiss., Vol. 16, 1937, page 465. The expounding of many results of the theory of stellar gas models can be found in the book: Chandrasekar, Introduction to the Science of the Structure of Stars, translated into Russian by the Foreign Literature Press, Moscow, 1950.

** See L.I.Sedov, Doklady Akademii Nauk SSSR, No. 4, 1954.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

relation (3.1) two simple relations of the form (2.9) with the help of the considerations in the theory of dimensions in the case of the preservation of assumption 1 and in the case of more general assumptions than 2 and 3, which are included in the following formulas:

$$\mu = \mu_0 \rho^{1-\xi} (RT)^{1-\eta}, \quad \varkappa_1 = B\rho^w (RT)^v, \quad \varepsilon^* = \varepsilon_0 \rho^\alpha (RT)^\beta, \tag{3.2}$$

where ξ, η, w, v, α, β are constants.

Let us introduce the following new variables

$$p_1 = \frac{p}{f}, \quad \tau = \frac{RT}{(f\mu_0)^{1/\eta}}, \quad x = \frac{m}{\mathfrak{M}}$$

the following designations:

$$\frac{a(f\mu_0)^{4/\eta}}{3fR^4} = \Omega \text{ AND } B\varepsilon_0 (f\mu_0)^{\frac{v+\beta-4}{\eta}} = \omega.$$

It is easy to verify that the system of equations (II) and boundary conditions (2.6) for the case $l_0 = 0$ are equivalent to the following system of equations

$$\left.\begin{aligned} &\frac{dp_1}{dx} = -\frac{x\mathfrak{M}^2}{4\pi r^4}, \qquad r^3 = \frac{3}{4\pi}\mathfrak{M}\int_0^x \frac{dx}{\rho}, \\ &p_1 = \rho^{\xi}\tau^{\eta} + \Omega\tau^4, \qquad \tau^3\frac{d\tau}{dx} = -\frac{3\omega\mathfrak{M}^2}{16\pi^2 r^4}\rho^w\tau^v\int_0^x \rho^\alpha\tau^\beta\, dx \end{aligned}\right\} \tag{3.3}$$

and to the following formula for the calculation of the luminosity

$$\mathfrak{L} = \varepsilon_0 \mathfrak{M} (\mu_0 f)^{\beta/\eta} \int_0^x \rho^\alpha \tau^\beta\, dx. \tag{3.4}$$

In this manner the problem is reduced to the solution of the system (3.3) for the following boundary conditions* (see conditions (2.6')):

* The boundary conditions (3.5) can be modified. If in the modified conditions there do not appear new dimensional physical constants, then all the succeeding conclusions preserve their force.

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

at the surface of the star we have for x = 1 the values

$$\rho = 0 \text{ AND } \tau = 0. \tag{3.5}$$

Hence it follows that if the solution of the posed mathematical problem exists and possesses physical significance, then the sought for functions

$$\tau,\ \rho,\ r,\ p_1 \left([p_1] = \frac{M^2}{L^4} \text{ AND } [\tau] = M^{\frac{2-i}{\eta}} L^{\frac{3i-4}{\eta}} \right)$$

are determined by the following quantities

$$x,\ \mathfrak{M},\ \omega,\ \Omega,$$

here the following formulas of dimensionality hold:

$$[\mathfrak{M}] = M,\ [\omega] = M^{k_1} L^{k_2},\ [\Omega] = M^{2-i\frac{2-i}{\eta}} L^{-4-i\left(\frac{3i-4}{\eta}\right)}, \tag{3.6}$$

where

$$\left.\begin{aligned} k_1 &= -2-\omega-\alpha+\frac{(4-\upsilon-\beta)(2-\xi)}{\eta}, \\ k_2 &= 4+3\omega+3\alpha+\frac{(4-\upsilon-\beta)(3\xi-4)}{\eta}. \end{aligned}\right\} \tag{3.7}$$

Further we assume that $k_2 \neq 0$. If it is assumed that $\Omega = 0$ (assumption 1) or if $\xi = \eta = 1$ (assumption 2), then in the case of $k_2 = 0$ among the defining quantities there are no quantities that depend upon linear dimensions, whereas the sought for quantities τ, η, r, p_1 are connected with linear dimensions. In this case the above-indicated statement of the problem by necessity must be made more precise.

From the defining parameters it is possible to compose only two independent abstract combinations

$$x = \frac{m}{\mathfrak{M}} \text{ И } \delta = \frac{\Omega}{\mathfrak{M}^{k_3}} \omega^{\frac{4\eta+4(3\xi-4)}{(4+3\omega+3\alpha)\eta+(4-\upsilon-\beta)(3i-4)}},$$

where

$$k_3 = \frac{1}{\eta}\left\{2\eta+4(\xi-2)-\frac{[4\eta+4(3\xi-4)][(2+\omega+\alpha)\eta-(4-\upsilon-\beta)(2-\xi)}{(4+3\omega+3\alpha)\eta+(4-\upsilon-\beta)(3\xi-4)}\right\}.$$

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Obviously, for $\mu = \text{const.}$, i.e., $\xi = \eta = 1$, we have the following

$$k_3 = -2 \text{ AND } \delta = \Omega \mathfrak{M}^3.$$

If the equalities $\xi = 1$ and $\eta = 1$ are fulfilled simultaneously, then for certain values of the indices ξ, η, α, β, w, v it is possible to satisfy the equality

$$k_3 = 0.$$

For $k_3 = 0$ we obtain that the abstract parameter δ does not depend upon the mass M.

From general considerations of the theory of dimensions it follows that the sought for solution possesses the following form

$$\left.\begin{aligned} r &= \left(\frac{\omega}{\mathfrak{M}^{k_1}}\right)^{1/k_2} r_1(x, \delta), \\ \rho &= \mathfrak{M}\left(\frac{\omega}{\mathfrak{M}^{k_1}}\right)^{-3/k_2} \rho_1(x, \delta), \\ \tau &= \mathfrak{M}^{\frac{2-\xi}{\eta}} \left(\frac{\omega}{\mathfrak{M}^{k_1}}\right)^{\frac{3\xi-4}{k_2\eta}} \tau_1(x, \delta), \\ p_1 &= \mathfrak{M}^2\left(\frac{\omega}{\mathfrak{M}^{k_1}}\right)^{-4/k_2} (\rho_1^{\xi}\tau_1^{\eta} + \delta\tau_1^4). \end{aligned}\right\} \qquad (3.8)$$

Substitution of eqs.(3.8) into the system of equations (3.3) and the boundary conditions (3.5) lead to the following equations

$$\left.\begin{aligned} \frac{d}{dx}(\rho_1^{\xi}\tau_1^{\eta} + \delta\tau_1^4) &= -\frac{x}{4\pi r_1^4}, \\ r_1^3 &= \frac{3}{4\pi}\int_0^x \frac{dx}{\rho_1}, \\ \tau_1^{\alpha}\frac{d\tau_1}{dx} &= -\frac{3}{16\pi^2 r_1^4}\rho_1^{w}\tau_1^{v}\int_0^x \rho_1^{\beta}\tau_1^{\beta}\,dx \end{aligned}\right\} \qquad (3.9)$$

and to the following boundary conditions

$$\text{FOR } x = 1 \quad \rho_1 = 0 \text{ AND } \tau_1 = 0. \qquad (3.10)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Equations (3.9) and conditions (3.10) do not contain the dimensional constants M, ω and they determine the dimensionless functions $\tau_1(x,\delta)$, $\rho_1(x,\delta)$ and $r_1(x,\delta)$. The parameter of δ does not enter if one uses assumption 1°. In the latter case the functions $r_1(x)$, $\rho_1(x)$ and $\tau_1(x)$ are universal numerical functions that depend only upon the value of the indices ξ, η, w, v, α, β.

On the basis of formulas (3.8) and (3.4) taking into consideration that $\omega = B\varepsilon_0(f\mu_0)^{\frac{v+\beta-4}{\eta}}$ and formulas (3.7), we obtain for the radius R and the luminosity L* of a star the following expressions:

$$\left.\begin{aligned}
&\mathfrak{R} = [(B\varepsilon_0)^{\eta}(f\mu_0)^{v+\beta-4}\times\\
&\times \mathfrak{M}^{[(2+w+\alpha)\eta-(4-v-\beta)(2-\xi)]\frac{1}{(4+3w+3\alpha)\eta+(4-v-\beta)(3\xi-4)}} r_1(1,\delta),\\
&\mathfrak{L}^* = \varepsilon_0(B\varepsilon_0)^{\frac{(3\xi-4)\beta-3\alpha\eta}{(4+3w+3\alpha)\eta+(4-v-\beta)(3\xi-4)}}\times\\
&\times (f\mu_0)^{\frac{\beta}{\eta}+\frac{v+\beta-4}{\eta}\left[\frac{(3\xi-4)\eta-3\alpha\eta}{(4+3w+3\alpha)\eta+(4-v-\beta)(3\xi-4)}\right]}\times\\
&\times \mathfrak{M}^{1+\delta+\frac{2-\xi}{\eta}\beta+\frac{[(2+w+\alpha)\eta-(4-v-\beta)(2-\xi)][(3\xi-4)\beta-3\alpha\eta]}{[(4+3w+3\alpha)\eta+(4-v-\beta)(3\xi-4)]\eta}}\cdot\int_0^1 \rho_1^{\alpha}\tau_1^{\beta}\,dx.
\end{aligned}\right\} \qquad (3.11)$$

If one disregards the light pressure in the equation of state (assumption 1), then it is necessary to assume $\delta = 0$, after which the formulas (3.11) determine completely the dependence of R and L* upon ε_0, $f\mu_0$, B and upon the mass M of a star.

If we have $\delta \neq 0$ but $k_3 = 0$, then δ does not depend upon the mass of the star, and therefore in this case the formulas (3.11) also completely determine the dependence of R and L* upon the mass M of the star.

If we eliminate ε_0 from the formulas (3.11), then we obtain the following relation:

$$B\mathfrak{L}^* = \mathfrak{D}\cdot(f\mu_0)^{\frac{4-v}{\eta}}\cdot\mathfrak{R}^{4+3w+(4-v)\frac{3\xi-4}{\eta}}\cdot\mathfrak{M}^{-1-w+(4-v)\frac{2-\xi}{\eta}}, \qquad (3.12)$$

where D is an abstract constant, which for $\delta = 0$ can depend only upon ξ, η, α, β, w, v.

It is curious to note that in formula (3.12) only the constant D can depend up-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

on the law governing the release of energy.

Formula (3.1) is obtained as a particular case of formula (3.12).

It is not difficult to persuade oneself immediately that the formula (3.12) preserves its force for the case of the model of a star with point source of energy at the star's center, when the power of the point source of energy L^* is assigned arbitrarily.

Formulas (3.12) and (3.11) can be utilized for the processing of data of observations. In comparing these formulas with empirical data one can obtain certain principles for judging the laws (3.2) and also for judging the correctness of the assumed statement of the problem.

Section 4. Certain Simple Solutions of the System of Equations of Equilibrium of Stars

In the theory of the outburst of novae during the investigation of the non-steady motions of gaseous masses of a star it is necessary to utilize in the initial conditions data on the distribution of the characteristics of state of the gas within the star during equilibrium. For this purpose utilization of the solutions to the system of equations (II) with boundary conditions (2.6) and (2.7) is inconvenient because of the complexity of these solutions and excludes the possibility of obtaining any effective solutions concerning non-steady motions.

In addition to this, for a deeper clarification of the role of the various physical factors it is useful to consider the solutions to the system (II), which are not subject to the boundary conditions on the surface of the star.

Let us consider the exact solutions to the equations of equilibrium (II) (in which we disregard the light pressures); here we shall rely on certain additional hypotheses instead of the boundary conditions on the surface of the star. Proceeding from considerations of dimensionality we shall treat the very simple hypothesis that the distribution of the characteristics of state besides the forces of gravitation which are connected with the values of the gravitational constant f depends

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

essentially upon still another physical law or other the influence of which can be realized by means of only one characteristic physical constant, which we will designate by the letter A.

More definitely we shall consider the equilibrium of a gas for which the distribution of the density and of the pressure depends only upon the following three dimensional parameters:

$$[r] = L, \quad [f] = \frac{L^s}{MT^2}, \quad [A] = ML^kT^s. \tag{4.1}$$

The dimensionality of the parameter A (namely the values of the constants k and s) is fixed by the above indicated hypothesis, which we shall not make more concrete and definite. Let us consider the case where the dimensionality of the supplementary physical constant A contains the symbol of mass. Obviously, without limiting the generality, we can assume that in the formula for the dimensionality of the constant A the symbol for mass enters in the first power*.

It is easy to verify that from hypothesis (4.1) proceed the following formulas:

$$\left.\begin{aligned} \rho &= \alpha_1 \frac{A^{\frac{2}{2-s}} f^{\frac{s}{2-s}}}{r^{\frac{2k+6}{2-s}}} \\ m &= \alpha_2 A^{\frac{2}{2-s}} \cdot f^{\frac{s}{2-s}} \cdot r^{-\frac{2k+3s}{2-s}}, \\ p &= \alpha_3 \frac{A^{\frac{4}{2-s}} f^{\frac{2+s}{2-s}}}{r^{\frac{8+2s+4k}{2-s}}}, \\ RT &= \alpha_4 \frac{A^{\frac{2}{2-s}} f^{\frac{2}{2-s}}}{r^{\frac{k+s+1}{2-s}}}, \end{aligned}\right\} \tag{4.2}$$

where the values α_1, α_2, α_3, α_4 are abstract constants and s does not equal 2:

* If the additional assigned constant A* is a kinematic quantity, then one can take as the quantity A the constant equal to A*/f.

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$s \neq 2$. If $s = 2$ and $k \neq -3$, then it is impossible to establish the dependence upon r, since in this case there exists an abstract combination Af/r^{k+3}; if $s = 2$ and $k = -3$, then the dimensionalities of f and A are dependent and therefore in this case the system (4.1) is incomplete and connot determine p and ρ.

According to its physical significance the mass m(r) is a positive monotone quantity which does not decrease with increasing r; therefore it must be :

$$-\frac{2k+3s}{2-s} > 0. \qquad (4.3)$$

Formulas (4.2) indicate that the center of symmetry is a singular point at which the density, pressure and temperature generally speaking are infinite. On the one hand this fact in the familiar sense can reflect the actual status of things, since at the center of a star the density, pressure and temperature possess a maximum and attain very large values. On the other hand it follows from physical considerations that the pressure, density and temperature at the center of a star are essentially finite. Hence it follows that in the immediate neighborhood of the center of a star, in those cases where infinite values of the characteristics of state obtain, the assumed hypothesis concerning the existence of only two physical constants f and A is inadmissible. This serves also to indicate the fact that close to the center of a star, besides the quantities f and A, still other supplementary physical factors become essential. However, if one admits that such greater refinement and precision is necessary only in the immediate neighborhood of the center of the star, then formulas (4.2) can be used for modeling of the actual equilibrium outside the neighborhood of the center of the star*.

Formulas (4.2) indicate also that the pressure and density can be converted

---

* We note by the way that close to the center of the star the gravitational constant apparently does not exist in explicit form, since close to the center of the star the uniformly acting force of Newtonian gravity is close to zero. However, we shall not touch upon here the deeper phenomena close to the center of the star.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

into zero only at infinity and that the mass tends to infinity together with r if $-\frac{2k+3s}{2-s} > 0$. This fact indicates that the hypothesis under consideration also requires correcting at remote distances from the center of the star.

If we have $\frac{2k+3s}{s-2} = 0$, then $k = -\frac{3}{2}s$; in this case the combination $\frac{2}{A^{2-s}}\frac{s}{f^{2-s}} = m_0$ possesses the dimensionality of mass. This constant can be taken in place of A, and therefore the formulas (4.2) acquire the following form*:

$$\rho = \alpha_1 \frac{m_0}{r^3}, \quad m = m_0, \quad p = \alpha_2 \frac{m_0^2 f}{r^4}, \quad RT = \alpha_3 \frac{m_0}{r}. \tag{4.4}$$

It is obvious that in this case the following equation

$$m = 4\pi \int_0^r r^2 \rho \, dr$$

is not satisfied; therefore this case cannot reduce to the exact solution of system (II). It is not difficult to discern that we are satisfied by the first three equations of system (II):

$$\frac{1}{\rho}\frac{dp}{dr} + \frac{fm}{r^2} = 0, \quad m = 4\pi \int_0^r r^2 \rho \, dr, \quad p = \frac{\rho RT}{\mu}, \tag{4.5}$$

where μ is a constant, if we define the constants α_2, α_3, and α_4 in terms of the constant α_1 in accordance with the formulas

$$\alpha_1 = -4\pi \frac{2-s}{2k+3s}\alpha_1^3, \tag{4.6}$$

$$\alpha_2 = -2\pi \frac{(2-s)^2\alpha_1^2}{(2k+3s)(4+s+2k)}, \tag{4.7}$$

$$\alpha_3 = \frac{\mu\alpha_2}{\alpha_1} = -2\pi\mu \frac{(2-s)^2\alpha_1}{(2k+3s)(4+s+2k)}. \tag{4.8}$$

It is not difficult to see that the exponential index $2\frac{4+s+2k}{2-s}$ in the formula for the pressure must be different from zero, since in the opposite case the

---

* We set $m = m_0$ and consequently $\alpha_4 = 1$.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

pressure would be obtained as a constant relative to the radius, which excludes the possibility of satisfying the equation of equilibrium.

We now turn our attention to the problem of the construction of a solution of the desired type which satisfies the last two equations of system (II), namely the equations of the theory of radiation:

$$\frac{d}{dr} R^4 T^4 = -\frac{3\varkappa_1 \mathfrak{L}\rho}{4\pi r^2} \quad \text{и} \quad \frac{d\mathfrak{L}}{dr} = 4\pi r^2 \rho \varepsilon^*. \tag{4.9}$$

These equations can be satisfied by means of a solution of the form (4.2), if the sources of energy are concentrated at the center of the star $\varepsilon^* = 0$ for $r > 0$ and if for the coefficient of absorption the formula of the following form is true

$$\varkappa_1 = B\rho^w (RT)^v, \tag{4.10}$$

where v, w and B are constants. It is easy to verify that the dimensionality of the constant B is given by the following formula

$$[B] = \mathrm{M}^{-(2+w)} \mathrm{L}^{10+3w-2v} \mathrm{T}^{2v-5}.$$

From equations (4.9) we obtain

$$\mathfrak{L} = \mathfrak{L}_0 = \text{const.} \quad \text{и} \quad \varkappa_1 = -\frac{4\pi}{3\mathfrak{L}_0} \frac{r^2}{\rho} \frac{d}{dr} (RT)^4. \tag{4.11}$$

Comparison of (4/10) and (4.11) in the case of the employment of the formulas (4.2) gives that the equations of radiation are satisfied if the following relations are fulfilled:

$$k(2w + 2v - 6) + s(2v - 9) + 6w + 2v = 0 \tag{4.12}$$

and

$$\frac{32\pi}{3} \cdot \frac{\alpha_3^{1-v}}{\alpha_1^{1-w}} \cdot \frac{s+k+1}{2-s} = \frac{B\mathfrak{L}_0}{A^{\frac{6-2w-2v}{2-s}} f^{\frac{8-s-sw-2v}{2-s}}}. \tag{4.13}$$

Equation (4.12) gives the connection between k, s, w, and v. On the basis of (4.6), (4.7), (4.8), (4.12) and (4.13) the coefficients α_1, α_2, α_3 and α_4 are ex-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

pressed in terms of B, L_0, μ, A, f, w and v. After substitution of these expressions into (4.2) we find the exact solution of the complete system of equations of equilibrium of a gas (II), which solution is represented by the following formulas:

$$\left.\begin{aligned}
\rho &= \left\{\frac{(3-w-v)}{16\pi(2w+3)}\left[\frac{(3w+v)(v-w-6)}{(3-w-v)^2 2\pi\mu f}\right]^{4-v}\frac{3B\varrho_0}{r^{v-2w}}\right\}^{\frac{1}{3-w-v}},\\
m &= 4\pi\frac{w+v-3}{3w+v}\left\{\frac{3-w-v}{16\pi(2w+3)}\right.\times\\
&\qquad\times\left.\left[\frac{(3w+v)(v-w-6)}{(3-w-v)^2 2\pi\mu f}\right]^{4-v}\frac{3B\varrho_0}{r^{3w+v}}\right\}^{\frac{1}{3-w-v}},\\
p &= \left\{\frac{3-w-v}{16\pi(2w+3)}\mu^{v-4}\left[\frac{(3w+v)(v-w-6)}{(3-w-v)^2 2\pi f}\right]^{\frac{5+w-v}{2}}\frac{3B\varrho_0}{r^{6+w-v}}\right\}^{\frac{2}{3-w-v}},\\
RT &= \left\{\frac{(3-w-v)}{16\pi(2w+3)}\left[\frac{(3w+v)(v-w-6)}{(3-w-v)^2 2\pi\mu f}\right]^{w+1}\frac{3B\varrho_0}{r^{3+2w}}\right\}^{\frac{1}{3-w-v}}.
\end{aligned}\right\} \qquad (4.14)$$

In the thus constructed exact solution of the problem concerning the equilibrium of a mass of gas that gravitates in accordance with the law of Newton, the laws governing the variation of p, ρ and RT according to the radius are determined completely in terms of the exponential indices w and v in the formula for the coefficient of absorption and in terms of the product of constant B entering into the law for the absorption coefficient and power of the radiation source L_0 located at the center of symmetry.

In the obtained solution (4.14) the constant A does not figure; this fact is explained by the circumstance that the constants α_1 and A enter only in the combination

$$\alpha_1 A^{\frac{2}{2-s}},$$

which in virtue of the equations of equilibrium in the problem under consideration is expressed in terms of the product BL_0. The power dependence upon BL_0 is obtained in explicit form in terms of the exponential indices w and v.

The solution (4.14) permits one to evaluate the influence of the laws for the

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

coefficient of absorption of x upon the equilibrium of gases with sources of radiation.

If we are given the dimensionality of the constant A (that is, the exponential indices k and s), then the formulas (4.2) determine all the laws for the variation of the characteristics of state according to the radius, whereas the exponential indices w and v in formula (4.10) can vary in satisfaction of condition (4.12). If the exponential indices w and v are given, then the laws governing the variation in accordance with the radius are also determined, but the constants k and s can vary in satisfaction of the same condition (4.12).

In accordance with the solution (4.14) the corresponding gas model of a star occupies all of infinite space and possesses infinite mass. It is obvious that the mass is finite within any sphere S of finite radius at the surface of which the pressure p_S can be very small. In the case where the pressure p_S at the surface S is fixed the presence of infinite mass outside the sphere S does not exert any influence upon the equilibrium of the masses within the sphere S. Thus the equilibrium of a finite mass within the sphere S is not connected in an essential manner with the laws governing the distribution of the characteristics of equilibrium outside the sphere S. In order to obtain the approximate solutions with finite mass one can make use of a solution of the type (4.14) within a certain sphere S, but outside this sphere the solution can be continued with continuous variation of the pressure* and with a certain other law for the variation of the density which ensures the finiteness and assigned magnitude of the mass.

In particular, if the constant A possesses the dimensionality of energy, that is $[A] = ML^2T^{-2}$ (hence k = 2 and s = -2, then the condition (4.12) gives

$$v = -3 - 5w. \tag{4.15}$$

* In the approximate construction of the equilibrium of a finite mass of gas one can admit a discontinuity in the distribution of density.

STAT

34

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In this case the formulas (4.14) acquire the following form:

$$\left.\begin{aligned} \rho &= \frac{1}{2^{2.5}}\left[\left(\frac{3}{2\pi\mu f}\right)^{7+5w}\frac{3B\varrho_0}{4\pi}\right]^{\frac{1}{6+4w}}\cdot\frac{1}{r^{2.5}}, \\ m &= 2\pi\left[\left(\frac{3}{2\pi\mu f}\right)^{7+5w}\frac{3B\varrho_0}{2^{5+2w}\pi}\right]^{\frac{1}{6+4w}}\cdot r^{0.5}, \\ p &= \frac{1}{8}\left[\left(\frac{3}{2\pi\mu f}\right)^{4+3w}\frac{3B\varrho_0}{\mu^{3+2w}4\pi}\right]^{\frac{1}{3+2w}}\cdot\frac{1}{r^3}, \\ RT &= \frac{1}{\sqrt{2}}\left[\left(\frac{3}{2\pi\mu f}\right)^{1+w}\frac{3B\varrho_0}{4\pi}\right]^{\frac{1}{6+4w}}\cdot\frac{1}{r^{0.5}}. \end{aligned}\right\} \qquad (4.16)$$

For the complete determinacy of the solution it is necessary to select the value of the exponential index w or, in agreement with (4.15), the exponential index v.

By means of formulas (4.14) it is easy to write down the solutions which correspond to the various requirements of particular formulas for the coefficient of absorption in astrophysics.

For example, if for the coefficient of absorption one takes the formula of Kramers

$$\varkappa = B\cdot\frac{ca}{R^4}\cdot\rho T^{-7/2} \quad (w=1,\ v=-3{,}5), \qquad (4.17)$$

then the solution (4.14) assumes the form

$$\left.\begin{aligned} \rho &= 0{,}00457\,\frac{(B\varrho_0)^{2/11}}{(\mu f)^{15/11}}\,\frac{1}{r^{32/11}}, \qquad m = 0{,}6319\,\frac{(B\varrho_0)^{2/11}}{\mu f^{15/11}}\,r^{4/11}, \\ p &= 0{,}000756\,\frac{(B\varrho_0)^{6/11}}{\mu(\mu f)^{10/11}}\,\frac{1}{r^{12/11}}, \qquad RT = 0{,}1654\,\frac{(B\varrho_0)^{2/11}}{(\mu f)^{4/11}}\,\frac{1}{r^{10/11}}. \end{aligned}\right\} \qquad (4.18)$$

Let us consider still another case, one where $[A] = [L] = ML^2T^{-3}$ (that is, $k = 2$, $S = -3$), which can correspond to the assumption that the distribution of states according to radius is determined by the total luminosity. This case is special and it is necessary to consider it separately, since from formulas (4.2),

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

(4.6), (4.7) and (4.8) we obtain:

$$\left.\begin{aligned} p &= \alpha_1 \frac{A^{2/5} f^{-2/5}}{r^2}, \qquad m = 4\pi\alpha_1 A^{2/5} f^{-2/5} \cdot r, \\ p &= 2\pi\alpha_1^2 \frac{A^{4/5} f^{-1/5}}{r^2}, \quad RT = 2\pi\alpha_1 \mu (Af)^{2/5} = \text{const.} \end{aligned}\right\} \qquad (4.19)$$

Hence it follows that the temperature is obtained as a constant relative to the radius. In this case the following equation

$$\frac{d(RT)^4}{dr} = -\frac{\varkappa_1 Q \rho}{4\pi r^2},$$

if $L \neq 0$, can be satisfied only for the assumption that $\varkappa_1 = 0$. After this the second equation of the theory of radiation

$$\frac{dQ}{dr} = 4\pi r^2 \rho \varepsilon^*$$

can always be satisfied and employed for the determination of L, if ε^* is known as a function of r, ρ and T*.

In particular, if $\varepsilon^* = 0$, then $L = L_0 = \text{const}$. In the formulas (4.19) there is contained only one essential arbitrary parameter, which is equal to $\alpha_1 A^{2/5}$.

Section 5. Concerning the Dependence Between the Period of Oscillation of Brightness and the Mean Mass Density for the Cepheids

The data of observations on the period of variation in brightness and on the velocities of gas radiating particles testifies conclusively to the fact that the observed fluctuations in luminosity are due to radial pulsations with large amplitudes and to velocities of gaseous masses forming the star.

Utilizing the considerations expounded in Section 2, we can assume that these

---

* It is evident that these considerations can refer also to the more general case where $s \cdot k \cdot l = 0$, which reduces in accordance with formulas (4.2) to constancy of the temperature.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

pulsations represent an adiabatic process which is described by the system of equations (I) in Section 2.

The motion under consideration can be an essentially nonlinear process. Within the gas are possible shock waves which are propagated toward the center of stars and from the center toward the periphery. It is possible that in certain intervals of time the boundary of the luminous photosphere during its expansion coincides with the shock wave. Since the velocity of the shock wave is greater than the velocity of the particles of the gas behind its front, then during expansion of the photosphere which limits the shock wave its maximum diameter will be considerably larger than the transverse linear dimension computed by integration of the radial velocities measured in observations.

In the propagation of the shock waves toward the center of a star, at the moment of reflection from the center there is the possibility of the occurence of the motions considered in Section 4 of Chapter IV in the case of the fixing of the flow of gas at a point. Variation in the brightness of cepheids can be explained by the large variations in temperature that occur during the pulsations in the photosphere and by the variations in the magnitude of absorption in the gaseous atmosphere surrounding the cepheid. For various diameters of the photosphere the magnitude of the path of the light beam in the surrounding atmosphere will vary.

For the description of the pulsations of gaseous balls of cepheid models, we shall keep in mind besides the system of equations (I) and the conditions at the jump discontinuities, which possess the same form as in the absence of the forces of gravity, the following boundary conditions:

$$\left.\begin{array}{l} \text{at the center of the star for } r = 0 \text{ we have: } m = 0 \text{ and } u = 0 \\ \text{at the surface of the star } r = R + \Delta R: \; m = M,\; p = 0 \text{ and } \rho = 0. \end{array}\right\} \quad (5.1)$$

Let us assume still another hypothesis, namely that the steady state regime of pulsations is determined completely only by the equations of motion, by the conditions at the jump discontinuities, by the boundary conditions and other conditions

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

which do not contain new dimensional constants. Under such assumptions it is not difficult to write down the system of defining parameters. This system is represented by the table

$$r,\ t,\ \gamma,\ f,\ R,\ M \qquad (5.2)$$

The mean radius R of the star and its mass M in a given case we shall consider as the independent parameters; as was indicated in Section 2, during equilibrium the radius R is a function of the mass M. However, in the case of pulsations such a relation between R and M cannot even be*, which fact can even be the cause of the absence of equilibrium, namely the cause for the pulsation of cepheids.

The mathematical nonlinear problem of finding u, ρ and p in the above indicated statement of the problem is very difficult. In the investigation of this problem there are only individual results obtained with the aid of additional essential assumptions and based in most cases upon the linearization of the equations of motion**.

However, without carrying out the solution one can in the most general statement of the problem establish by means of the considerations of the theory of dimensions the fundamental result that relates the period of pulsations of a cepheid with its mass and radius.

In fact, let us designate by the letter τ the period of pulsations of a

---

* In the case of the presence of such a relation in the form of a power dependence, besides the mass M there appears also a dimensional constant contained in this relation. In a system of defining parameters, besides the mass M and this constant (which can be different for different series of cepheids) one can take directly M and R.

** An exposition of these results is contained in the book: Rosseland, Theory of the Pulsations of Variables, translated into Russian by Foreign Literature Press, Moscow, 1952.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

cepheid; since this is the characteristic of motion on the whole, then on the basis of the established table of defining parameters (5.2) we derive that tau is

$$\tau = f(\gamma, f, \mathfrak{R}, \mathfrak{M}). \tag{5.3}$$

Since f, R and M possess independent dimensionalities, then it follows from this that

$$\frac{1}{\tau} = f(\gamma)\sqrt{\frac{f\mathfrak{M}}{4\pi\mathfrak{R}^3}} = f_1(\gamma)\sqrt{f\rho_{\text{ср}}}. \tag{5.4}$$

Such are the simple considerations that permit one without carrying out the detailed mathematical solution to substantiate for rather general assumptions the relation (5.4), which was established at first by means of empirical observations.

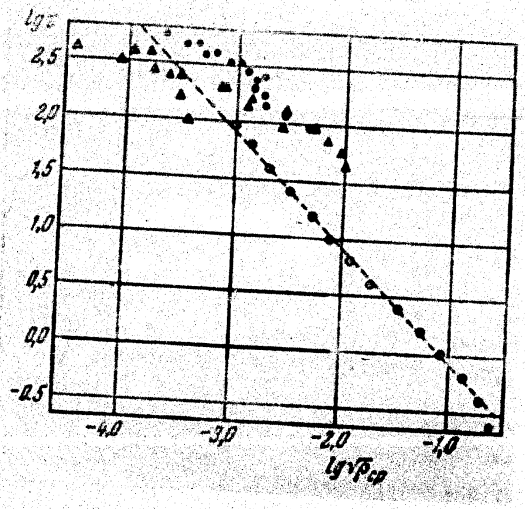


Fig.88. Empirical Data on the Relation Period–density for Different Variables

●) long-period variables ▲) semi-regular variables ●) cepheids

Figure 88 shows in logarithmic scales log τ and $\log\sqrt{\rho_{av}}$ diagrams of the results of the processing of observations; the individual points are plotted as the mean

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

values for groups of stars, and the dotted straight line is according to the theoretical formula (5.4). The disposition of the points which were obtained from the observations of cepheids indicates remarkable agreement with the slope of the theoretical dotted straight line.

Relation (5.4) evidently reflects the fundamental laws in the pulsations of gaseous masses that form the variable stars.

Processing of the observations of semi-regular and long-period variables indicates that the corresponding points on a similar diagram are arranged along certain straight lines which are parallel to the corresponding straight line for cepheids.

Section 6. Contributions to the Theory of the Outbursts of Novae and Supernovae

1. The equations of motion and the boundary conditions.

In agreement with the data of observations presented in Section 1 it follows that the outbursts of novae and supernovae represent the nonsteady motions of large masses of gas which are caused by an abrupt explosion, namely by a violent release of tremendous energy.

For the theoretical comprehension of such grandiose cosmic catastrophes we shall study examples of the exact solutions of equations of nonsteady motion of a gas taking into consideration the gravitational forces which can be considered as schematic models reflecting certain essential outlines of the actual phenomena of stellar outbursts.

Let us consider the solutions of the system of equations (I) of nonsteady adiabatic motions of a gas with spherical symmetry, described in Section 2 and possessing the following form

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

values for groups of stars, and the dotted straight line is according to the theoretical formula (5.4). The disposition of the points which were obtained from the observations of cepheids indicates remarkable agreement with the slope of the theoretical dotted straight line.

Relation (5.4) evidently reflects the fundamental laws in the pulsations of gaseous masses that form the variable stars.

Processing of the observations of semi-regular and long-period variables indicates that the corresponding points on a similar diagram are arranged along certain straight lines which are parallel to the corresponding straight line for cepheids.

Section 6. Contributions to the Theory of the Outbursts of Novae and Supernovae

1. The equations of motion and the boundary conditions.

In agreement with the data of observations presented in Section 1 it follows that the outbursts of novae and supernovae represent the nonsteady motions of large masses of gas which are caused by an abrupt explosion, namely by a violent release of tremendous energy.

For the theoretical comprehension of such grandiose cosmic catastrophes we shall study examples of the exact solutions of equations of nonsteady motion of a gas taking into consideration the gravitational forces which can be considered as schematic models reflecting certain essential outlines of the actual phenomena of stellar outbursts.

Let us consider the solutions of the system of equations (I) of nonsteady adiabatic motions of a gas with spherical symmetry, described in Section 2 and possessing the following form

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\left.\begin{aligned}
&\frac{\partial \rho}{\partial t}+\frac{\partial \rho u}{\partial r}+\frac{2\rho u}{r}=0,\\
&\frac{\partial m}{\partial r}=4\pi r^2\rho,\\
&\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{fm}{r^2}=0,\\
&\frac{\partial \frac{p}{\rho^\gamma}}{\partial t}+u\frac{\partial \frac{p}{\rho^\gamma}}{\partial r}=0.
\end{aligned}\right\} \qquad (6.1)$$

Let the nonsteady motion arise at the moment t = 0 in the resting mass of gas as a result of the liberation of energy at the center of symmetry.

The initial distribution of the characteristics of the gas which is defined by the conditions of equilibrium we conceive in the form obtained in Section 4

$$\left.\begin{aligned}
&u=0,\quad p=\alpha_1\frac{A^{\frac{2}{2-s}}f^{\frac{s}{2-s}}}{r^{\frac{2k+6}{2-s}}},\quad m=4\pi\frac{s-2}{2k+3s}\alpha_1 A^{\frac{2}{2-s}}f^{\frac{s}{2-s}}r^{\frac{2k+3s}{s-2}},\\
&p=-4\pi\frac{(2-s)^2\alpha_1^2}{(2k+3s)(8+2s+4k)}\frac{A^{\frac{4}{2-s}}f^{\frac{2+s}{2-s}}}{r^{\frac{8+2s+4k}{2-s}}}.
\end{aligned}\right\} \qquad (6.2)$$

The constant A possesses the dimensionality ML^kT^s. The abstract constant α_1, the constant A and the exponential indices k and s can be expressed in terms of the power of the energy source located at the center of the star (the source is active during equilibrium before the outburst) and in terms of the constants entering into the formula (4.10).

If the sought for nonsteady motion of the gas, on the whole determined by the system (6.1), by the initial conditions (6.2) and by the conditions at the jump discontinuities which can occur in the flux, depends only upon two constants with independent dimensionalities*

$$[f]=\frac{L^3}{MT^2}\ \text{AND}\ \ [A_1]=ML^kT^s, \qquad (6.3)$$

* The constant A in eq.(6.2) and the constant A_1 have equal values.

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

then the energy which begins to be liberated at the moment of time t = 0 at the center of symmetry must be determined by the dimensional quantities f, A_1 and t; therefore this energy given by the additional conditions must be represented by the following formula

$$E = \alpha f^{\frac{2-k}{3+k}} A_1^{\frac{5}{3+k}} \cdot \frac{1}{f^{\frac{5s+4k+2}{k+3}}}, \tag{6.4}$$

where α is a certain abstract constant, which in particular can be equal to infinity. If the constant A_1 possesses the dimensionality of energy [the solution concerning the equilibrium of the gas is determined by the formulas (4.16)], then

$$k = 2, \quad s = -2, \quad \frac{5s+4k+2}{k+3} = 0, \tag{6.5}$$

and the formula (6.4) gives:

$$E = \alpha A_1. \tag{6.6}$$

The case where α is finite corresponds to the instantaneous release of finite energy in the center of the star.

If we have

$$\frac{5s+4k+2}{k+3} < 0, \tag{6.7}$$

then starting at the moment of time t = 0 the energy being released grows continuously.

It is obvious that the sought-for field of disturbed motion is determined by the following system of parameters

$$A, \ A_1, \ f, \ \gamma, \ r, \ t, \tag{6.8}$$

therefore the motion will be automodeling and consequently the following formulas will be true for the distribution of the characteristics of motion of the gas:

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\left.\begin{aligned} u &= \frac{r}{t} V(\lambda), \\ \rho &= \frac{1}{\mathcal{A} t^2} R(\lambda), \\ m &= \frac{r^3}{\mathcal{A} t^2} M(\lambda), \\ p &= \frac{r^2}{\mathcal{A} t^2} P(\lambda), \end{aligned}\right\} \tag{6.9}$$

where V, R, M and P are dimensionless quantities which can depend upon certain abstract constants, in particular upon γ and only upon one variable parameter λ, which can be determined by the formula

$$\lambda = \frac{r}{(\mathcal{A}_1 f)^{\frac{1}{k+3}} t^{\frac{2-s}{k+3}}}.$$

For each characteristic phase of the motion we have λ = const., which determines the dependence of the radius r upon the time t at points with the given phase.

After the substitution of (6.9) into the system of equations with partial derivatives we obtain a system of four ordinary differential equations for the functions V (λ), R (λ), M (λ) and P (λ):

$$\left.\begin{aligned} \lambda\left\{\left(\frac{2-s}{k+3} - V\right)V' - \frac{P'}{R}\right\} &= M + V^2 - V + \frac{2P}{R}, \\ \lambda\left\{V' - \left(\frac{2-s}{k+3} - V\right)\frac{R'}{R}\right\} &= 2 - 3V, \\ \lambda\left(\frac{2-s}{k+3} - V\right)\left(\frac{P'}{P} - \gamma\frac{R'}{R}\right) &= 2V + 2(\gamma - 2), \\ \lambda M' &= -3M + 4\pi R. \end{aligned}\right\} \tag{6.10}$$

Instead of the function we can introduce the function $z = \frac{\gamma P}{R} = \frac{\gamma p}{\rho} : \frac{r^2}{t^2}$ and transform this system of equations into the following form

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$\left.\begin{aligned}
\lambda\left\{\left(\frac{2-s}{k+3}-V\right)V'-\frac{z}{\gamma}\left(\frac{z'}{z}+\frac{R'}{R}\right)\right\} &= \\
&= M+V^2-V+2\frac{z}{\gamma}, \\
\lambda\left\{V'-\left(\frac{2-s}{k+s}-V\right)\frac{R'}{R}\right\} &= 2-3V, \\
\lambda\left(V-\frac{2-s}{k+s}\right)\left[\frac{z'}{z}+(1-\gamma)\frac{R'}{R}\right] &= -2V-2(\gamma-2), \\
\lambda M' &= -3M+4\pi R.
\end{aligned}\right\} \qquad (6.11)$$

In the occurrence of the abrupt release of energy at the center of the star there is formed a shock wave which is propagated from the center toward the periphery of the star.

At the surface of the shock wave which is propagating in the gas at rest we must fulfill the following conditions of conjointness*

$$\begin{aligned}
v_2 &= \frac{2}{\gamma+1}c\left[1-\frac{a_1^2}{c^2}\right], \\
\rho_2 &= \frac{\gamma+1}{\gamma-1}\rho_1\left[1+\frac{2}{\gamma-1}\frac{a_1^2}{c^2}\right]^{-1}, \\
m_2 &= m_1, \\
p_2 &= \frac{2}{\gamma+1}\rho_1 c^2\left[1-\frac{\gamma-1}{2\gamma}\frac{a_1^2}{c^2}\right].
\end{aligned}$$

Here c is the velocity of propagation of the jump discontinuity, and $a_1^2 = \gamma p_1/\rho_1$ is the square of the velocity of souhd in the unperturbed state.

The radius of the shock wave r_2 is determined only by three dimensional parameters with independent dimensionalities f, A_1, t; therefore we have

$$r_2 = \lambda^*(A_1 f)^{\frac{1}{k+3}} t^{\frac{2-s}{k+3}} \quad \text{at} \quad \lambda = \lambda^* = \text{const}. \qquad (6.12)$$

We select the numerical value of the constant A_1 such that the abstract constant λ^* at the shock wave should equal unity.

In the case of such a definition of A_1 the following formulas hold true

$$\lambda^* = 1 \text{ and } \lambda = \frac{r}{r_2}. \qquad (6.13)$$

* See equation (7.1) of Chapter IV.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

On the basis of formula (6.12) it follows that

$$c_2 = \frac{dr_2}{dt} = \frac{2-s}{k+3}\frac{r_2}{t} = \frac{2-s}{k+3}\lambda^{\frac{k+3}{2-s}}(A_1 f)^{\frac{1}{2-s}} r_2^{-\frac{k+s+1}{2-s}} \qquad (6.14)$$

The shock wave slows down if $(k+s+1)/(2-s) > 0$ or $(2k+6)/(2-s) = \omega > 2$, and it

---

* The effect of the slowing down or accelerating of the propagation of the shock wave through a medium of variable density depends, besides upon the law governing the falling off of the density, again upon the law governing the falling off of the pressure. In particular, if the density in the undisturbed state ($u = 0$) varies in accordance with the law $\rho_1 = K/r^\omega$, $[K] = ML^{\omega-3}$, and the second dimensional constant A possesses the dimensionality of energy $[A] = ML^2T^{-2}$, where the initial pressure $P_1 = 0$ (we do not take into account the force of Newtonian attraction), then the following formula holds true for the velocity of propagation

$$c = \alpha\sqrt{\frac{A_1}{K}}\,\frac{1}{r_2^{\frac{3-\omega}{2}}},$$

where α is a constant.

In the case under consideration ω can vary independently of the exponential indices of the dimensionality of the constant A_1, which was impossible in the case where forces of gravitation are taken into account. (The essential property of the constant f. In the case under consideration the dimensionality of the constant K in general is independent of the dimensionalities of f and A_1).

In this statement of the problem, for all $\omega < 3$ there takes place a slowing down of the shock wave. Only for the case of $\omega > 3$, which fact is connected with the presence of infinite mass close to the center of symmetry, the shock wave will be accelerated.

The presence of laws governing the decrease in the density itself is insufficient for obtaining any conclusions concerning the possibility of the accelerated motion of a shock wave.

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

is accelerated if $(k+s+1)/(2-s) < 0$ or $(2k+6)/(2-s) = \omega < 2$, where ω is the exponential index in formula (6.2) which determines the initial distribution of the density.

In particular, if we have $k = 2$ and $s = -2$ (A_1 is energy), then the shock wave is slowed down. In this case the following formula holds true for the velocity c

$$c = \frac{4}{5}\lambda^{*5/4}(fA_1)^{1/4}\frac{1}{r_2^{1/4}}. \qquad (6.15)$$

Under the conditions at the jump discontinuity the following quantity figures:

$$q = \frac{a_2^2}{c^2} = \frac{\gamma p_1}{\rho_1 c^2}.$$

On the basis of the formulas (6.2) and (6.14) for $\lambda^* = 1$ we have the following expressions:

$$q = -\frac{2\pi\gamma(k+3)^2}{(2k+3s)(4+s+2k)}a_1\left(\frac{A}{A_1}\right)^{\frac{2}{2-s}}. \qquad (6.16)$$

The quantity q was obtained as a constant; this follows also directly from considerations of the theory of dimensionality*.

The product $a_1 A^{\frac{2}{2-s}}$ is determined by the conditions of equilibrium, initial luminosity and absorption properties. The constant A_1 is determined by the condition $\lambda^* = 1$, which in the final account is connected with the law and properties of the quantity of energy released at the center of the star during an outburst.

---

* In the problem concerning a powerful explosion (see Chapter IV) we had $q = 0$; taking into account the counter pressure the quantity q varied and grew in the course of time. In the given problem we have $q = \text{constant} \neq 0$. From the constancy of the quantity q it follows that the temperature behind the front of the jump discontinuity T_2 is proportional to the temperature T_1 in front of the discontinuity front.

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

On the basis of the formulas (6.2), (6.9), (6.12), (6.14), (6.16) and of the conditions on the shock wave we obtain for the quantities V_2, R_2, M_2, P_2, and z_2 behind the front of the shock wave the following expressions:

$$\left.\begin{aligned} V_2 &= \frac{2}{\gamma+1}\,\frac{2-s}{k+3}\,(1-q),\\ R_2 &= -\frac{(\gamma+1)(2k+3s)(4+s+2k)}{2\pi\gamma(k+3)^2}\,\frac{q}{\gamma-1+2q},\\ M_2 &= \frac{2(2-s)(4+s+2k)}{\gamma(k+3)^2}\,q,\\ P_2 &= -\frac{(2-s)^2(2k+3s)(4+s+2k)}{2\pi\gamma^2(\gamma+1)(k+3)^4}\,q\,[2\gamma-(\gamma-1)q],\\ z_2 &= \left(\frac{2-s}{k+3}\right)^2\frac{[2\gamma-(\gamma-1)q][\gamma-1+2q]}{(\gamma+1)^2}. \end{aligned}\right\} \qquad (6.17)$$

These formulas for $k = 2$ and $s = -2$ reduce to the following form

$$\left.\begin{aligned} V_2 &= \frac{2}{\gamma+1}\cdot\frac{4}{5}\,(1-q),\\ R_2 &= \frac{6(\gamma+1)}{25\pi\gamma}\,\frac{q}{\gamma-1+2q},\\ M_2 &= \frac{48}{\gamma\cdot 25}\,q,\\ P_2 &= \frac{96}{625}\cdot\frac{[2\gamma-(\gamma-1)q]q}{(\gamma+1)\pi\gamma^2},\\ z_2 &= \frac{16}{25}\,\frac{[2\gamma-(\gamma-1)q][\gamma-1+2q]}{(\gamma+1)^2}. \end{aligned}\right\} \qquad (6.18)$$

In formulas (6.17) the parameter q is defined by the equation (6.16). This parameter possesses obvious mechanical significance and it can be taken in place of the parameter $\alpha_1(A/A_1)^{2/(2-s)}$.

If q is given, then the conditions (6.17) can be utilized as Cauchy data for the integration of the system of equations (6.11), after which the constant alpha α in the law governing the release of energy (6.4) is defined.

2. Finding of the Finite Integrals of the System of Equations (6.11)

Investigations of the one-dimensional nonsteady self-modeling motions of a gas with the help of considerations of the theory of dimensionality were carried out in

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

our work* published in 1945. In this same work we had introduced the sought for dimensionless functions V, R, P, z satisfying ordinary differential equations. Later on, with the help of methods developed and variables introduced by us, we set up and solved the problems on a powerful explosion, on the motion of a piston, and on certain other matters**.

Making use of these variables, the American astronomers Carrus, Fox, Gaas and Kopal*** treated the problem concerning the outbursts of novae. They investigated the case of "progressive waves" which reduces to self-modeling motion. These authors made use of the equations (6.11) and of the conditions on the shock wave (6.18) in the case corresponding to $k = 2$ and $s = -2$; here the solution was obtained directly by way of numerical integration.

The authors of this work did not note the existence of two simple definite algebraic integrals for the system of four ordinary differential equations which were considered by us. These integrals can easily be established**** with the help of indirect considerations of the theory of dimensionality just as this was done in Section 7 of Chapter IV, in the theory of powerful explosions.

Let us now turn our attention to obtaining the integrals of the system of equations (6.11).

Let us conceive of a mobile volume O bounded by two mobile spheres on which the

---

* L.I.Sedov, "Concerning Certain Nonsteady Motions of a Compressible Fluid", Prikladnaya Matematika Mekhanika (Applied Mathematics and Mechanics), Volume IX, No. 4, 1945.

** See Chapter IV.

*** P.Carrus, P.Fox, F.Gaas, Z.Kopal, Propagation of Shock Waves in a Stellar Model with Continuous Density Distribution, The Astrophysical Journal, Volume 113, 1951, p. 496-519.

**** The investigation of the problem for the case $k = 2$ and $s = -2$ and the establishing of these integrals was done by I.M.Yavorskaya.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

parameter λ assumes the constant values λ' and $\lambda'' > \lambda'$.

The mass of the gas between these spheres is equal to $m''-m'$ and is a variable quantity. It is obvious that the quantity $m''-m'$ depends upon λ'', λ', A_1, f, t and possibly upon certain other constant abstract parameters.

From considerations of the theory of dimensionality we obtain

$$m'' - m' = \int_{r'}^{r''} \rho 4\pi r^2 dr = \frac{C}{f} (A_1 f)^{\frac{3}{k+3}} \cdot t^{\frac{3s+2k}{k+3}},$$

where C is a constant that does not depend upon the time. From this formula it follows that

$$\frac{d}{dt} \int_{r'}^{r''} \rho 4\pi r^2 dr = -\frac{(m''-m')}{t} \left(\frac{3s+2k}{k+3} \right). \tag{6.19}$$

On the other hand, the following formula holds true for any function $F(r,t)$ and for any mobile volume O between two spheres of radii $r''(t)$ and $r'(t)$:

$$\frac{d}{dt} \int_{r'}^{r''} F(r,t)\, dr = \frac{\tilde{d}}{dt} \int_{r'}^{r''} F(r,t)\, dr + F(r'',t)\left(\frac{dr''}{dt} - u'' \right) - F(r',t)\left(\frac{dr'}{dt} - u' \right), \tag{6.20}$$

where the derivative $\frac{\tilde{d}}{dt}$ designates the variation of the integral for a mobile volume which is moving together with the particles of the gas and which coincides at a given moment of time with the mobile volume O under consideration.

Since $\frac{\tilde{d}}{dt} \int_{r_1}^{r''} \rho 4\pi r^2 dr = 0$, then on the basis of the formulas (6.19) and (6.20) we obtain

$$-\frac{m''-m'}{t}\left(\frac{3s+2k}{k+3} \right) = \rho'' 4\pi r''^2 \left[\frac{2-s}{k+3} \frac{r''}{t} - u'' \right] - \rho' 4\pi r'^2 \left[\frac{2-s}{k+3} \frac{r'}{t} - u' \right].$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Hence on the basis of (6.9) and (6.10) we find the first of the sought for integrals in the following form

$$\lambda^{s}\left[\left(\frac{3s+2k}{k+3}\right)M+4\pi R\left(\frac{2-s}{k+3}-V\right)\right]=$$
$$=\lambda'^{s}\left[\left(\frac{3s+2k}{k+3}\right)M'+4\pi R'\left(\frac{2-s}{k+3}-V'\right)\right]. \quad (6.21)$$

If the continuous motion under consideration is bounded by a shock wave that is being propagated through a gas at rest, then in virtue of the conditions on the shock wave (6.17) we obtain that the constant on the right in formula (6.21) is equal to zero, and therefore in this case the integral (6.21) must be taken in the form

$$M=4\pi R\frac{k+3}{3s+2k}\left(V-\frac{2-s}{k+3}\right). \quad (6.22)$$

Relying on the law governing the conservation of energy, we find in a similar way also the second integral. For the total energy of the gas within the above defined mobile volume Ω we can write down the following expression:

$$E=\int_{r}^{r'}\left[\frac{\rho u^2}{2}+\rho\varepsilon-\frac{\rho f(m-m')}{r}\right]4\pi r^2\,dr.$$

In the expression behind the integral sign the first term defines the kinetic energy, the second term defines the internal energy, and the third term determines that part of the internal energy which is due to the internal forces of gravitational attraction.

In accordance with the law governing the conservation energy we have:

$$\frac{dE}{dt}=\frac{dA^{(e)}_{sur}}{dt}+\frac{dA^{(e)}_{mass}}{dt}+\frac{dQ^{(e)}}{dt},$$

where $\frac{dA^{(e)}_{sur}}{dt}$ is the work of the external surface forces, which is defined by the equality

$$\frac{dA^{(e)}_{sur}}{dt}=-p''u''4\pi r''^2+p'u'4\pi r'^2;$$

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$\frac{dA^{(e)}_{mass}}{dt}$ is the work of the external mass forces of gravitation, for which the following equality holds true

$$\frac{dA^{(e)}_{mass}}{dt} = fm' \frac{d}{dt} \int_{r'}^{r} \frac{\rho 4\pi r^2}{r} dr;$$

and $\frac{dQ^{(e)}}{dt}$ is the external flow of heat, which is equal to zero since the process is adiabatic in accordance with hypothesis.

Hence it follows that for the particles of the gas which are included at a given moment of time within the volume Q under consideration the law governing the conservation of energy can be written in the following form

$$\frac{d}{dt} \int_{r'}^{r''} \left[\frac{\rho u^2}{2} + \rho\varepsilon - \frac{\rho fm}{r} \right] 4\pi r^2 dr = 4\pi r'^2 p' u' - 4\pi r''^2 p'' u''. \qquad (6.23)$$

It is obvious that for self-modeling motion in the case of λ' = const. and λ'' = const. the formula of the following form holds true

$$E^* = \int_{r'}^{r''} \left[\frac{\rho u^2}{2} + \rho\varepsilon - \frac{\rho fm}{r} \right] 4\pi r^2 dr = \alpha^* f^{\frac{2-k}{3+k}} A_1^{\frac{5}{3+k}} t^{-\frac{5s+4k+2}{k+3}}, \qquad (6.24)$$

where α^* is a constant that depends upon the λ' and λ''.

On the basis of (6.20), taking into account (6.23) and (6.24), we obtain the relation

$$\frac{5s+4k+2}{k+3} \frac{E^*}{t} = \left\{ 4\pi r^2 \left[pu + \left(\frac{\rho u^2}{2} + \rho\varepsilon - \frac{\rho fm}{r} \right) \left(u - \frac{2-s}{k+s} \cdot \frac{r}{t} \right) \right] \right\}^{\cdot}.$$

This relation in dimensionless variables, if we take into consideration the

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

equality $\rho\varepsilon = p/(\gamma-1)$, takes the following form

$$\left\{4\pi\lambda^5\left[pV-\left(\frac{RV^2}{2}+\frac{p}{\gamma-1}-RM\right)\left(\frac{2-s}{k+3}-V\right)\right]\right\}'_\lambda = \alpha^*(\lambda'',\lambda')\cdot\frac{5s+4k+2}{k+3}. \quad (6.25)$$

The relation (6.25) taking into account the relation $P = zR/\gamma$ gives the integral of the system of equations (6.11), if the right part of (6.25) is converted into zero, that is, if we have

$$\frac{5s+4k+2}{k+3}=0. \quad (6.26)$$

In particular, this equality is fulfilled when $[A_1] = ML^2T^{-2}$ ($k = 2$, $s = -2$).

If (6.26) is satisfied, then the second integral will be obtained in the form*

$$\lambda^5\left[\frac{zRV}{\gamma}+\left(\frac{RV^2}{2}+\frac{zR}{\gamma(\gamma-1)}-RM\right)\left(V-\frac{2-s}{k+3}\right)\right]=\text{const}. \quad (6.27)$$

If the continuous motion under consideration is bounded by a shock wave which is being propagated through a gas that is at rest, then in virtue of the conditions on the shock wave (6.17) we can represent by imposing $\lambda^* = 1$ on the shock wave the integral (6.27) in the following form

$$\frac{zRV}{\gamma}+\left(V-\frac{2-s}{k+3}\right)\left(\frac{RV^2}{2}+\frac{zR}{\gamma(\gamma-1)}-RM\right)=k\lambda^{-5}, \quad (6.28)$$

where the constant k is determined by the equality

$$k=\frac{384}{3125\pi}\,\frac{3\gamma-4}{\gamma^3(\gamma-1)}\,q^2.$$

If we have $\gamma = 4/3$, then $k = 0$, which introduces an additional simplification.

By means of the found integrals the system of ordinary equations of the fourth

---

* For the evaluation of the effectiveness of the applied method of obtaining the integrals (6.21) and (6.27) of the system of equations (6.11) we can recommend to the reader his carrying out the verification of the correctness of these integrals by computations. This problem is nontrivial and was solved by I.M.Yavorskaya.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

order can be reduced to a system of equations of the second order.

3. The Solution in the Case $\gamma = 4/3$

In the case where gamma is $\gamma = 4/3$ the equations of continuity and adiabaticity

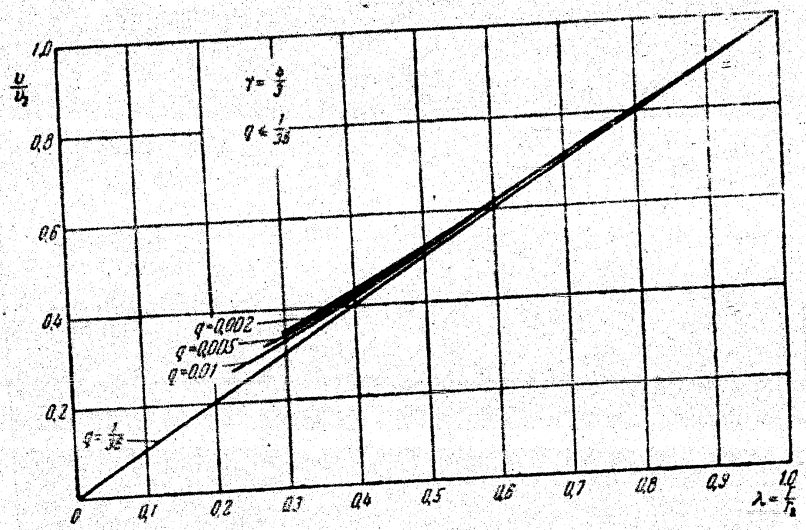


Fig.89. The Distribution of Velocities in the Field of the Disturbed Motion of a Gas. Around a Center an Empty Sphere is Formed with Increasing Radius $r^* = \lambda^*(q)r_2$.

give still another simple integral. In fact we have the following

$$\lambda\left[V' - \left(\frac{2-s}{k+3} - V\right)\frac{R'}{R}\right] = 2 - 3V,$$

$$\lambda\left(V - \frac{2-s}{k+3}\right)\left[\frac{P'}{P} - \frac{4}{3}\frac{R'}{R}\right] = -\frac{2}{3}(3V-2).$$

Hence after exclusion of the term 3V-2 and after integration we find:

$$\frac{P}{\left(\frac{2-s}{k+3} - V\right)^{2/3} R^s} = C =$$

$$= -\frac{2\pi(2-s)^{4/3}(k+3)^{2/3}[\gamma-1+2s]^{4/3}[2\gamma-(\gamma-1)s]}{(\gamma+1)^{7/3}(2k+3s)(4+s+2k)q}; \qquad (6.29)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

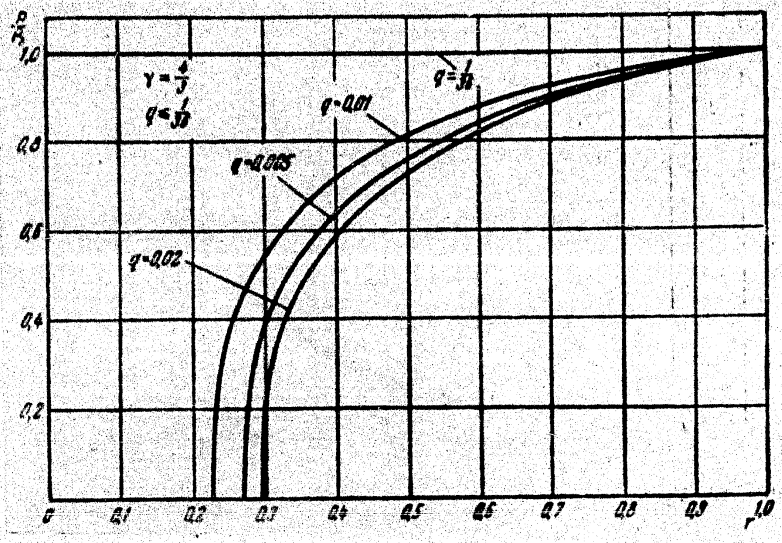


Fig.90. The Distribution of the Density in the Field of the Disturbed Motion of a Gas. On the Internal Boundary the Density is Equal to Zero

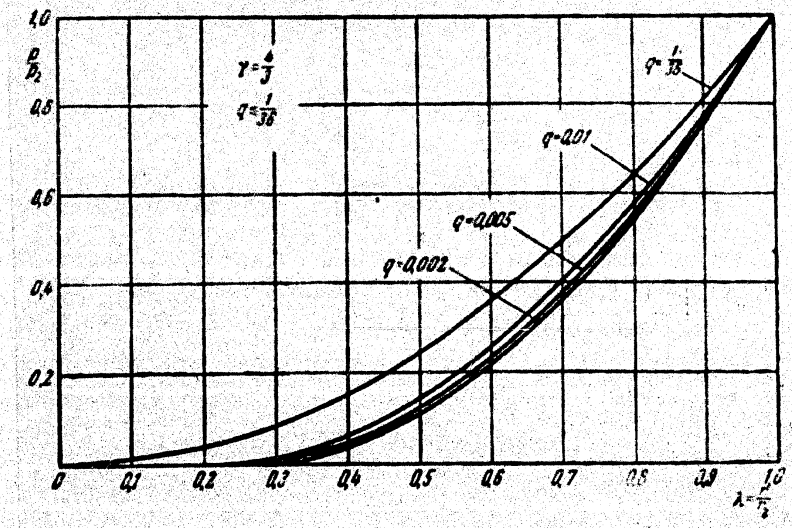


Fig.91. The Distribution of the Pressure in the Field of the Disturbed Motion of a Gas. On the Internal Boundary the Pressure is Equal to Zero

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

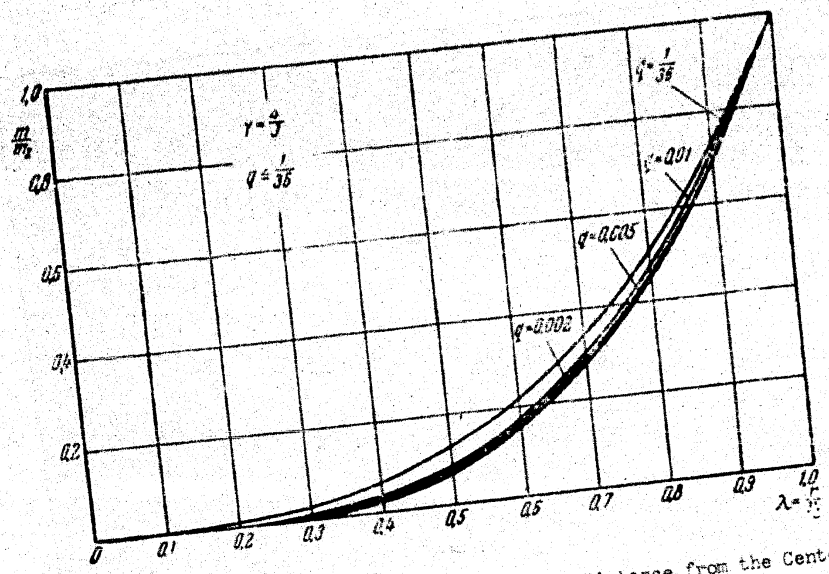


Fig.92. The Mass of the Gas as a Function of the Distance from the Center of Symmetry. For r = r* we have: m = 0

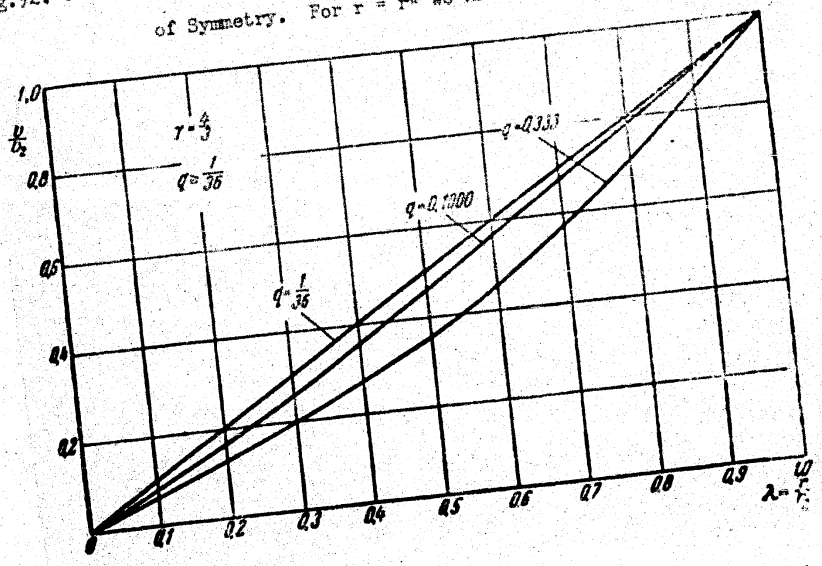


Fig.93. The Distribution of the Velocities in the Field of the Disturbed Motion of a Gas. The Motion is Prolonged to the Center of Symmetry

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

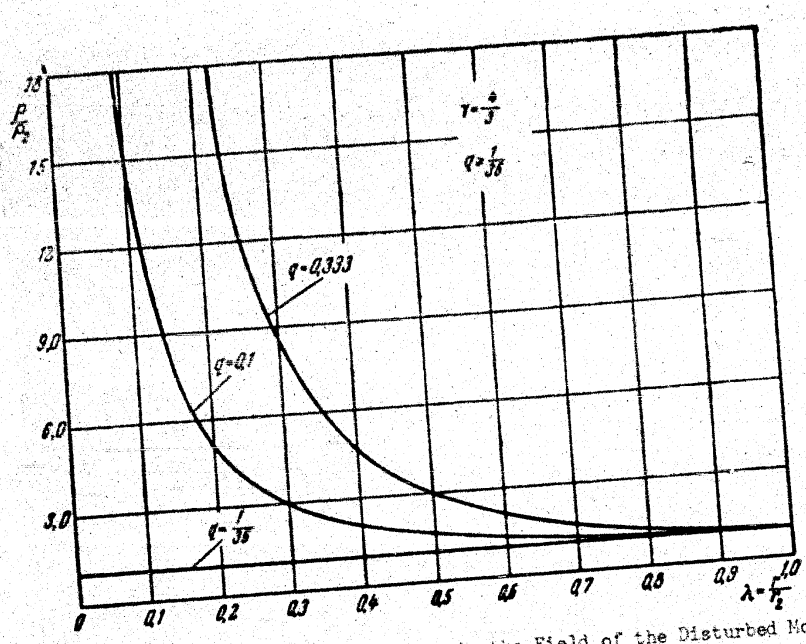


Fig.94. The Distribution of the Density in the Field of the Disturbed Motion of a Gas. At the center of symmetry the density is infinite for q > 1/36.

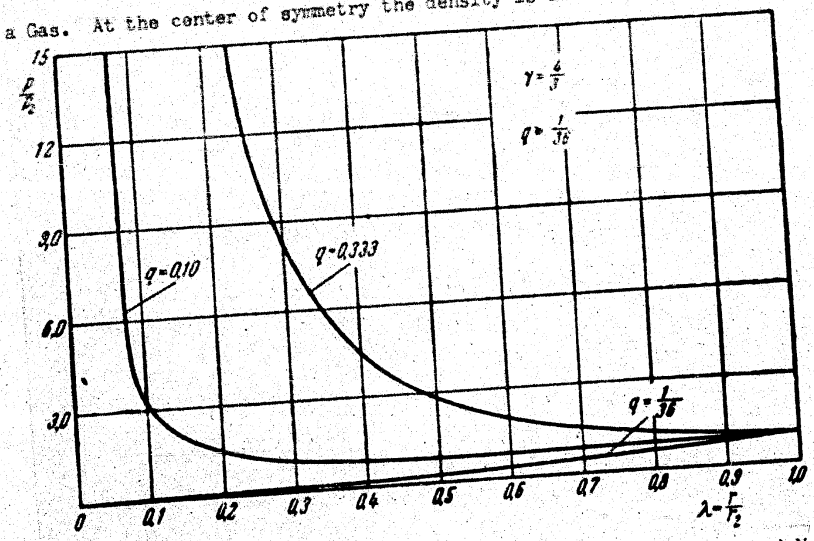


Fig.95. The Distribution of the Pressure in the Field of the Disturbed Motion of a Gas. At the center of symmetry the pressure is infinite for q > 1/36.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

the constant C is determined by the right side of the equality from the conditions on the shock wave (6.17).

Everywhere in what follows we shall set k = 2 and s = -2. Employing the integrals (6.22), (6.28) and (6.29), we easily express M, P and R by means of the simple formulas in V.

Substitution of these expressions into equation (6.10) leads to one equation for λ, which is determined by means of quadrature computed in finite definite form.

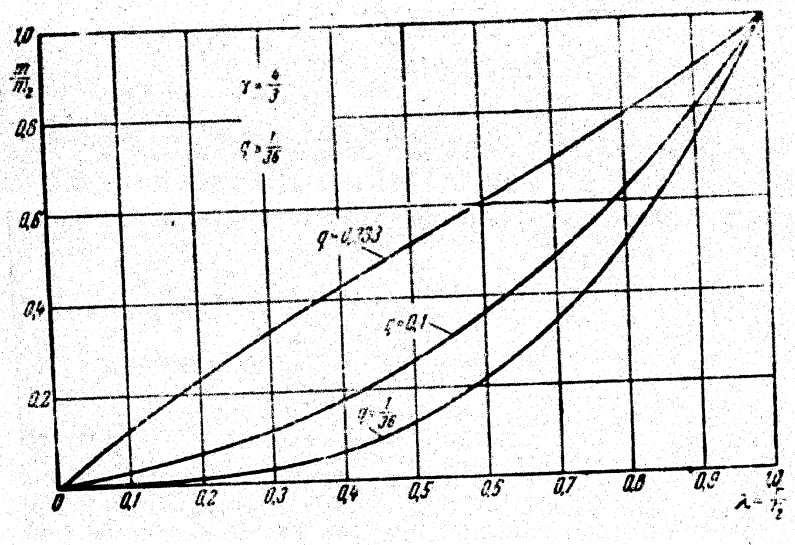


Fig.96 - The Mass of the Gas as a Function of the Distance to the Center of Symmetry

Analysis of this solution indicates that for q < 1/36 at the center of explosion a hollow cavity occurs which expands in accordance with the following law

$$r^* = \lambda^* r_2,$$

where the constant λ* is less than unity and depends only upon q. At the boundary of the expanding sphere the pressure and density are converted into zero.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

If we have q much greater than 1/36 ($q \geq 1/36$), then the disturbed motion of the gas after explosion occupies the entire internal spherical shock wave. For the case $q > 1/36$ the density and the pressure at the center are converted into infinity. Typical distributions of the characteristics of the pressure for various q are represented in Figs. 89, 90, 91, 92, 93, 94, 95, and 96.

It is easy to discern that for $\gamma = 4/3$ the system of ordinary differential equation (6.10) possesses the solution

$$\left.\begin{aligned} V = \frac{2}{3}, \quad R = R_0 = \text{const}, \quad M = M_0 = \frac{4\pi}{3} R_0, \\ P = P_0 = \frac{R_0}{9} - \frac{2\pi}{3} R_0^2. \end{aligned}\right\} \tag{6.30}$$

In agreement with the formulas (6.9) the corresponding exact solution of the equations in partial derivatives (6.1) possesses the form

$$\left.\begin{aligned} v = \frac{2}{3}\frac{r}{t}, \quad \rho = \frac{R_0}{t^2}, \quad m = \frac{r^3}{t^2}\cdot\frac{4\pi}{3} R_0, \\ p = \frac{r^2}{t^4}\left[\frac{R_0}{9} - \frac{2\pi}{3} R_0^2\right], \end{aligned}\right\} \tag{6.31}$$

where R_0 is an arbitrary positive constant. In the corresponding motion of the gas the density depends only upon the time and does not depend upon the radius.

For $\gamma = 4/3$ the formulas (6.18) give:

$$\left.\begin{aligned} V_2 = \frac{24}{35}(1-q), \quad R_2 = \frac{21}{50\pi}\cdot\frac{q}{\frac{1}{3}+2q}, \\ M_2 = \frac{36}{25} q, \quad P_2 = \frac{54(8-q)q}{7\cdot 625\pi}. \end{aligned}\right\} \tag{6.32}$$

From (6.32) and from the equality $V_2 = 2/3$ it follows that $q = 1/36$; if now we set $q = 1/36$ and equate the arbitrary constant R_0 to the value R_2, which is equal to $3/\pi\cdot 100$ behind the jump discontinuity, then we find that $P_2 = P_0$ and $M_2 = M_0$. Thus for $\gamma = 4/3$ and $q = 1/36$ the formulas (6.31) in the simple form give the exact solution of the problem on outburst under consideration.

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

In this case the solution (6.31) can be written in the following form

$$\frac{v}{v_2}=\lambda,\quad \frac{\rho}{\rho_2}=1,\quad \frac{p}{p_2}=\lambda^2,\quad \frac{m}{m_2}=\lambda^3,\quad \lambda=\frac{r}{r_2}. \tag{6.33}$$

The solution obtained separates the solutions for the motions with hollow cavity formed in the case of $q < 1/36$ and for the motions with continuation to the center of symmetry in the case of $q > 1/36$. In the graphs of Figs.89-94 are also drawn the curves for the solution (6.33).

The solution obtained in Section 10 of Chapter IV and represented by the formulas (10.26) can for the case $\gamma = 4/3$ be generalized to the case of the equations of motion (6.1), in which the forces of Newtonian gravity are taken into consideration.

It is not difficult to verify that for $\gamma = 4/3$ the system (6.1) admits the following exact solution:*

$$\left.\begin{aligned} &u=\frac{Z'(t)}{Z(t)}\cdot r,\quad p=\frac{\Phi(\xi)}{Z^4(t)},\\ &\rho=\frac{R(\xi)}{Z^3(t)},\qquad \xi=\frac{r}{Z(t)},\\ &\text{where}\qquad Z''(t)\cdot Z^2=\chi\\ &\text{and}\qquad \frac{d\Phi}{d\xi}\xi^2+4\pi\int R(\xi)\int_0^{\xi}\eta^2R(\eta)\,d\eta=-\chi\xi^3R(\xi), \end{aligned}\right\} \tag{6.34}$$

where χ is an arbitrary constant and $R(\xi)$ is an arbitrary function. The solution (6.31) is a particular case of the solution (6.34) for the case

$$R(\xi)=\frac{R_0}{f},\quad Z(t)=t^{2/3}\ \text{and}\ \chi=-\frac{2}{9}. \tag{6.35}$$

In the general case the solution (6.34) describes nonself-modeling motions similar to the motions studied in detail in Section 10 of Chapter IV.

* This solution was obtained by M.L.Lidov, for which see Doklady Akademii Nauk SSSR, Volume XCVII, No. 3, 1954, pp 409-411.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

4. Investigation of the Solution in the General Case K = 2 and s = -2.

In the general case where $\gamma \neq 4/3$ the motion of the gas behind the front of the shock wave which is being propagated through a gas that is at rest with characteristics distributed in accordance with the law (6.2) can be constructed with the help of the integrals found and with the help of the equations of motion (6.10).

The results of computations for the case $\gamma = 5/3$ for various values of q are represented in Figs. 97, 98, 99 and 100.

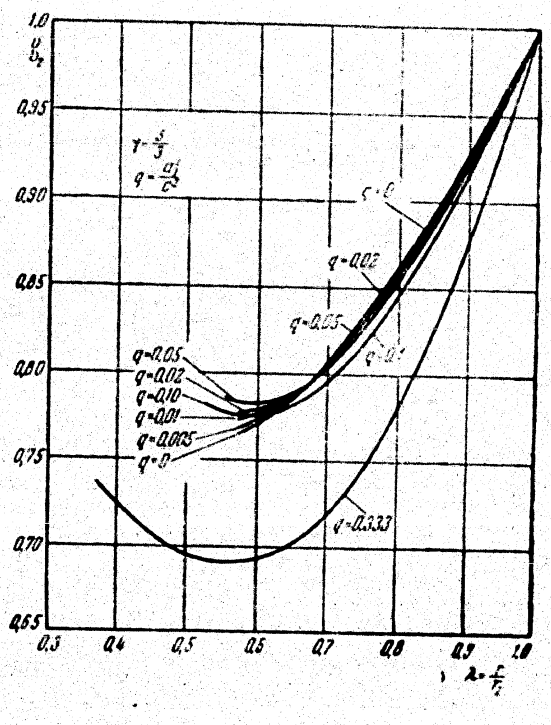


Fig.97. The Distribution of Velocities for the Case $\gamma = 5/3$

The left edges of the values of $\lambda = r/r_2$ for the curves of distribution correspond to m = 0, and consequently these points can be considered as the internal boundaries that arise at the center of symmetry at the initial moment of time. The

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

disturbed motion of the gas can be considered as the result of the displacement of the gas by a special spherical piston - by a sphere whose radius r* increases from zero in accordance with the law

$$r^* = \lambda^* \cdot r_2, \text{ WHERE } \lambda^*(\gamma, q) = \text{const}. \qquad (6.36)$$

(the function $\lambda^*(\gamma,q)$ for $\gamma = 4/3$ and 5/3 is represented in Fig.101).

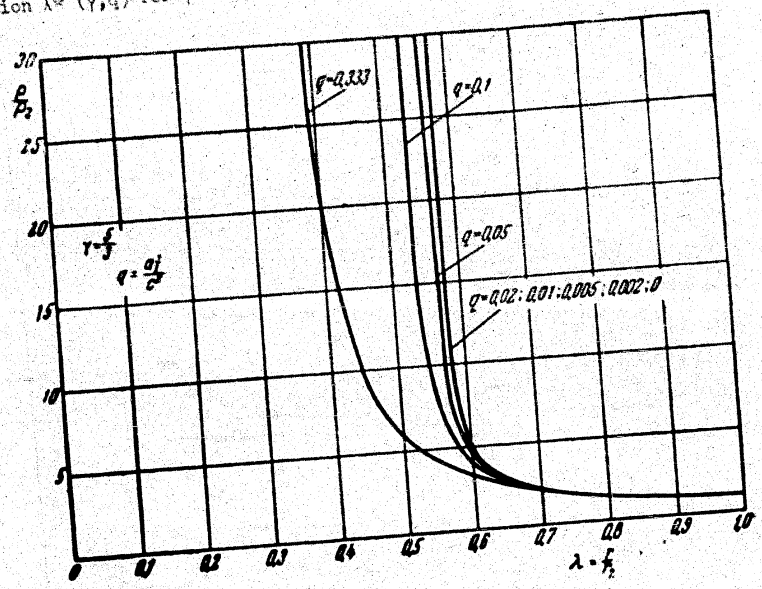


Fig.98 - The Distribution of Density for the Case $\gamma = 5/3$

In the case where $q \to 0$ the solution under consideration tends to the solution found in Section 9 of Chapter IV in the absence of the forces of Newtonian gravitation.

For the case $q = 0$ we have the fact that $\lambda^* \neq 0$ and depends only upon γ; the corresponding limiting values $r^*/r_2 = \lambda^*(0,\gamma)$ for the cases $\gamma = 5/3$ and 4/3 when $\omega = 2.5$ are also given in Fig.72.

Let us find the asymptotic behavior of the solution close to the internal boun-

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

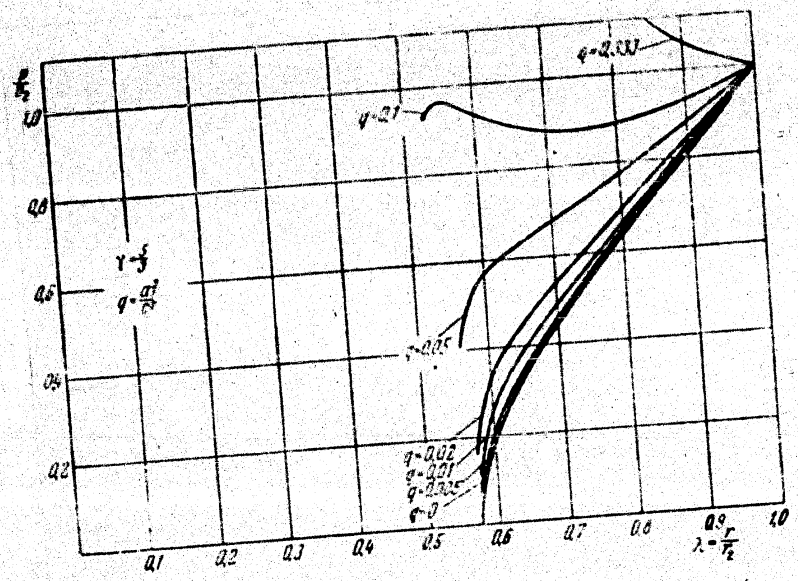


Fig.99. The Distribution of the Pressure for the Case γ = 5/3

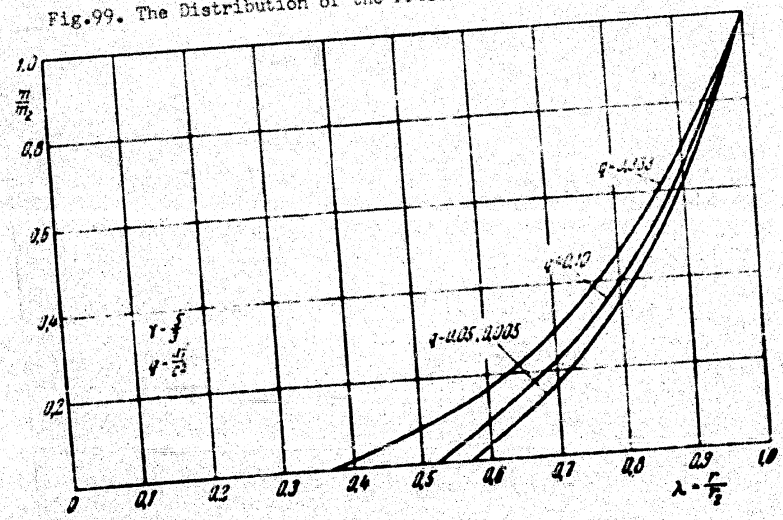


Fig.100. The Mass of the Gas as a Function of the Distance from the Center of Symmetry

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

dary. On this boundary we have that $m \to 0$, and since $r = r^*$ is a finite quantity it is evident from eq.(6.9) that, as $r \to r^*$, we have $N \to 0$. Therefore the equations (6.10) close to $r \approx r^*$ tend to the equations (9.6) of Chapter IV. Physically this is connected with the fact that the forces of Newtonian attraction close to the internal boundary are small and in the limit are equal to zero.

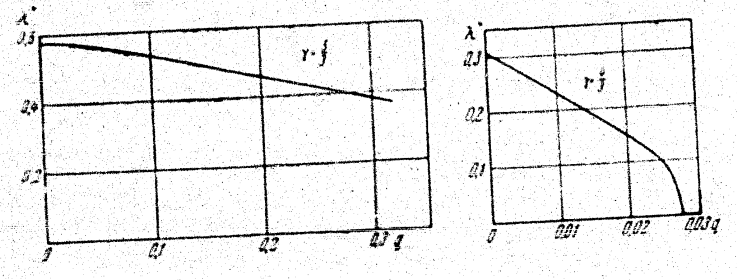


Fig.10i - Radius of the Internal Boundary as a Function of q at $\gamma = \frac{5}{3}$ and $\gamma = \frac{4}{3}$

In order to obtain the asymptotic formulas it is sufficient to study the solutions of the ordinary equations (9.7), (9.8) and (9.9) of Chapter IV close to the singular point $V = \delta = \frac{2}{s-3}$, $z = 0$. In the considered case, $\psi = 2.5$ and therefore $\delta = 4/5$.

For the case $q = 0$ the asymptotic formulas are given in Section 9 of Chapter IV [see eqs.(9.23)]. The equation (9.7) of Chapter IV, in which $\delta = 4/5$, possesses the following form

$$\frac{dz}{dV} = \frac{z\left\{\left[12(V-1)+3(\gamma-1)V\right]\left(V-\frac{4}{5}\right)^2-(\gamma-1)V(V-1)\left(V-\frac{4}{5}\right)-\left[2(V-1)+\frac{12}{5\gamma}(\gamma-1)\right]z\right\}}{\left(V-\frac{4}{5}\right)\left[V(V-1)\left(V-\frac{4}{5}\right)+\left(\frac{12}{5\gamma}-3V\right)z\right]}. \quad (6.37)$$

There exists only one unique integral curve which arrives at the singular point $V = 4/5$ and $z = 0$ with the asymptotic law

$$z \approx -\frac{2}{5}\gamma\left(V-\frac{4}{5}\right). \quad (6.38)$$

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

This solution corresponds to the case q = 0.

The equations of the other integral curves of equation (6.37) which arrive at the singular point V = 4/5, z = 0 close to this point possess the following form

$$z \approx A\left(\frac{4}{5} - V\right)^{\frac{5\gamma-6}{6(\gamma-1)}}, \tag{6.39}$$

where A is an arbitrary constant. It is evident that $\frac{5\gamma-6}{6(\gamma-1)} < 1$. Formula (6.39) is easily verified by substitution into equation (6.37).

From the equation (6.11) we easily obtain, on the basis of (6.39), the following:

$$\frac{\lambda}{\lambda^*} \approx e^{-\frac{5\gamma}{12(\gamma-1)}\left(V-\frac{4}{5}\right)} \quad \text{or} \quad \frac{4}{5} - V \approx \frac{12(\gamma-1)}{5\gamma}\,\frac{r-r^*}{r^*},$$

$$R \approx \frac{C}{\left(\frac{4}{5}-V\right)^{\frac{5\gamma-6}{6(\gamma-1)}}}, \quad P = \frac{zR}{\gamma} = \frac{AC}{\gamma}, \tag{6.40}$$

where C is a constant of integration*.

Hence when formulas (6.9) and (6.18) are taken into account it follows that when r → r* the following asymptotic formulas hold true

$$\left.\begin{aligned} \frac{v}{v^2} &\approx \frac{\gamma+1}{2(1-q)}\,\frac{r^*}{r_2}, \\ \frac{\rho}{\rho_2} &\approx \frac{25\pi\gamma(\gamma-1+2q)C}{6(\gamma+1)q\left[\frac{12(\gamma-1)}{5\gamma}\,\frac{r-r^*}{r^*}\right]^{\frac{5\gamma-6}{6(\gamma-1)}}}, \\ \frac{p}{p_2} &\approx \frac{625\pi\gamma(\gamma+1)AC}{96q[2\gamma-(\gamma-1)q]}\left(\frac{r^*}{r_2}\right)^2. \end{aligned}\right\} \tag{6.41}$$

* From the established integrals of the system of ordinary equations it is easy to deduce the following relations between the constants A, C, λ*, k and q:

STAT

14

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

The asymptotic behavior of the solution in the case q = 0 was studied in Section 9 of Chapter IV; there it was shown that for $\omega < 3$ the pressure on the internal boundary is equal to zero. In the case under consideration we have $\omega = 2.5$.

For the density in agreement with formula (9.23) of Chapter IV the following asymptotic formula holds true:

$$\frac{\rho}{\rho_2} \approx A\left(\frac{r-r^*}{r_2}\right)^{-\frac{2.5\gamma-3.5}{3\gamma-3.5}}. \tag{6.42}$$

Hence it follows that for $r \to r^*$ and $\gamma = 4/3$ the ratio ρ/ρ_2 is equal to zero; of course for $\gamma = 1.4$ and for $\gamma = 5/3$ the quantity ρ/ρ_2 is infinite, in which case the formula (6.42) assumes the form

$$\frac{\rho}{\rho_2} \approx A\left(\frac{r_2}{r-r^*}\right)^{4/9}.$$

We note that in all the cases for $r \to r^*$ the mass m tends to zero.

5. The Energy of the Initial State and of the Disturbed Motion of a Gas.

In agreement with paragraph 2 we can write for the total energy E between the spheres of radius r' and radius r'' the following:

$$E = \int_{r'}^{r''} \left[\frac{\rho u^2}{2} + \rho\varepsilon - \frac{f\rho\,(m-m')}{r}\right] 4\pi r^2\,dr.$$

For the initial state of equilibrium of a perfect gas with characteristics distributed in accordance with the law (6.2) the energy E is easily computed.

Since $\rho\varepsilon = \rho c_v T$, then from formula (6.2) we can find:

$$E = 16\pi^2 \frac{s-2}{2k+3s}\,\alpha_1^2 A^{\frac{4}{2-s}} f^{\frac{2+s}{2-s}} \left\{\frac{10+s+4k-\gamma(8+2s+4k)}{(\gamma-1)(8+2s+4k)} \times \right.$$
$$\left. \times \int_{r'}^{r''} r^{-\frac{4(k+s+1)}{2-s}}\,dr + r'^{\frac{2k+3s}{s-2}} \int_{r'}^{r''} r^{\frac{2k+6}{2-s}}\,dr\right\}. \tag{6.43}$$

For k = 2 and s = -2 this expression assumes the form

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

$$E = 32\pi^2 a_1^2 A \left\{ \frac{4-3\gamma}{3(\gamma-1)} \ln \frac{r''}{r'} + 2 - 2\sqrt{\frac{r'}{r''}} \right\}. \qquad (6.44)$$

From the formula (6.44) it follows that for fixed r'' as r' approaches zero ($r' \to 0$) we have E approaches negative infinity ($E \to -\infty$) in the case $\gamma > 4/3$ and we have E = constant for the case $\gamma = 4/3$.

On the basis of the obtained solutions it follows that the total energy of the disturbed field of motion of the gas within the shock wave is finite.

Thus it is possible to deduce that in the assumed idealized statement of the problem the obtained pressures of gas in the case $\gamma > 4/3$ are caused by the release of infinitely great energy at the center of symmetry at the initial moment of time. This energy is expended in compensation for the initial infinite negative energy and in creation of the disturbed motion of the gas. From the considerations of dimensionality it is evident that the energy of the disturbed motion is constant. The additional negative energy which is possessed by the masses of gas which are captured by the shock wave is compensated by the influx of energy due to the work of the forces of pressure which are different from zero in the case $q > 0$ on the internal boundary sphere.

If we have $\gamma = 4/3$, then the energy of the disturbed motion is finite and constant, but the pressure on the internal boundary does not perform any work.

It is interesting to note that in all the cases considered for the motion of the gas the complete dispersion of the entire mass of the gas to infinity occurs for $t \to +\infty$. It is evident that we cannot apply to gas motions of such a kind the well known criteria governing the magnitude of the initial velocity of the particles which is necessary for the overcoming of the force of Newtonian attraction and for their rupture from the star. It is evident that because of the pressure gradients the dispersion and rupture of the masses making up the composition of the star are possible, and also in those cases where the entire initial velocities are equal to zero.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

If the initial distribution of the type (6.2) holds true only at finite distances r_0 from the center of symmetry, then the studied motions can be considered only up to those times when the radius of the shock wave r_2 has not become equal to r_0. If r_2 becomes larger than r_0, then q can turn out to be a variable.

The infiniteness of the values of the energy of the outburst, the infiniteness of the pressure at the center, the instantaneousness of the release of energy etc., which are all consequences of the assumed idealization, indicate that at the center of the star the studied solution must be corrected.

However, it is possible to assume that at certain distances far from the center of the star and for certain limited integrals of time the found laws governing the motion of the gas can be utilized for clarifying and describing the actual phenomena of the outbursts of novae and supernovae.

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Biot-Savart law

Brightness of a star

Boltzmann's constant

Weber's law

Quantity, dimensionless (abstract)

Magnitude, stellar

- - , absolute visual

-- , bolometric

- , dimensional (nominal)

Quantities, principal (primary)

- , derivative (secondary)

Explosions, powerful

Screw in a fluid

Tide

Wave of Riemann, traveling

- , powerful shock

Waves on the surface of an incompressible fluid

Flare-up of a star

Hagen-Poiseuille flow

Gliding

Combustion

Boundary of ricocheting

- of stability of gliding

Hooke's law

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

ORIGINAL

Motion of a screw in a fluid

- of a gas, self-modeling

- - , one-dimensional

- - , periodic pulsating

- of a fluid in pipes

- - - -, laminar

- - - - , turbulent

- - , viscous incompressible

- - , one-dimensional

- - , turbulent

- - - , homogeneous

- - - - , isotropic

- - - , steady-state

- of a ship

Motion in an incompressible fluid

- - - - , compressible

Motions, similar

Detonation

Defect of velocity

Diffusion of vortices in a viscous fluid

Law of Biot-Savart

- of Weber

- of universal gravitation

- of Hooke

- of Coulomb

- of Newton, second

- of similarity of Froude

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

FOR OFFICIAL

- of Poiseuille
- of conservation of mechanical energy
- of friction of Coulomb
Laws, asymptotic, of damping of shock waves
Laws of Kepler
Novae and supernovae
Stars, variable

Sources of energy of stars, distribution

Kepler's laws
Oscillations of an elastic wing in a progressive flow
Clapeyron's equation
Classification of stars according to brightness
Coefficient of acuteness
- of completeness, prismatic
- , propulsive
- of heat capacity
Criterion of similarity
Coulomb's law
— of friction

Mass of a star
Machine, centrifugal
Pendulum, mathematical
Modeling (simulating)
Model, hydrostatic, of an aerostat
Model of a star with infinite radius and mass
- - - with point source of energy

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

FOR ORIGINAL

Unstable gliding, drawn out

Newton's second law

Incidence upon water, vertical

Parameters defining the class of phenomena

Overloading

Period of pulsations of cepheids

Pi-theorem

Submersion into a fluid of a cone and wedge

Similarity, physical

Completeness of a system of defining parameters

Piston

Constant of Boltzmann

Constant, gas

Constant, gravitational

Advance of a screw, relative

Curretn (see Motion)

Prandtl-Schlichtinger formula

Counter-pressure during point explosion

Poiseuille's law

Pulsation of a star, radial

Path of intermixing

Radius of a star

Scattering of a gas

- - for one-dimensional motion

Dimensionality

Disruption of an arbitrary discontinuity in a burning mixture

Regime of motion

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

ORIGINAL

Reynolds number

Ricochet, aqueous

Luminosity of a star

Force

- of inertia

System of units, technical

- - , physical

System of units, universal unique

Velocity of tangential stress of friction

- , radial

- of flutter, critical

Layer, boundary, in the case of circulation around a flat plate

Resistance during motion of ship (tugboat) without screws

- - - - , residual

- - - - in the presence of screws

- - - - of friction

Strukhal' number

Schematization of phenomenon

Heat emission of a body in fluid flow

Flow of Hagen-Poiseuille

Pipe, aerodynamic

Turbulence

-, isotropic

-, homogeneous

Shock against water

Equations of Clapeyron

Equations of nonsteady motions of stars

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2

ORIGINAL

- of equilibrium of stars

Stability of gliding

Flutter of a wing

Focussing of gas at point

- - for one-dimensional motion

Formula of Prandtl-Schlichting

- of dimensionality

Froude's number

Characteristics of phenomena

Cepheids

Number of Reynolds

- of Strukhal'

- of Froude

Equivalent of heat, mechanical

Energy of stars

- , kinetic

- , potential

- of powerful explosion

Brightness of a star

STAT

Declassified in Part - Sanitized Copy Approved for Release 2012/09/25 : CIA-RDP81-01043R001200180003-2